Unit M4.8
Final Notes on 1-D Structural Members

Readings:

16.003/004 -- "Unified Engineering"
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

FINAL BLOCK **M4** OBJECTIVES via **M4.8**

Through participation in the lectures, recitations, and work associated with Block M4 as emphasized in Unit M4.8, it is intended that you will be able to………

- ….**describe** the key aspects composing the models of the one-dimensional structures of a rod, a beam, a shaft, and a column, and **recognize** their similarities
- ….**identify** the limitations associated with these models
- ….**apply** the basic equations of elasticity to **derive** the solutions for the general cases of various one-dimensional structures
- ….**apply** these models to **examine** the behavior of various structural configurations

Remember that all we have done are models of actual structural configurations (which are, by nature, always three-dimensional). The key is in the assumptions and resulting limitations and how good an answer is needed

It is useful to look at the forms of the relations for the models we considered:

One-dimensional Model	Rod	Beam	Shaft
Equilibrium Equation(s)	$\frac{dP}{dx_1} = \square p_x$	$\frac{dS}{dx} = q$ $\frac{dM}{dx} = S$	$\frac{dT}{dx_1} = \square t$
Stress	$\square_{11} = \frac{P}{A}$	$\square_{xx} = \square \frac{Mz}{I}$ $\square_{xz} = \square \frac{SQ}{Ib}$	$\square_{13} = \frac{Tx_2}{J}$ $\square_{12} = \square \frac{Tx_3}{J}$ $\square_{res} = \frac{Tr}{J}$
Deflection	$\frac{du_1}{dx_1} = \frac{P}{EA}$	$\frac{d^2 w}{dx^2} = \frac{M}{EI}$	$\frac{d\square}{dx_1} = \frac{T}{GJ}$

Note: **same general form**

--> Equilibrium Equation $$\frac{d}{dx_1}\left[\begin{matrix}\text{stress}\\ \text{resultant}\end{matrix}\right] \sim \text{loading}$$

--> Stress: $$\sigma \sim \frac{\text{stress resultant}}{\text{geometry}}$$

--> Deflection: $$\frac{d^n}{dx^n}\left(\text{deflection}\right) \sim \frac{\text{stress resultant}}{\text{structural stiffness}}$$

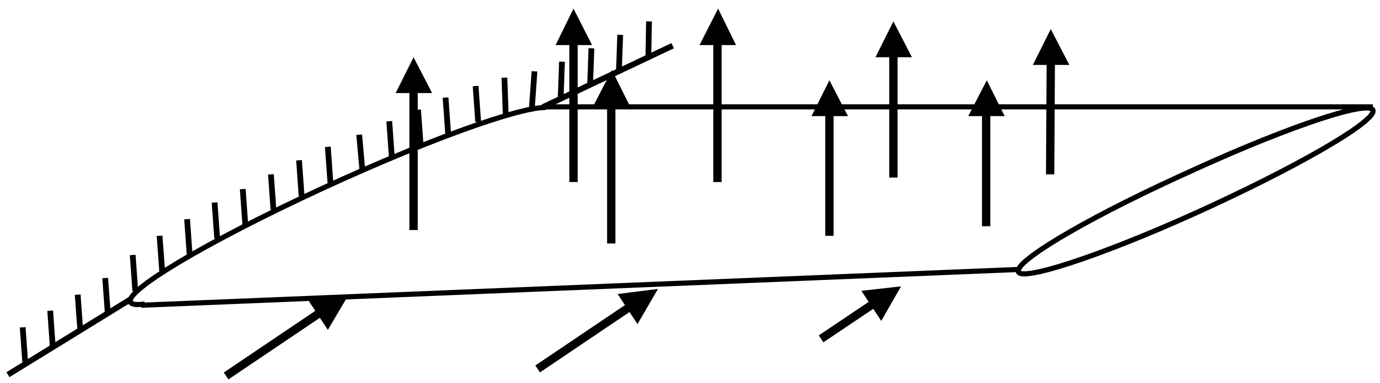
--> Two parts to structural stiffness = (material contribution) x (geometrical contribution)

--> What can we now do?

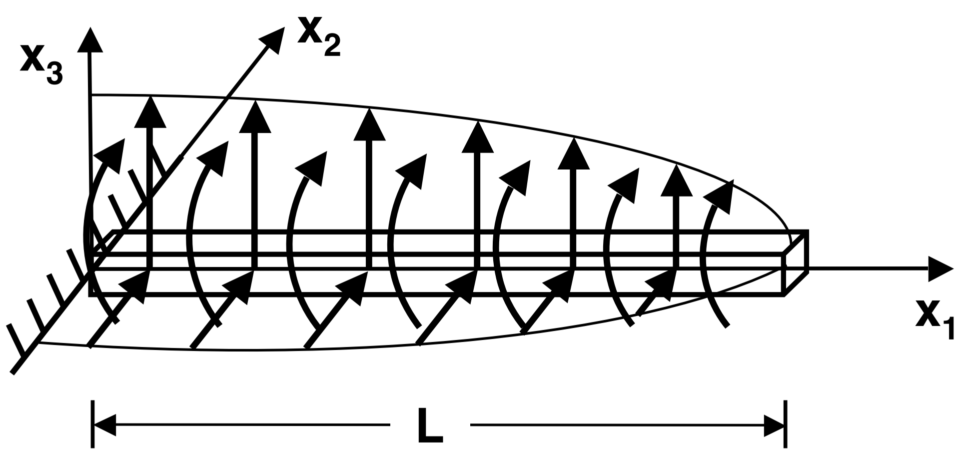
Make simple models of real structure by superposing our simple one-dimensional models

Example: an airplane wing

Get aerodynamic lift, drag, moments, etc., form fluids



Model as a constant cantilevered cross-section beam with distributed loads and moments:



etc.