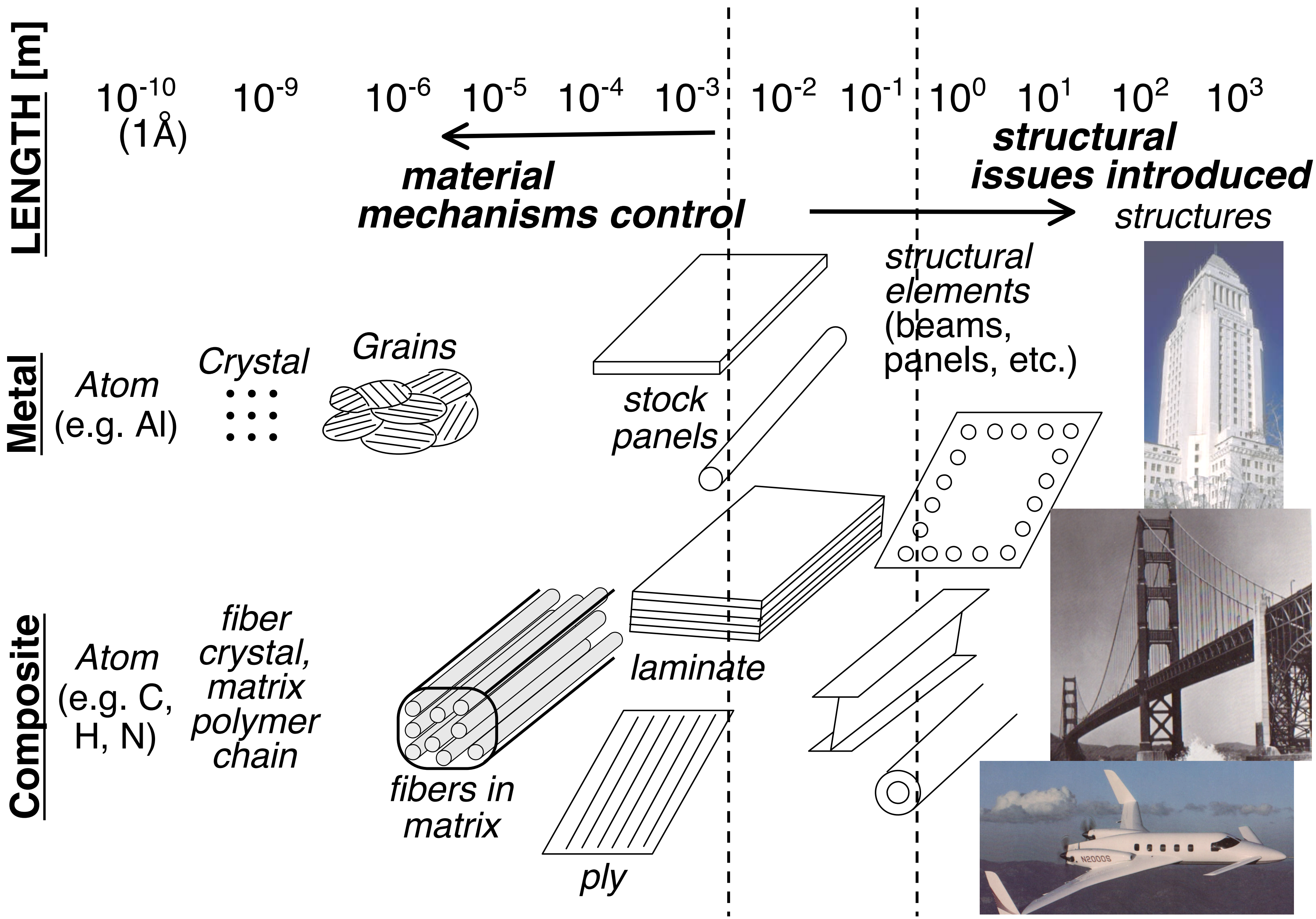
Block 5:

Failure: The Material's Role

LEARNING OBJECTIVES FOR BLOCK **M5**

Through participation in the lectures, recitations, and work associated with Block M5, it is intended that you will be able to………

- ….**summarize** and **describe** the key aspects leading to and engaged in material failure at both the macroscopic and microscopic levels
- ….**apply** developed models of material behavior to assess failure in various scenarios


