Block 4:

Structural Behavior of Slender Members (1-D)

OVERALL OBJECTIVES FOR BLOCK **M4**

Through participation in the lectures, recitations, and work associated with Block M4, it is intended that you will be able to………

- ….**describe** the key aspects composing the model of the one-dimensional structures of a rod, a beam, a shaft, and a column
- ….**identify** the limitations associated with these models
- ….**apply** the basic equations of elasticity to **derive** the solutions for the general cases of various one-dimensional structures
- ….**apply** these models to **examine** the behavior of various structural configurations

Unit M4.1
Summary of Equations of Elasticity

Readings:
CDL 5.6

16.003/004 -- “Unified Engineering”
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

LEARNING OBJECTIVES FOR UNIT **M4.1**

Through participation in the lectures, recitations, and work associated with Unit M4.1, it is intended that you will be able to………

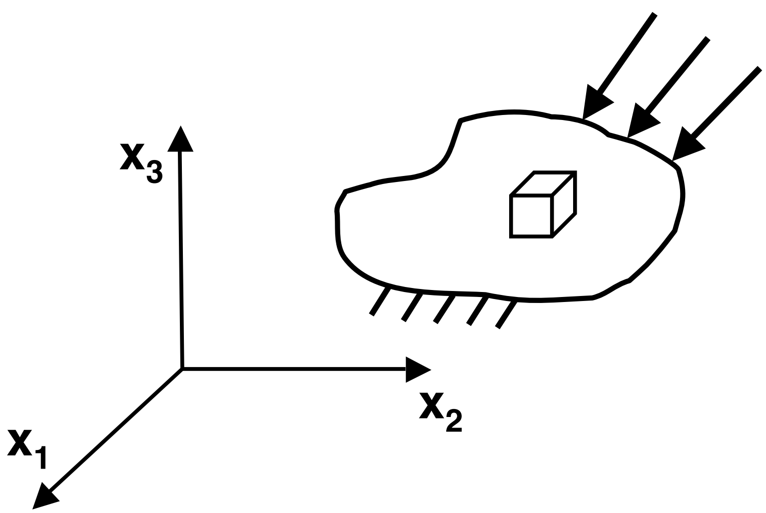
- ….**summarize** the governing equations of elasticity and **explain** the fundamental concepts upon which they are based and that they represent
- ….**describe** the logical steps to be taken in **applying** these equations for a given structural configuration

Let's summarize where we've been in terms of elasticity and then see where we can go with this.

Governing Equation Summary

Given general body under applied surfaces forces and body forces

Figure M4.1-1 **Illustration of general body**



Look at infinitesimal element

What are states of deformation, strain and stress at any point:

u_1	ε_{11}	ε_{12}	σ_{11}	σ_{12}
u_2	ε_{22}	ε_{13}	σ_{22}	σ_{13}
u_3	ε_{33}	ε_{23}	σ_{33}	σ_{23}

3 displacements + 6 strains + 6 stresses = 15 unknowns

Using the 3 "great principles of solid mechanics", we arrived at 3 sets of equations:

(3) Equilibrium:
$$\frac{\partial \sigma_{mn}}{\partial x_m} + f_n = 0$$

(6) Strain-Displacement: *(from compatibility)*
$$\varepsilon_{mn} = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right)$$

(6) Stress-strain: *(constitutive)*
$$\sigma_{mn} = E_{mnpq}\,\varepsilon_{pq}$$

or

$$\varepsilon_{pq} = S_{pqmn}\,\sigma_{mn}$$

=> 15 equations in 15 unknowns (see handout M-4 for fully written out)

But, recall assumptions/limitations:

e.g., - small strain
- linear behavior
.
.
.

Also, we had the 6 equations of strain-compatibility

These equations then have to be solved for the particular configuration.

This involved a basic logical set of steps:

1. Modeling Assumptions

Modeling assumptions are made for the particular class of structural member

In general these are on the:

a) Geometry
b) Loading /Stress State
c) Deformation/Strain State

These assumptions result in limitations on the behavior

Again, a key issue is ***consistency***.

Note: A component may actually be classified "differently" if it is used differently although it physically is the same piece (example: a *rod* and a *beam* can look exactly the same)

2. Boundary Conditions

but

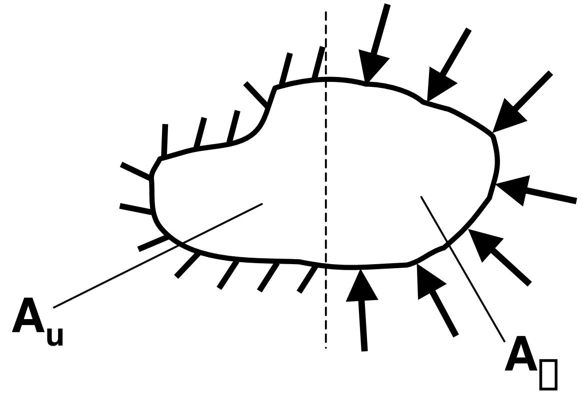
There are two types of boundary conditions (B.C.):

(a) on stresses (call such area $\mathbf{A}_{\sigma}$)

(b) on displacements (call such area $\mathbf{A}_{U}$)

All surface area must have one or the other (note that a value of 0 is a boundary condition)

Figure M4.1-2 **Generic illustration of boundary conditions on a body**



Given the 15 equations of elasticity and the boundary conditions, we then need… (this is the "Theory of Elasticity")

3. Solution Approaches for Structures

…to solve the formulated problem

Once the problem is modeled, there are basically two approaches:

a. Exactly/Analytical -- satisfy all equations and B.C.'s

--> Can be done for certain special models (will see in second term of Unified, 16.20)

b. Numerical Techniques -- … come as "close as possible"

--> energy methods, etc., utilizing computers (16.21, etc.)
--> most often used nowadays, but analytical solutions still important in preliminary design or for checking some modeling

--> All bodies are 3-dimensional, but can often model some as 1-dimensional (our focus this term)

<u>1-Dimensional structures</u>:

- Rods
- Beams
- Columns

} (Unified & 16.20)

- Torsional rods (more generally 16.20)

<u>2-Dimensional structures</u>:

- Plates
- Shells

} (16.20 & 16.21)

Final Words: Arbitrary body (continuum) is generally statically indeterminate. Thus, must use deflection of body with static equilibrium, compatibility, and stress-strain relations to solve the problem.

Unit M4.1 (New) Nomenclature

A_u -- surface area of body over which displacement is prescribed (e.g., a boundary condition)

A_σ -- surface area of body over which stress is prescribed (e.g., a boundary condition)

B.C. -- boundary condition

u_i -- displacement of point of body in x_i direction (i = 1, 2, or 3)

ε_{ij} -- strain of point of body (i, j = 1, 2, 3)

σ_{ij} -- stress of point of body on x_i-face in x_j - direction (i, j = 1, 2, 3)

x_1, x_2, x_3 -- coordinate directions