Fluids Quiz 2 Solution   Spring '05

1) Compare total enthalpy:

$$h_{0_i} = h_i + \frac{1}{2}V_i^2 = c_p T_i + \frac{1}{2}V_i^2$$

$$= 1000\ \mathrm{J/kg\,K} \cdot 300\,\mathrm{K} + \frac{1}{2}\, 200^2\ \mathrm{m^2/s^2}$$

$$= 320000\ \mathrm{m^2/s^2}$$

$$h_{0_e} = h_e + \frac{1}{2}V_e^2 = c_p T_e + \frac{1}{2}V_e^2$$

$$= 1000 \cdot 240 + \frac{1}{2}\,400^2$$

$$= 240000 + 80000$$

$$= 320000\ \mathrm{m^2/s^2} \quad \text{same as } h_{0_i}.$$

i) adiabatic

Test isentropic relation

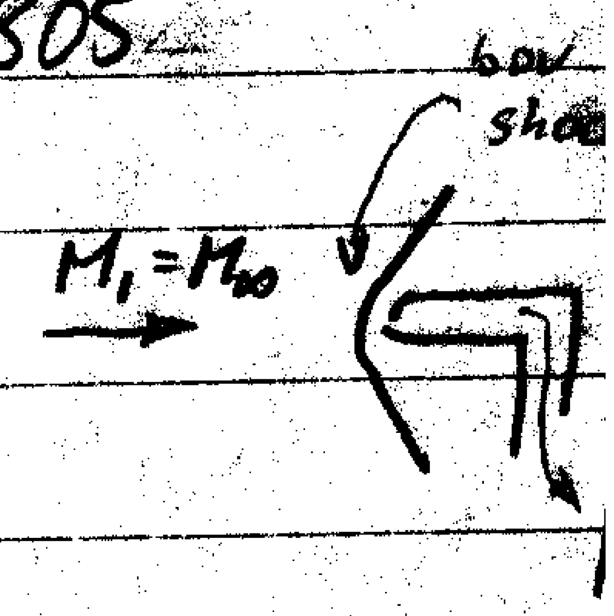
$$\frac{\rho_e}{\rho_i} \stackrel{?}{=} \left(\frac{T_e}{T_i}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{0.528}{1.000} \stackrel{?}{=} \left(\frac{240}{300}\right)^{2.5}$$

$$0.528 \neq 0.5724$$

iv) non-isentropic

2. a) From shock table: for $M_1 = 1.6 \ldots \quad \frac{p_{o_2}}{p_1} = 3.805$



Also, $p_1 = p_\infty$

so $\boxed{\frac{p_a}{p_\infty} = 3.805}$

b) $\theta = 8^\circ$, $M_1 = 1.6 \rightarrow \boxed{\beta = 48^\circ}$ from oblique shock chart.

$M_{n_1} = M_1 \sin 48^\circ = 1.189$

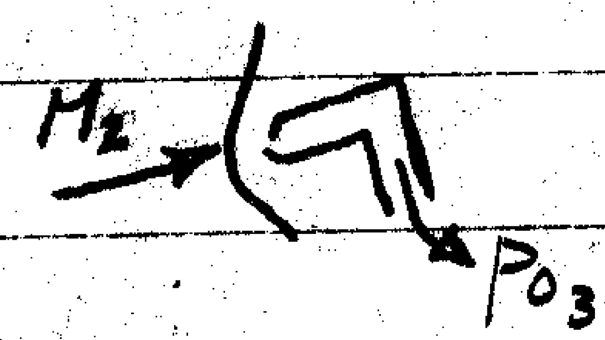
from shock table: $M_{n_2}(M_{n_1}) = 0.848$

$$\left[M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{M_{n_2}}{\sin(40^\circ)} = 1.319 \right.$$

c) Two weaker shocks will always have less loss than one strong shock to get to stagnation, so $p_c > p_a$.

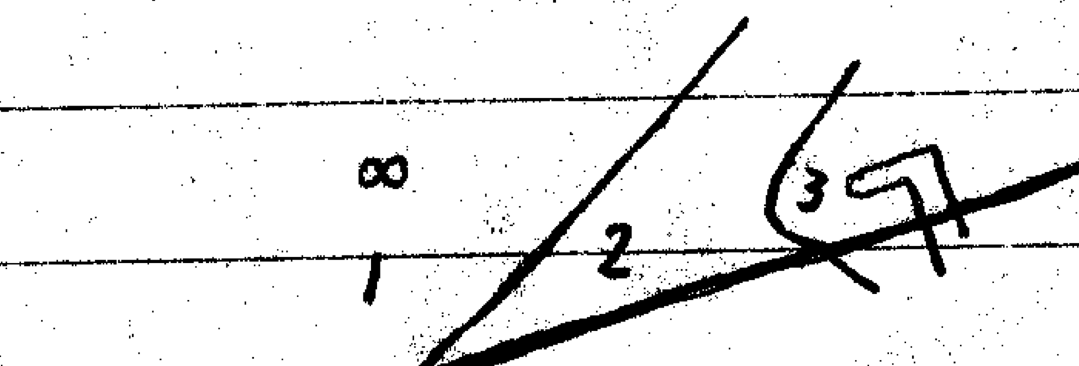
Long way: $p_2 = p_1 \cdot \left(\frac{p_2}{p_1}\right) = p_\infty \cdot f(M_{n_1}) = p_\infty \cdot 1.485$ ← "$\frac{p_2}{p_1}$" from shock table

Across pitot bow shock: $\frac{p_{o_3}}{p_2} = f(M_2) = 2.77$ ← "$\frac{p_{o_2}}{p_1}$" from shock table for "M" = M_2



$p_c = p_{o_3} = p_2 \cdot 3.61 = p_\infty \cdot 1.485 \cdot 2.77$

$\boxed{p_c = p_\infty \cdot 4.113}$



$p_c > p_a$ ✓

3. a) Isentropic expansion (no shocks): $\frac{p_r}{p_e} = \left[1 + \frac{\gamma-1}{2} M_e^2\right]^{\frac{\gamma}{\gamma-1}} = 4$ $(p_e = p_B)$

From isentropic table: for $\frac{p_0}{p} = 4$, $M_e = 1.56$, $\frac{A_e}{A^*} = 1.219$

Must also have $A_t = A^* = A/1.219 = 1\,m^2/1.219$

$$A_t = 0.82\ m^2$$

b) The above calculations are unaffected by $h_0 = h_r$.
Flow remains shock-free for any other h_r

c) $\dot{m} = \frac{\gamma p_e}{\sqrt{(\gamma-1) h_r}} f(M_e)$ p_e, M_e unchanged.

so $\frac{\dot{m}'}{\dot{m}} = \sqrt{\frac{h_r}{h_r'}} = \sqrt{\frac{1}{1.1}} = 0.953 = 1 - 0.047$ 4.7% decrease in $\dot{m}$

d) Throat is increased 1%, still choked → $\dot{m}$ increased 1%

But A_e is same, so flow is under-expanded.

→ expansion fans at exit.