LENGTH [m]
10^{-10} (1Å)
10^{-9}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
material mechanisms control
structural issues introduced
structures
Metal
Atom (e.g. Al)
Crystal
Grains
stock panels
structural elements (beams, panels, etc.)
Composite
Atom (e.g. C, H, N)
fiber crystal, matrix polymer chain
fibers in matrix
laminate
ply

Unit M5.1
Material Failure/Strength

Readings:

A&J 8, 11, 17

CDL 5.2, 5.3, 5.8, 5.18

16.003/004 -- “Unified Engineering”
Department of Aeronautics and Astronautics
Massachusetts Institute of Technology

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LEARNING OBJECTIVES FOR UNIT **M5.1**

Through participation in the lectures, recitations, and work associated with Unit M5.1, it is intended that you will be able to………

- ….**define** the concept of strain energy
- ….**describe** the various macroscopic stress-strain behaviors of materials prior to final failure
- ….**identify** the key points in material behavior leading to failure and **label** the associated properties
- ….**recall** the aspects of time-dependent material response

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We often use the word *"failure"*, but that term is often used rather "loosely". That leads us to ask the question:

What is failure?

In terms of structures, failure has a very specific meaning:

--> structural failure: the inability of the structure to perform as intended

In Unit M1.1 we talked about needs to

- carry loads (strength)
- resist deformation (rigidity)
- have sufficient lifetime (longevity)

There are a number of different structural mechanisms by which failure can occur. Here we want to focus on material failure which leads to structural failure.

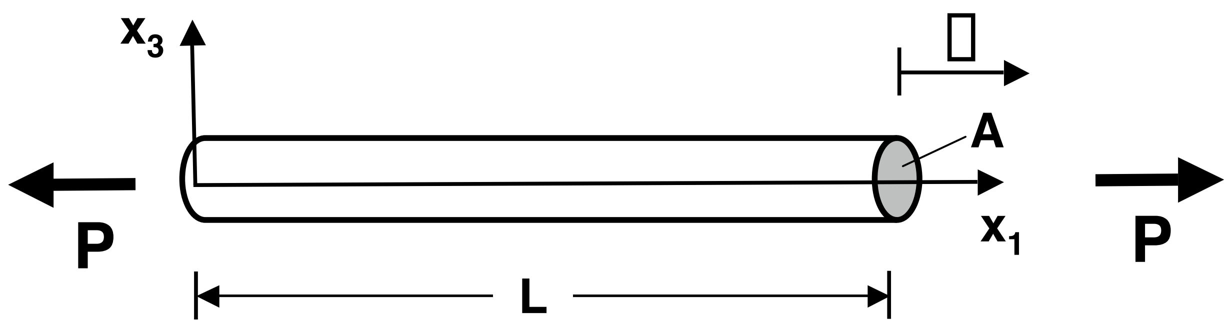
So we need to return to the stress-strain curve which we defined in Block 3 and go beyond the initial linear range where we defined modulus. But it is useful to first define the concept of…..

Strain Energy

Deforming a body adds energy to it as work is done on the body

--> consider a rod under load

Figure 5.1-1 **Rod under uniaxial load**



P = Applied load

δ = deformation

Work = W = Pδ

(more generally: $W = \int_0^{\delta} P \, d\delta$)

How to get energy?

$$\varepsilon = \frac{\delta}{L} \qquad \sigma = \frac{P}{A}$$

via 1st Law of Thermodynamics:

W - U = 0

So if this is a non-dissipative process (i.e., all energy stored, none lost as heat, etc.), then:

$$W = U = \int_0^{\delta} P \, d\delta$$

$$\Rightarrow \quad U = \int_0^{\varepsilon} \sigma A \, dL \, d\varepsilon$$

with: $AL = V =$ Volume

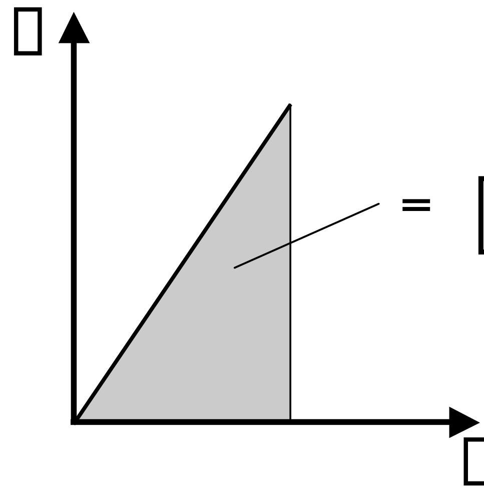
So get:

$$\boxed{U = \int_0^{\varepsilon} \sigma \, d\varepsilon}$$

<u>Strain</u> <u>Energy</u> per unit volume

Think, graphically, as area under the stress-strain curve:

Figure 5.1-2 **Representation of strain energy on stress-strain curve**



$$= \int_0^{\varepsilon} \sigma \, d\varepsilon = U$$

Note that if σ - ε relation is linear, $\sigma = E\varepsilon$

$$\Rightarrow \quad U = \frac{1}{2} E \varepsilon^2$$

Armed with this, we can now consider the full…..

