UNIFIED HANDOUT

MATERIALS AND STRUCTURES - #M-11

Spring, 2007

Concept Review Sheet

for Unified Q7M: Units M4.6, M4.7, M4.8, M5.1-M5.4

TORSION

- A shaft is long compared to its other two dimensions (L >> b,h).
- The load applied is a pure torque about the axis of the long direction.
- The cross-section rotates as a rigid body (i.e. no distortion of the cross-section).
- All displacements and deformations can be defined through the twist angle ϕ.
- The pertinent internal resultant is the torque, T.
- The torque resultant at any location along the shaft is equipollent to the torque caused by the shear stresses acting and integrated over the "cut" surface.
- The change in torque along the shaft is equal to the negative of the applied torsional load.
- The polar moment of inertia, J, defines the geometric contribution of the structural configuration to the resistance to torsional deformation.
- The shear modulus defines the material contribution of the structural configuration to the resistance to torsional deformation.
- The total torsional stiffness is GJ.
- Maximum magnitude of shear stress occurs at a point furthest from the centerline of the shaft.
- The model gives exact solutions for circular cross-sections (including tubes), but approximations for other sections.

COLUMNS AND BUCKLING

- "*Effective structural stiffness*" is a linear change in restoring force with deflection
- "A system becomes unstable when a negative stiffness overcomes the natural stiffness of the structure"
- Terminology: *eigenvalue* is the value of the load for static instability; *eigenvector* is the displacement shape/mode of the structure in the static instability
- **Bifurcation** is a mathematical concept. The manifestations in actual systems are altered due to physical realities/imperfections
- A column is long compared to its other two dimensions (L >> b,h) and is loaded in compression along its long axis (x_1)
- The transverse deflection is governed by the bending equation
- Basic equation is:

$$EI \,(d^2u_3/dx_1^2) + Pu_3 = 0$$

This is a reduction of a fourth order equation for the case of EI and P not dependent on x_1

- General homogenous solution to equation is:
  $u_3 = A \sin \lambda x_1 + B \cos \lambda x_1 + C + D\, x_1$ where $\lambda = (P/EI)^{1/2}$
- Obtain constants by using boundary conditions (2 at each end needed); Solve the resulting set of equation for roots of the set of equations -- this results in eigenvalues and associated eigenvectors; Smallest eigenvalue is the buckling load
- General form of critical buckling load is:
  $P_{cr} = c\pi^2 EI/L^2$
- c = coefficient of edge fixity (dependent on boundary conditions)
- If there are eccentricities and imperfections, an inhomogeneous solution also arises and this provides a transverse deflection versus load that approaches the buckling bifurcation line as an asymptote
- Columns can fail by yielding/squashing/crushing prior to reaching critical buckling load. This is governed by yield and ultimate stresses in compression.
- The more slender a column is, the more likely it is to buckle (characterized by *slenderness ratio*)
- A column buckles in the direction with the smallest moment of inertia

FAILURE: THE MATERIAL'S ROLE

- *Structural failure* is the inability of the structure to perform as intended
- Strain energy per unit volume is the area under the stress-strain curve
- *Proportional stress* is the stress where the stress-strain behavior deviates from linear
- *Yield* stress is the stress where the stress-strain behavior deviates from elastic
- *Ultimate* stress is the stress where the material breaks or the material holds its highest stress
- Stress-strain behavior is characterized as *linear* (if modulus/stiffness stays the same), *softening* (if modulus/stiffness decreases), or *stiffening* (if modulus/stiffness increases)
- Stress-strain behavior is characterized as *elastic* (if unloading occurs along the same path as loading and energy is conserved), *plastic* (if unloading occurs along a different path from loading and energy is expended)
- True strain involves considering the actual length at any time versus considering the original length in engineering/nominal strain
- True stress involves considering the actual cross-sectional area at any time versus considering the original cross-sectional area in engineering/nominal stress
- True strain equals nominal strain, and true stress equals nominal stress for small values of strain
- Material can have an increase in strain under stress over a period of time. This phenomenon is known as creep (this is a viscoelastic response).
- The creep phenomenon increases with increased temperature
- Material can *relax* over time -- requiring less load to maintain a given displacement (this is also a manifestation of viscoelastic response)

- The theoretical value of material strength due to atomic attraction overestimates the actual values by a considerable factor (estimate is E/15) except in rare, perfect cases
- Failure is a weakest link phenomenon and occurs at imperfections, etc. such as dislocations in materials with crystal structure
- There are multiple lengthscales at which these imperfections can occur
- Strengthening methods are linked to the method by which materials (and structures) fail, e.g. keep the imperfection from propagating

- In crystals, failure occurs by shear stress along planes of slippage
- Hydrostatic stress does not cause failure
- The Tresca criterion is based on failure by maximum shear stresses and thereby compares the difference of principal stresses
- The von Mises criterion also considers the difference of principal stresses but adds up the effects (interactive criterion) rather than looking at each of the three differences separately

- Stress concentrations cause failure load to be lowered due to the increase in stress local to the stress concentration
- Failure in fracture mechanics is governed by the length of the crack and the critical stress intensity factor (also known as *fracture toughness*)
- The (strain) energy expended in the fracture process is consumed by the surface energy of the new crack area created
- Fatigue is “the tendency of a material to break under repeated stress”
- The greater the number of stress cycles, the lower the repeated stress needed to cause fatigue
- There are crack growth rules (e.g. Paris law) that characterize the extension of cracks under repeated (cyclic) loads
- Safe-life design is based on “Miner’s rule” and the use of S-N diagrams
- Damage tolerant design is based on crack growth to critical crack size