Unit M5.3
Yield (and Failure) Criteria

Readings:

CDL 5.11, 5.13, 6.9

16.003/004 -- “Unified Engineering”
Department of Aeronautics and Astronautics
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LEARNING OBJECTIVES FOR UNIT **M5.3**

Through participation in the lectures, recitations, and work associated with Unit M5.3, it is intended that you will be able to………

- ….**explain** why shearing is a key mechanism in material failure (yielding) in many cases
- ….**describe** typical failure/yield criteria, their origin, and the importance of hydrostatic stress
- ….**use** these criteria in assessing the failure for cases with multiaxial stress fields

Thus far we have talked about the manifestation of yielding in the overall stress-strain response and the mechanisms/origins of yielding. We would like to move forward and be able to predict yielding (and failure) in structures under general (multiaxial) Load/Stress states.

Before we look at two (classic) criteria which have been devised to do this, let us consider two key facts. First the…..

Maximum Shear Plane

Crystals (grains) slip along certain planes.

In a material with many crystals and thus randomly oriented grains, overall slip occurs along a more or less oriented plane

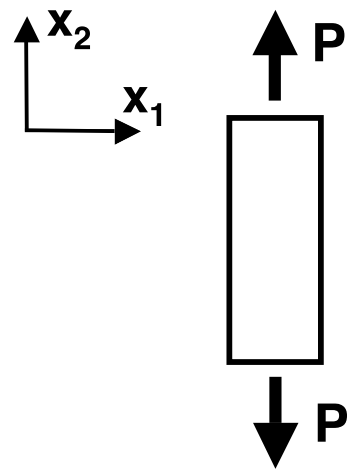
Thus, the material property is actually τ_{yield} (shear yield stress)

This "yields" the question…..

How is this related to σ_{yield}?

--> Consider the uniaxial tensile test

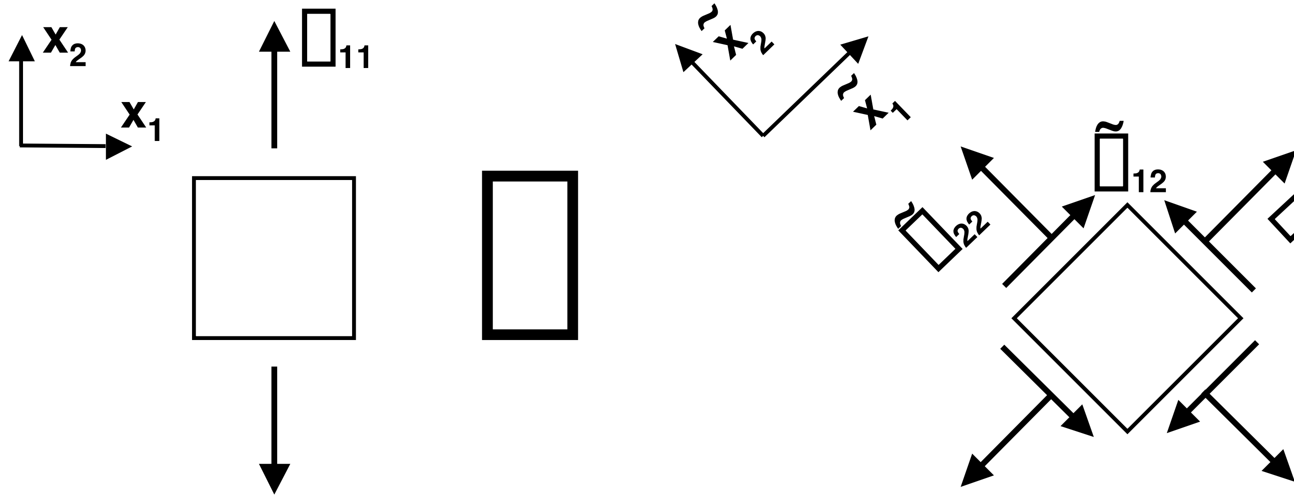
Figure 5.3-1 **Coupon under uniaxial tension**



Stress state:
$$\sigma_{11} = P/A$$
$$\sigma_{22} = 0$$
$$\sigma_{33} = 0$$

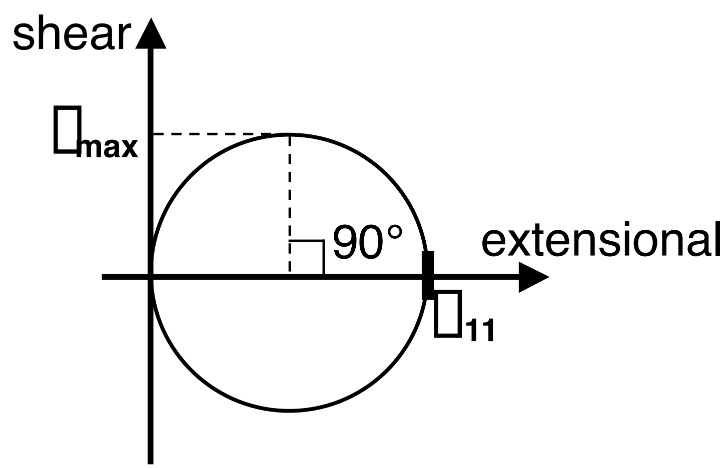
<u>Recall</u> stress transformation (Mohr's circle)

Figure 5.3-2 **General in-plane transformation of uniaxial stress state**



and maximum shear stresses!