Stress-Strain Curve: Yield and Ultimate Stress

The overall stress-strain behavior that a material shows, gives it overall characteristics. In identifying these, it is first useful to identify three key stresses/strains/points: (in tension or compression)

--> ***proportional*** (   ) -- where behavior deviates from linear

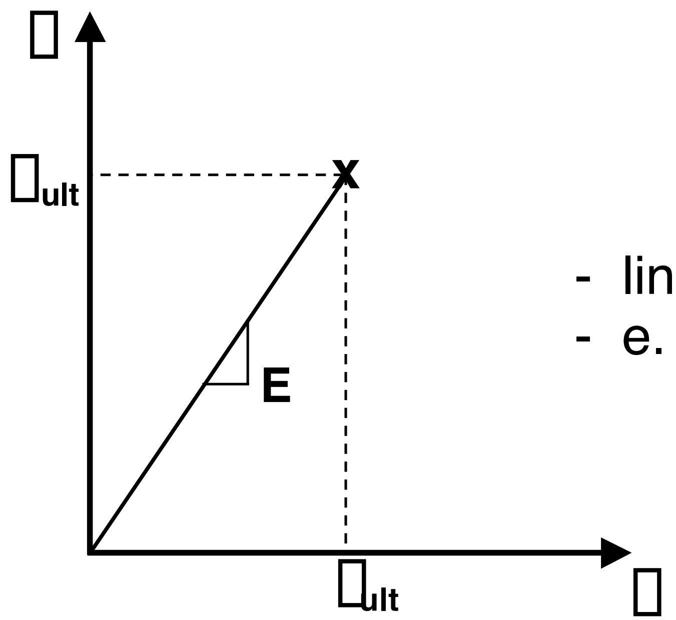
--> ***yield*** (   ) -- where behavior deviates from elastic   (σ_y)
(no permanent deformation prior to this)

--> ***ultimate*** (   ) -- where material breaks (into two pieces or more!) (σ_{ult}) [i.e., rupture]

We can now classify various stress-strain behavior.

--> Brittle

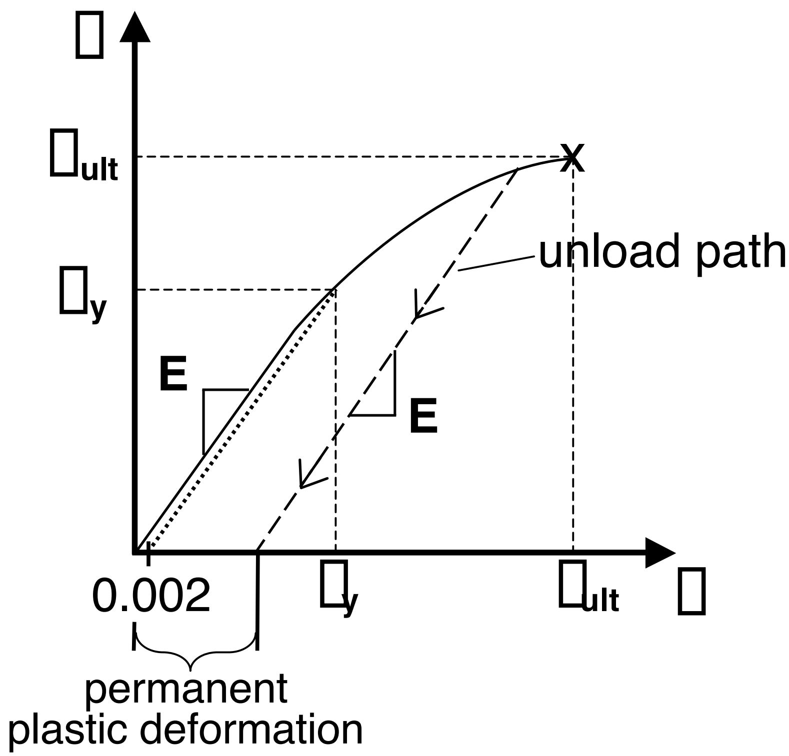
Figure 5.1-3 **Representation of stress-strain behavior for brittle material**



