Unit M5.4
Other Considerations in Failure

Readings:

A & J 13, 14, 15, 16

CDL 5.9, 5.14, 5.15

A & J 17-27

16.003/004 -- "Unified Engineering"
Department of Aeronautics and Astronautics
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LEARNING OBJECTIVES FOR UNIT **M5.4**

Through participation in the lectures, recitations, and work associated with Unit M5.4, it is intended that you will be able to………

- ….**describe** stress concentrations and their effects
- ….**explain** the basic concepts associated with fracture mechanics
- ….**employ** the basic fracture mechanics model to assess fracture
- ….**discuss** the concept of fatigue and key associated issues

There are many other ways in which a material/structure can fail. We'll look at a few key ones here

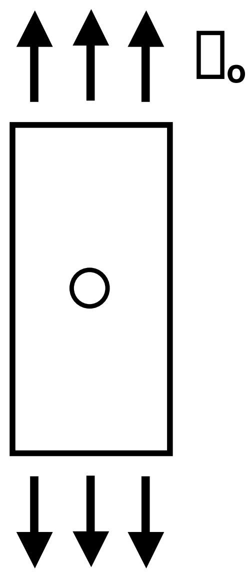
Stress Concentrations

There are often "structural details" that cause the stress to go above the far-field applied value. These are stress concentrations

(K_T - stress concentration)

Example: a hole

Figure M5.4-1 **Piece of material with a hole under stress**



Stress "lines" can't go by hole but must go around it. This causes stress to concentrate at edge of hole

isotropic material: $K_T = 3$ = stress concentration at hole

failure occurs depending on the <u>notch</u> <u>sensitivity</u> of the material:

(Pure) Notch sensitive: failure at $\sigma_o = \dfrac{\sigma_{ult}}{K_T}$

perfectly sensitive to notch

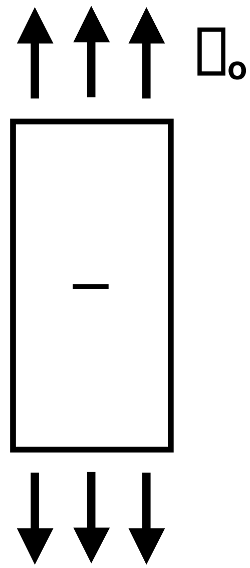
Notch insensitive: failure at $\sigma_o = \sigma_{ult}$

insensitive to presence of notch due to yielding

There are many types of notches. Can find associated stress concentrations via handbooks.

--> Consider the special case of a crack!

Figure M5.4-2 **Piece of material with crack under stress**



Solution shows $K_T = \infty$! (theoretically)

but there is strength. Need to resort to…..

<u>Fracture Mechanics</u>

In the presence of cracks, materials can "fast fracture".

This occurs if there is the proper energy balance:

energy released by fracture process = energy created in new crack surfaces

Griffith Criterion (1923)

$$dW - dU \geq G_c \, dA \quad \text{(Thermobalance)}$$

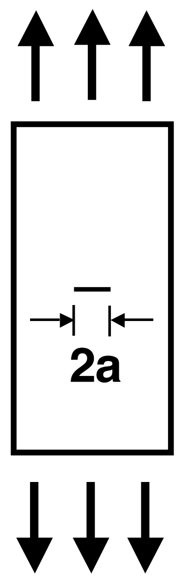
- dW: external work by loads
- dU: internal elastic strain energy
- G_c: energy per unit area of new crack $\left[\frac{energy}{Area}\right] = \left[\frac{J}{m^2}\right]$
- dA: Area of new crack created

This is generally expressed in the more usable form:

$$Y \sigma \sqrt{\pi a} = K$$

- Y: geometrical factor
- σ: stress
- a: half-crack size
- K: stress intensity factor

$\alpha = 1$ for center crack:

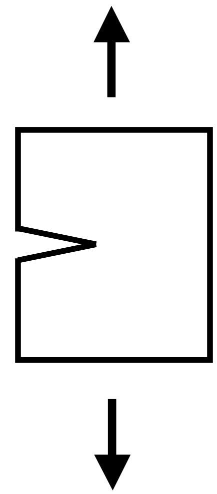


At fracture, $\sigma = \sigma_f$, $K = K_c$ = critical stress intensity factor (also known as fracture toughness)

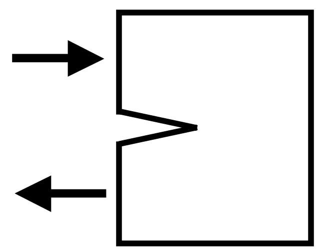
Note: Fracture depends on stress and on size of crack in structure

--> Note **modes** of crack propagation:

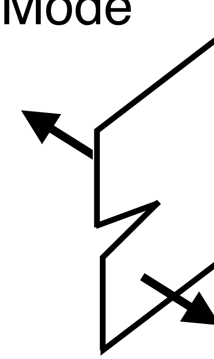
Mode I - Opening Mode



Mode II - Sliding/Shearing Mode



Mode III - Antiplane/Tearing Mode



--> Uses of Fracture Mechanics

A. Find static strength for known crack size

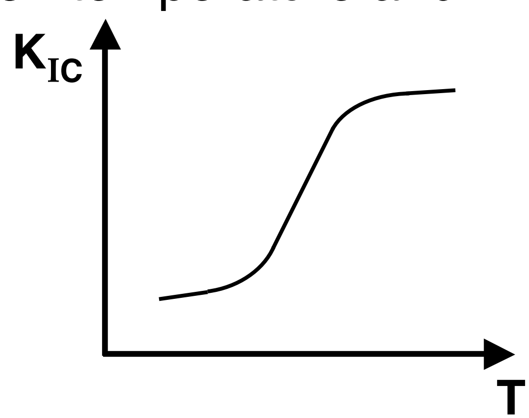
$$\sigma_f = \frac{K_{Ic}}{\sqrt{\pi a}}$$

B. Determine critical crack size in a material

$$2a = \frac{2}{\pi} \left(\frac{K_{Ic}}{\sigma_{ult}} \right)^2$$

--> Notes on K_{Ic}

- "funny units"
  [stress] • [length]$^{1/2}$
- material parameter -- often determines use in tensile field
- glass transition temperature and "Liberty boats"



K_c, G_c are material properties

Thus far, all the "failures" we've talked about have been due to the one-time application of load. However, we must also consider….

Fatigue

--> Definition of *fatigue* - "the tendency of a material to break under repeated stress"

Types of fatigue:

1. Low cycle fatigue
   - number of cycles less than 10^4
   - for originally uncracked (macroscopically) materials
   - massive yielding/damage in each cycle
   - sometimes heat created from energy dissipation

(Example: paper clip)

2. <u>High</u> <u>cycle</u> <u>fatigue</u>
   - number of cycles greater than 10^4
   - for originally uncracked (macroscopically) materials
   - at stresses well below yield/ultimate stress
   - microscopic damage generated and accumulates overtime

   (Example: axles, vibrating parts)

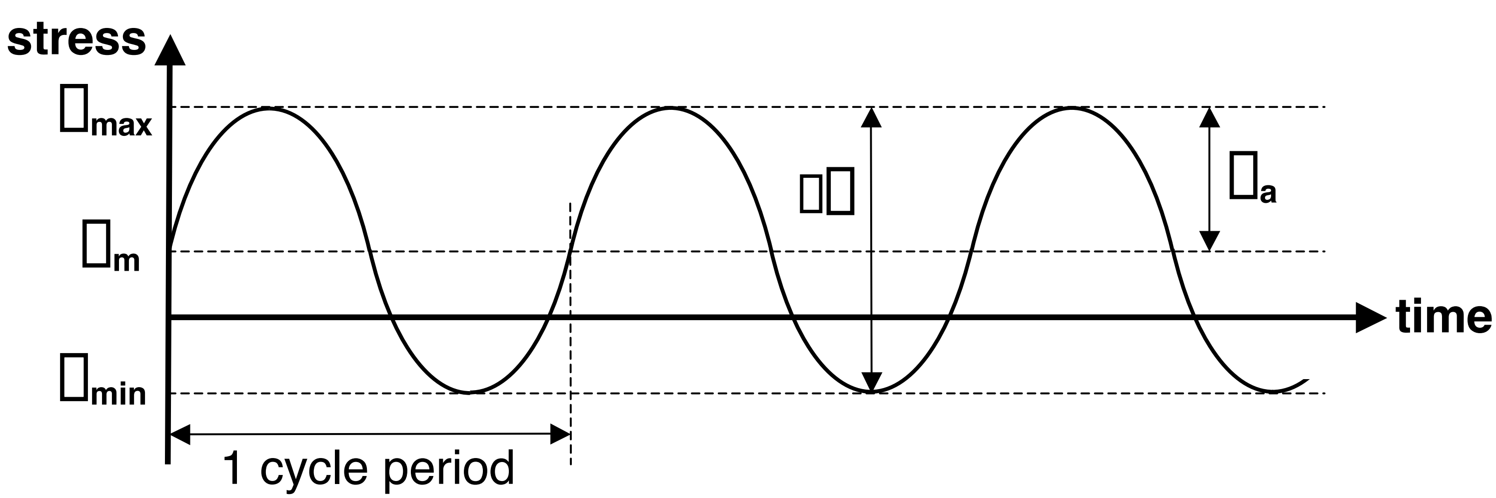
3. <u>Damage growth from stress concentration</u>
   - based on fracture mechanics
   - crack extends on each stress cycle