Figure 5.3-3 **Mohr's circle and maximum shear stress for in-plane uniaxial stress state**



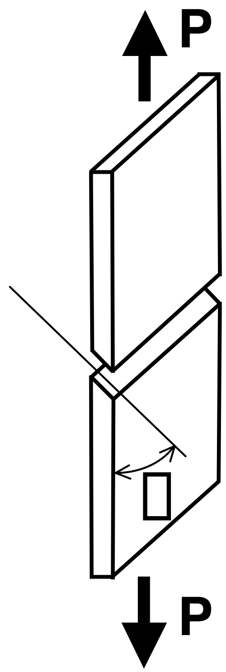
Thus, yield occurs when: $\frac{\sigma_{11}}{2} = \tau_y$

$$\Rightarrow \quad \boxed{\sigma_{yield} = 2\,\tau_{yield}}$$

What angle does τ_{max} occur at? 45° to principal stress direction σ_{11}

$\Rightarrow$ slip/yield should occur along 45° lines

Figure 5.3-4 **45° slip line in coupon under uniaxial stress**



(shows in failure modes)

The second fact deals with…

The Importance of Hydrostatic Stress

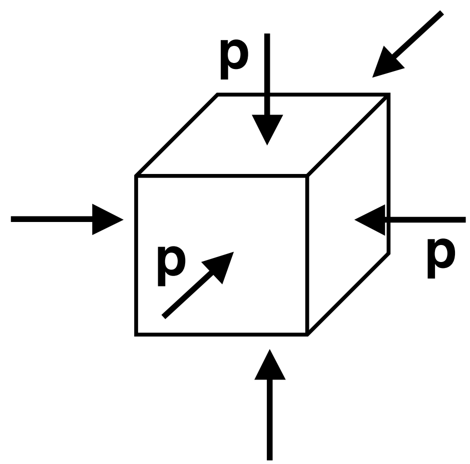
Hydrostatic stress is a state of stress such that:

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = p$$

(principal stresses all equal $\Rightarrow$ no shear)

Experimental data shows no yielding under hydrostatic stress

Figure 5.3-5 **Unit cube under state of hydrostatic stress**



Mohr's circle collapses to a point!

Conclusion: Any yield criterion must not allow yielding under hydrostatic stress/pressure

There are two classic criteria we will consider devised for isotropic materials (σ_{yield} is one value)

The first is the…

Tresca Criterion

(1868)

"Material yields if the maximum shear stress exceeds τ_{yield}"

Generalizing this to three dimensions gives:

$$\left.\begin{array}{c} \left|\sigma_{I} - \sigma_{II}\right| = \sigma_{yield} \\ \underline{\underline{\text{or}}} \\ \left|\sigma_{II} - \sigma_{III}\right| = \sigma_{yield} \\ \underline{\underline{\text{or}}} \\ \left|\sigma_{III} - \sigma_{I}\right| = \sigma_{yield} \end{array}\right\} \text{ for yield}$$

Recall $\sigma_y = 2\tau_y$

Check case of hydrostatic stress:

$$\sigma_{I} = \sigma_{II} = \sigma_{III} = C \Rightarrow \quad \text{no yield}$$

--> Look at the case of plane stress ($\sigma_{III} = 0$):

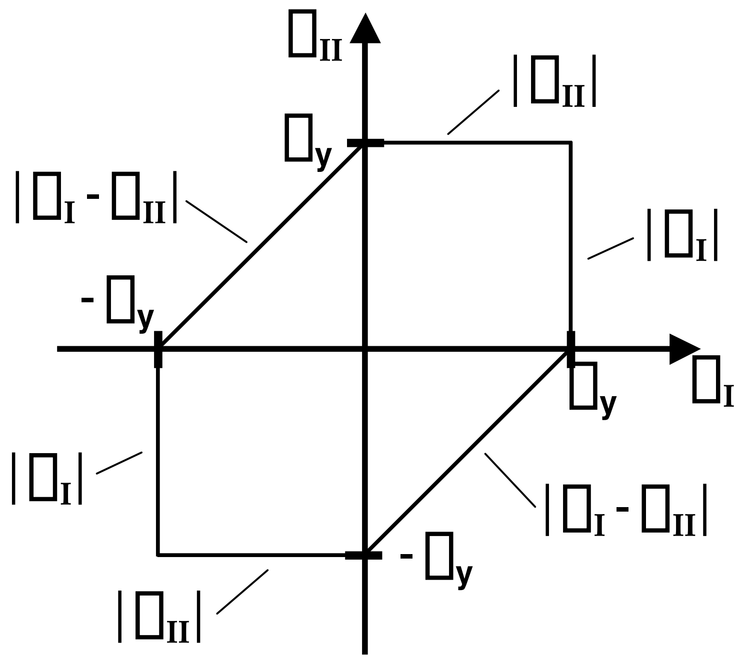
$$\Rightarrow \quad \left|\sigma_{I} - \sigma_{II}\right| = \sigma_y$$

