Unified Engineering Propulsion Quiz  Number: ___SOLUTIONS_ ___
Spring 2005

# Unified Propulsion Quiz
## March 18, 2005

**Closed Book – no notes other than the equation sheet provided with the exam**
**Calculators allowed.**

- **Put your name on each page of the exam.**
- **Read all questions carefully.**
- **Do all work for each problem on the pages provided.**
- **Show intermediate results.**
- **Explain your work --- don't just write equations.**
- **Partial credit will be given (unless otherwise noted), but only when the intermediate results and explanations are clear.**
- **Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.**
- **Show appropriate units with your final answers.**
- **Box your final answers.**

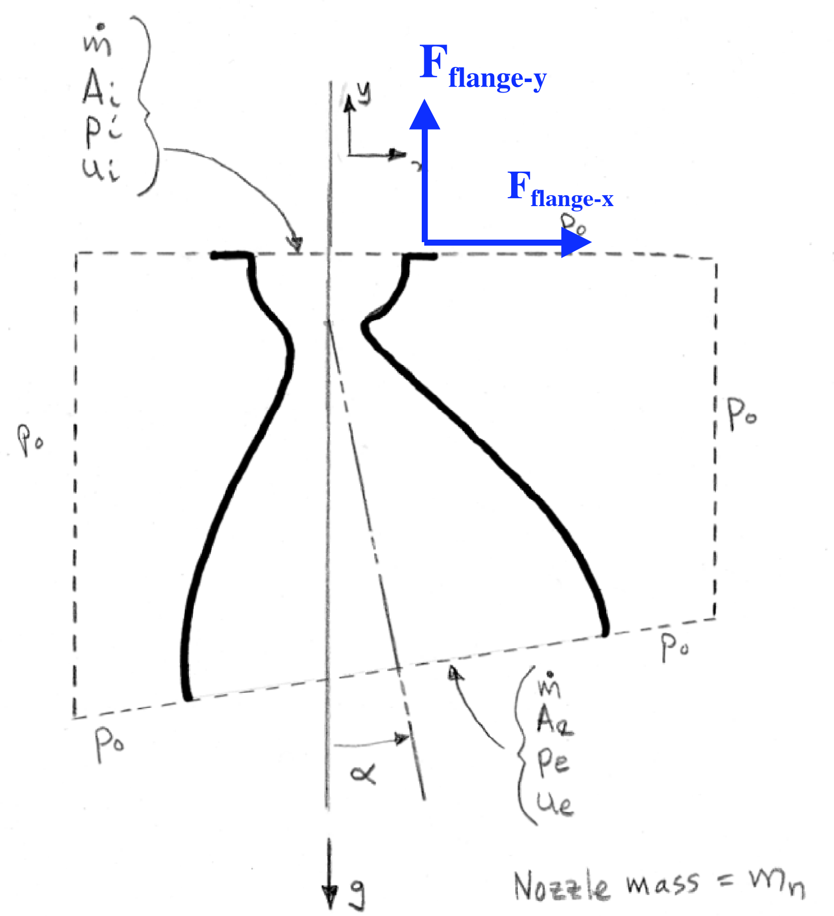
## Exam Scoring

| | |
|---|---|
| **#1 (25%)** | |
| **#2 (25%)** | |
| **#3 (15%)** | |
| **#4 (35%)** | |
| **Total** | |

Unified Engineering Propulsion Quiz Number: _SOLUTIONS___

Spring 2005

**1.** **(25 points, partial credit given, L.O.'s A & B)** Below is a schematic of a gimbaled rocket nozzle. Write equations for the x- and y-components of force on the nozzle flange in terms of the parameters given.

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Starting with the integral form of the momentum equation for a control volume of fixed mass:

$$\underbrace{\sum \vec{F} - \vec{F}_o}_{\text{sum of forces on C.V.}} = \underbrace{\int_V \left[\frac{\partial(\rho\vec{u})}{\partial t}\right] dV}_{\text{change in momentum of mass contained in C.V. over time}} + \underbrace{\int_s \vec{u}(\rho\vec{u})\cdot\vec{n}ds}_{\text{change in momentum flux across surface of C.V.}}$$

- BLANK PAGE FOR EXTRA WORK -

Assume steady flow and no acceleration of the reference frame (2nd and 3rd terms are zero). Then the x- and y-components are:

$$\sum F_x = \int_s u_x(\rho\vec{u})\cdot\vec{n}ds \quad and \quad \sum F_y = \int_s u_y(\rho\vec{u})\cdot\vec{n}ds$$

$F_{flange\text{-}x} - (p_e - p_o)A_e\sin\alpha = 0 + u_e(u_e\sin\alpha)\,\rho A_e$

$\boxed{F_{flange\text{-}x} = \dot{m}[u_e\sin\alpha] + (p_e - p_o)A_e\sin\alpha}$

$F_{flange\text{-}y} - mg - (p_i - p_o)A_i + (p_e - p_o)A_e\cos\alpha = -u_i(-u_i)\,\rho A_i + u_e(-u_e\cos\alpha)\,\rho A_e$

$\boxed{F_{flange\text{-}y} = mg + \dot{m}[u_i - u_e\cos\alpha] + (p_i - p_o)A_i - (p_e - p_o)A_e\cos\alpha}$

Unified Engineering Propulsion Quiz Number: SOLUTIONS
Spring 2004

**2. (25 points, partial credit given, L.O.'s C, D & F)** What are the principal design parameters and constraints for a gas turbine engine? How are these related to the principal figures of merit and to mission performance? In particular, describe how changes in the various design parameters and constraints lead to changes in mission performance.