- linear elastic to failure
- e. g. ceramics, glass

--> Ductile (also…plastic)

Figure 5.1-4 **Representation of stress-strain behavior for ductile material**

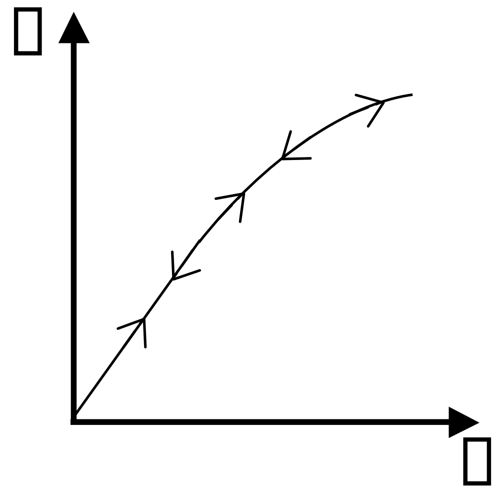


In metals and common engineering materials, yield defined by 0.002 offset (empirically)

- plastic (permanent) deformation
- unloading is elastic
- e.g., metals, polymers

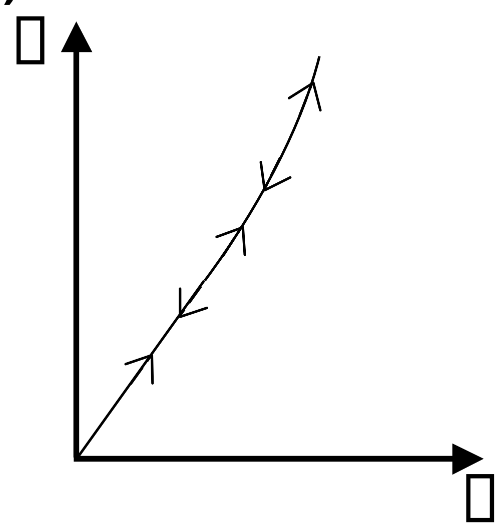
--> Elastic (also…plastic)

Figure 5.1-5 **Representation of stress-strain behavior for elastic material (with softening)**



e.g. rubber

Figure 5.1-6 **Representation of stress-strain behavior for elastic material (with stiffening)**



e.g. graphite fiber

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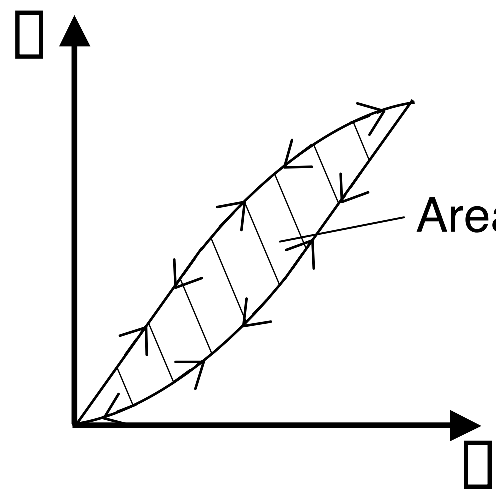
For both cases:

- Load and unload along same path
- no energy consumed

--> Anelastic

Figure 5.1-7 **Representation of stress-strain behavior for anelastic material**



- Load and unload paths not the same
- Energy dissipated (generally as heat -- e.g., paper clip)
- provides damping
- e.g., soft metals, polymer

Note: yield is one form of this (e.g., plasticity)

We generally measure the stress-strain behavior by a tensile test or a compressive test. However, the behavior between these two loading cases can (appear to) differ. We thus need to consider the concept of….