$$\left|\sigma_{I}\right| = \sigma_y$$

$$\left|\sigma_{II}\right| = \sigma_y$$

Plotting the "failure envelope"…..

Figure 5.3-6 **Tresca failure envelope for case of plane stress**



A second criterion is the…

von Mises Criterion

Very similar to Tresca but <u>not</u> discontinuous

$$\left(\sigma_I - \sigma_{II}\right)^2 + \left(\sigma_{II} - \sigma_{III}\right)^2 + \left(\sigma_{III} - \sigma_I\right)^2 = 2\sigma_y^2$$

<u>at yield</u>

- Still the differences of the principal stresses
- Now "sum up" the effects
- Still no yielding for case of hydrostatic stress

--> get a "rounded-off" Tresca

--> Look at case of plane stress ($\sigma_{III} = 0$):

$$\left(\sigma_I - \sigma_{II}\right)^2 + \sigma_{II}^2 + \sigma_I^2 = 2\sigma_y^2$$

$$\left.\Rightarrow \quad \sigma_I^2 - \sigma_I \sigma_{II} + \sigma_{II}^2 = \sigma_y^2 \quad \right\} \text{ at yield}$$

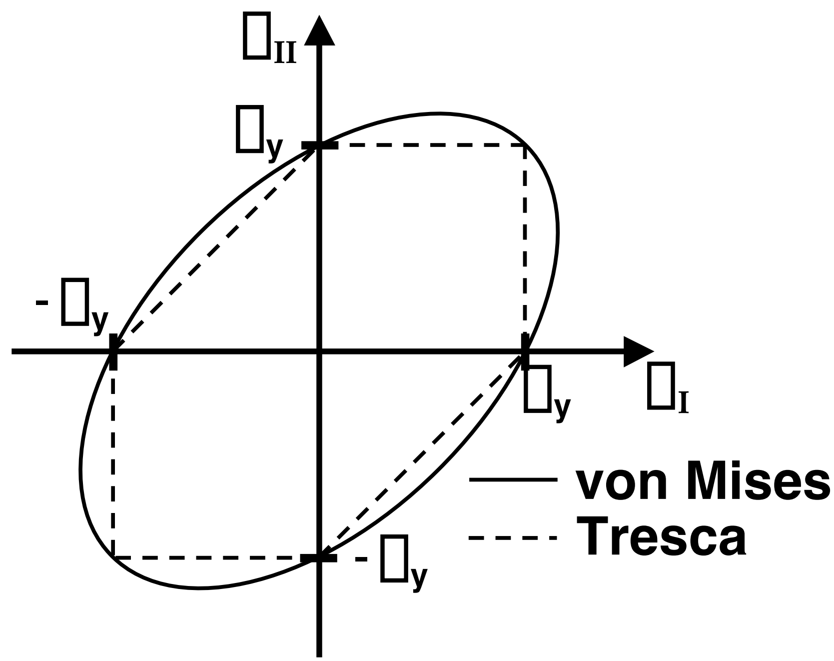
(<u>Note</u>: often write as:

$$\sqrt{\sigma_{11}^2 - \sigma_{11}\sigma_{22} + \sigma_{22}^2 + 3\sigma_{12}^2} = \sigma_y$$

by using transformations to non-principal axes)

Compare to Tresca….

Figure 5.3-7 **Comparison of Tresca and von Mises failure criteria for case of plane strain**



Finally, consider…

Using Yield Criteria

The steps for a general structure are:

1. Analyze structure to obtain stresses (σ_{ij}) and principal stresses (σ_I, σ_{II}, σ_{III})
2. Obtain yields/ultimates via handbook (e.g., MIL HDBK 5, 17) or experimentation
3. Choose yield/failure criterion
4. Utilize calculated stresses in failure/yield criteria with associated material yields/ultimates

NOTE: Failure criteria get far more complex for inhomogeneous, nonisotropic material

Thus far we've concentrated on material failure by yielding. We next look at the phenomenon of *fracture*.

Unit 5.3 (New) Nomenclature

τ_{yield} (τ_y) -- shear yield stress

σ_{yield} -- yield stress

σ_I, σ_{II}, σ_{III} -- principal stresses