The principal design parameters and constraints for a gas turbine engine are the turbine inlet temperature, the compressor pressure ratio, and the bypass ratio.

These can be related to the principal figures of merit using ideal cycle analysis. The principal figures of merit are the overall efficiency, the maximum thrust and any impacts on aircraft weight and drag. Increasing the turbine inlet temperature increases the thrust per unit mass flow. Increasing the compressor pressure ratio increases the thermal efficiency, but can also increase or decrease propulsive efficiency at fixed turbine inlet temperature and increase or decrease thrust per unit mass flow. Increasing the bypass ratio increases the propulsive efficiency, but can also increase drag and weight.

The principal figures of merit (max thrust, overall efficiency and impacts on aircraft weight and drag) are related to aircraft range and maneuverability by:

$$\text{Range} = \frac{h}{g}\eta_{overall}\frac{L}{D}\ell n\frac{W_{initial}}{W_{final}} \quad \text{and for maneuvering:} \quad TV - DV = W\frac{dh}{dt} + \frac{d}{dt}\left(\frac{1}{2}\frac{W}{g}V^2\right)$$

Increased efficiency (e.g. through increasing bypass ratio or compressor pressure ratio) leads to increased range (or more payload if you have to carry less fuel). However changes in thrust per unit mass flow (e.g. through changes in bypass ratio or turbine inlet temperature) can have significant effects on drag and the structural weight fraction.

For maneuvering, the higher the excess power the better, so increasing the thrust to weight and/or decreasing the drag are good. Therefore, increasing turbine inlet temperature and decreasing bypass ratio are generally favorable. (Increasing compressor pressure ratio is also beneficial if it leads to increased efficiency – since this means less fuel carried and therefore less weight.)

Unified Engineering Propulsion Quiz Number: _SOLUTIONS_______

Spring 2005

**3. (15 points, partial credit given, L.O.'s C & E)**

You are charged with the re-design of a liquid-propellant rocket motor for a deep space application ($p_o = 0$, $g = 0$, drag $= 0$). You have the option to upgrade the turbopumps and the combustion chamber, allowing a 10% increase combustion chamber temperature. The drawback is increased mass of the propulsion system.

What design trades to you anticipate will be important in making a decision on the attractiveness of this re-design option? Please refer to important equations and physical relationships to substantiate your answer.

The rocket is designed for operation in deep space so $p = 0$, $g = 0$, drag $= 0$. Increasing the chamber temperature will increase the exhaust velocity and increase the change in vehicle velocity (du) per unit of propellant mass flow ($dm_v$).

$$u_e = \sqrt{2C_pT_c\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} \qquad \text{and} \qquad du = -\frac{u_e dm_v}{m_v}$$

(This last equation assumes that pressure forces on the exit plane of the nozzle $(p_e-p_o)A_e$ are negligible relative to the change in momentum flux.)

However, any change in propulsion system mass will either mean decreased propellant or payload mass, or increased $m_v$. Increased vehicle mass, $m_v$, will act to reduce the change in vehicle velocity (du) per unit of propellant mass flow ($dm_v$).