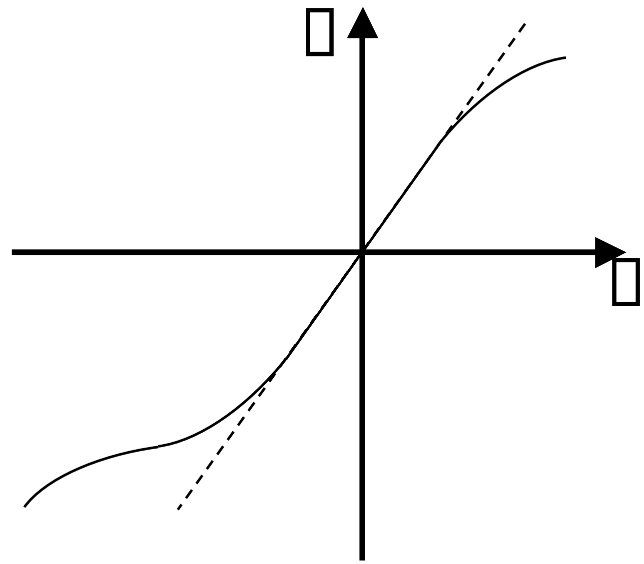
True Stress and True Strain

If we plot "nominal" (subscript "n") stress-strain using our engineering definition of stress and strain:

$$\sigma_n = \frac{P}{A_o} \qquad \varepsilon_n = \frac{\Delta L}{L_o}$$

subscript "o" indicates original (unloaded)

Figure 5.1-8 **Representation of uniaxial engineering stress-strain behavior over tensile and compressive regimes**



This is, in part, due to a phenomenon known as "necking" -- yielded region shrinks in cross-sectional area (in tension). So need to define true stress and true strain.

--> Start with incremental strain: $d\varepsilon = \dfrac{dL}{L}$

L is changing, so true strain from initial length, L_o, to final length, L_f is:

$$\varepsilon_{(t)} = \int_{L_o}^{L_f} d\varepsilon$$

$$\Rightarrow \quad \varepsilon_{(t)} = \int_{L_o}^{L_f} \frac{dL}{L} = \ln (L) \Big]_{L_o}^{L_f}$$

$$= \ln \left(L_f\right) - \ln \left(L_o\right) = \ln \left(\frac{L_f}{L_o}\right)$$

$$= \ln \frac{L_f}{L_o} = \ln \frac{L_o + \delta}{L_o} = \ln \left(1 + \frac{\delta}{L_o}\right)$$

for $\frac{\delta}{L_0}$ small:

$$\ln \left(1 + \frac{\delta}{L_o}\right) \approx \frac{\delta}{L_0}$$

$$\Rightarrow \quad \varepsilon_t \approx \varepsilon_n$$

Not true for larger strains (plastic)

Similarly, $\sigma_t = \frac{P}{A}$

where A is area at any point in time

For plastic deformation, volume is generally conserved, so:

$$A_f L_f = A_o L_o = AL = \text{constant}$$

$$\Rightarrow \quad A = A_f \frac{L_f}{L}$$

$$\Rightarrow \quad \sigma_{(t)} = \frac{PL}{A_f L_f} = \frac{PL}{A_o L_o}$$

Now plotting the same behavior except on a "<u>true</u>" basis

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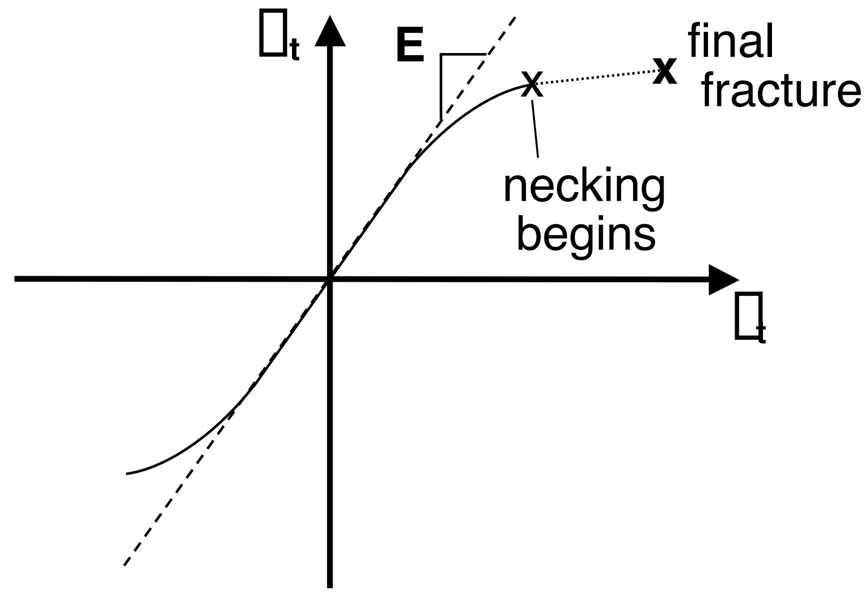
Figure 5.1-9 **Representation of uniaxial true stress-strain behavior over tensile and compressive regimes**



