Unified Quiz S6
April 27, 2005

One 8½” x 11” sheet (two sides) of notes
No calculators allowed.
No books allowed.

• Put your name on each page of the exam.
• Read all questions carefully.
• Do all work for each problem on the two pages provided.
• Show intermediate results.
• Explain your work --- don’t just write equations. Any problem without an explanation can receive no better than a “B” grade.
• Partial credit will be given, but only when the intermediate results and explanations are clear.
• Please be neat. It will be easier to identify correct or partially correct responses when the response is neat.
• Show appropriate units with your final answers.
• Box your final answers.

Exam Scoring

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A causal, LTI system, $G$, has impulse response $g(t)$. The Laplace transform of $g(t)$ is

$$G(s) = \frac{4}{s(s + 2)^2}$$

1. What is the region of convergence of the Laplace transform? Explain.
2. Find $g(t)$.
3. Is the system BIBO stable or unstable?
Problem 1

ID number (last four digits)________________________
A causal, LTI system, $G$, has impulse response $g(t)$ given by

$$g(t) = \frac{1}{1 + t}\sigma(t)$$

Is the system BIBO stable? Explain.
Problem 2

ID number (last four digits)________________________
Problem 3

Given the signals $g(t)$ and $u(t)$ as plotted below, find the signal $y(t)$ given by

$$y(t) = g(t) * u(t)$$

Sketch the result in the grid below, as accurately as possible. Explain your reasoning on the page that follows. The grid squares do not have to represent 1 unit — you can choose the units as appropriate to plot the result. Be sure to label the axes of the grid.
Problem 3 (25%)  
ID number (last four digits)_______________________
Problem 3  
ID number (last four digits)________________________
Consider an LTI system $G$ with input signal $u(t)$ and output signal $y(t)$.

1. What is the definition of the transfer function, $G(s)$?

2. Explain why the transfer function is the Laplace transform of the impulse response.
Problem 4

ID number (last four digits)____________________
Problem 5 (25%)

ID number (last four digits)

Find the step response of the circuit below. The component values are $R = 4\,\Omega$, $L = 2\,\text{H}$. 

![Circuit diagram](image)

$u(t)$ $L$ $R$ $y(t)$
Problem 5
ID number (last four digits)__________________________