Unified Engineering Propulsion Quiz Number: _SOLUTIONS_

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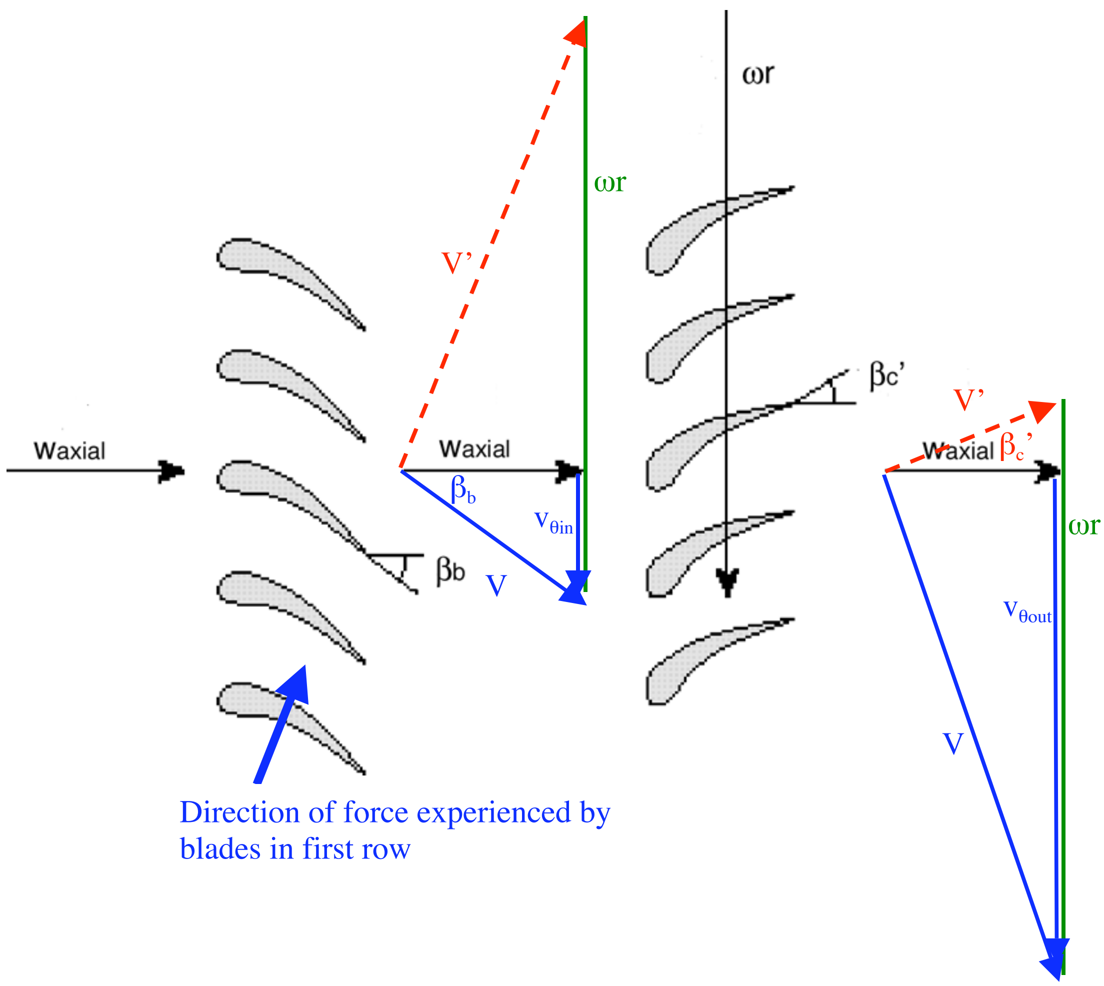
**4. (35 points, partial credit given, L.O.'s B & G)** A set of blade rows is shown.

a) Draw and label the velocity triangles for these blade rows on top of the axial velocity vectors that are given (i.e. show the velocities in the relative and absolute frames).




b) Say in words what the torque is equal to. Write an expression for the torque on the first blade row and draw the direction of the force experienced by the blades in the first row.

Torque is equal to the change in axial flux of angular momentum. Both blade rows experience a torque since the tangential velocity in the stationary frame changes across each of the blade rows (however, it is only the moving blade that does work changing the total enthalpy of the flow).

Torque =(mass flow)($rv_{\theta out}-rv_{\theta in}$)

c) What happens to the stagnation and static temperatures across the first blade row? Why?

The first blade row does no work. Therefore, assuming the flow is adiabatic, the stagnation enthalpy is constant from the steady flow energy equation and therefore the stagnation temperature is constant.

$$q_{1-2} - w_{s_{1-2}} = h_{T_2} - h_{T_1}$$

Since the velocity increases and the total enthalpy is constant, then the static temperature must decrease.

$$C_p T + \frac{V^2}{2} = C_p T_T = h_T$$

d) Write an expression for the power input or extracted from the flow. Is this a compressor or a turbine?

This is a compressor since the stationary frame tangential velocity across the moving blade row increases in the direction of rotor motion.

Power $=\omega(\text{Torque})=\omega$ (mass flow)$(r_{out}v_{out}-r_{in}v_{in})$.

f) Explain in words how a multistage axial compressor or turbine works.

The rotating blade rows do work on the flow (compressor) or have work done on them by the flow (turbine). This changes the total enthalpy of the flow—increasing (compressor) or decreasing (turbine) the energy in the flow. The stationary blade rows are nozzles that convert energy in the flow from one form to another. Since the axial velocity is relatively constant, a stator that adds swirl converts internal energy to swirling kinetic energy. A stator that removes swirl does the opposite. The purpose of the alternating blade rows in a compressor is to add energy (in the form of swirling kinetic energy) by doing work on the flow by applying a torque on the spinning blades that changes the axial flux of angular momentum. Then the swirling component of the kinetic energy is converted to internal energy so that the axial velocity through the machine remains roughly constant (or else viscous forces would increase too much). The opposite processes hold for a turbine.