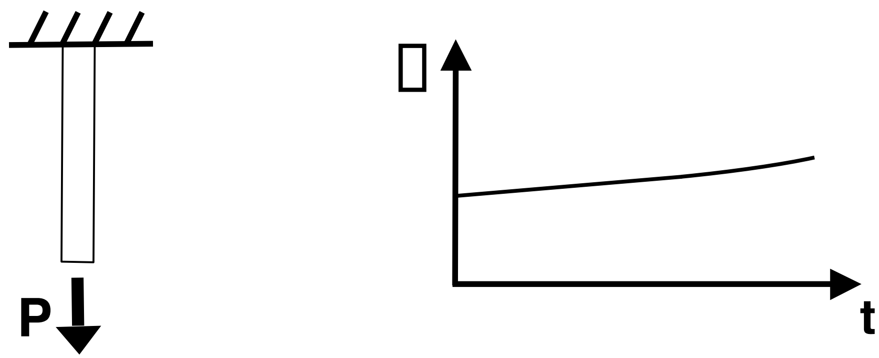
- The symmetry is recovered

Thus far we have considered only time-independent phenomena. However, there is also a time-dependence in strain response….

Viscoelasticity and Creep

In general, strain increases with time for a given load/stress level

Figure 5.1-10 **Representation of creep behavior for rod subjected to constant load**



This can either be

- viscoelasticity (due to "flow"…viscous)
- creep (due to plasticity)

(hang a weight off a support pipe)

--> Dependence of strain is on <u>time</u> and <u>temperature</u>

Generally:

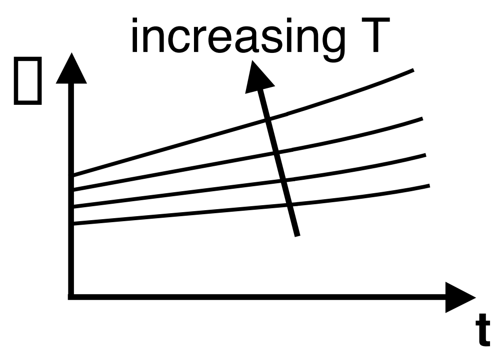
$$\frac{d\epsilon_{cr}}{dt} = A\sigma^{n} e^{-\frac{Q}{RT}}$$

creep strain rate stress

(A, n, Q, R = constants that are measured for a material)

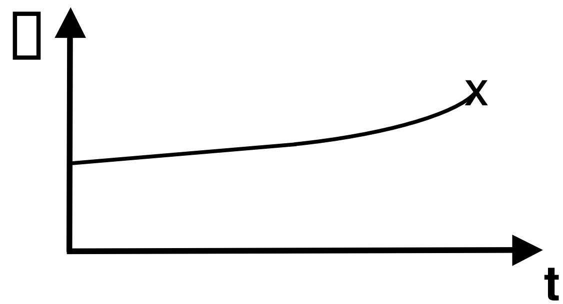
--> particularly important at high temps

- engine blades
- keeps aluminum off supersonic wings

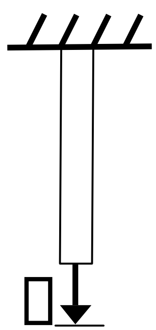


--> Related phenomena

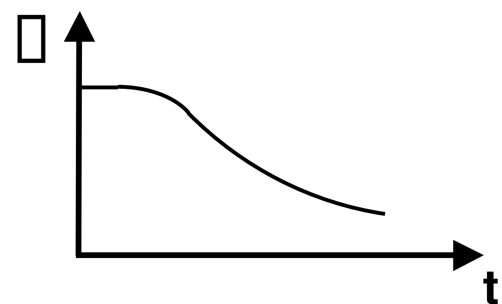
- creep rupture



- relaxation (for viscoelastic materials) for a given deformation, stress decreases



How much stress is needed to maintain displacement?



We'll next consider the mechanisms by which these phenomena occur

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Unit 5.1 (New) Nomenclature

t -- time
U -- strain energy per unit volume
W -- work
ε_{cr} -- creep strain
ε_n -- nominal strain
ε_t -- true strain
σ_n -- nominal stress
σ_t -- true stress

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