   (Example: pressure vessels, Aloha 737)

--> <u>Terminology</u> of fatigue

Cyclic stress can be caused by any macroscopic loading (e.g. beam, rod, shaft)

Figure M5.4-3 **Basic stress-time plot and associated fatigue terminology**



σ_{max} = maximum cyclic stress

σ_{min} = minimum cyclic stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \text{cyclic stress amplitude}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \text{mean stress}$$

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \text{stress ratio}$$

<u>Note</u>: only two needed to define loading

Also:

N = Number of cycles

N_f = Number of cycles to failure

$$\text{cyclic frequency} = \frac{1}{\text{cyclic time}} \quad [Hz]$$

--> Characterization of fatigue

There are two ways in which fatigue is characterized….

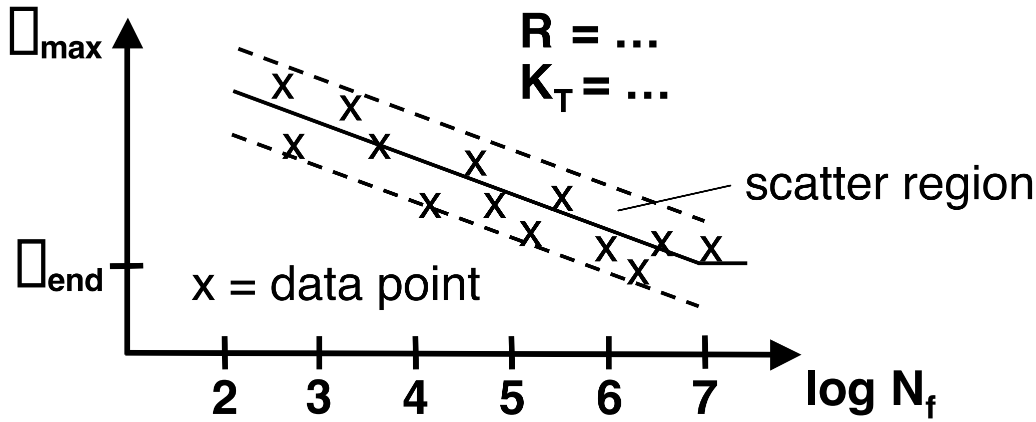
A. S-N Diagrams

"Classically", there is no macroscopic manifestation of fatigue until the last cycle at which it breaks (N_f)

So, fatigue of materials is explored experimentally:

1. Define a stress ratio, R
2. Set a σ_{max} value
3. Test material under defined stress cycle until failure
   → N_f determined experimentally
4. Repeat steps 2 and 3 for multiple values of σ_{max}
5. Plot results on an "*S-N diagram*"

Figure M5.4-4 **Typical S-N diagram**



Much data collected…..

- Done for different values of stress concentration, K_T (N_f decreases as K_T increases)
- Substantial scatter (can be orders of magnitude)
  $\Rightarrow$ significant uncertainty
- Much lower strength at larger number of cycles
  (so plotted on log scale for cycles)
- Same materials/cases have stress endurance limits (no fatigue failure) generally defined at $N > 10^7$
- Not defined for low cycles (generally $< 10^3 - 10^4$)
- Will find S-N curves with different defining stress parameters
  (recall, two needed to define cycle; σ_{max} and R used in Figure M5.4-4)
- Results depend on material, stress applied, stress concentration

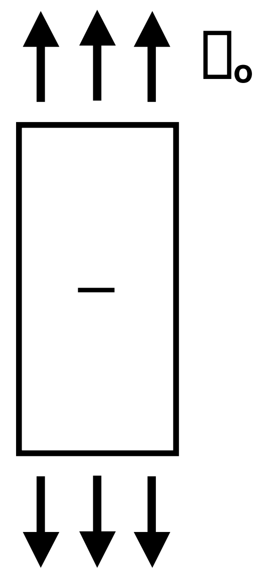
--> Multiple mechanisms at work

B. Crack Growth Rules

This approach is based on fracture mechanics and considers a macroscopic growth and its "*self-similar*" growth (growth maintaining the same shape)

Again, experimental data is key

1. Begin with a defined crack
2. Set at σ_{max} and σ_{min} values
3. Test material under desired stress cycle
4. Measure crack length (determine growth) at specified cycle
5. Plot data and correlate using a "growth law"



--> Most common: "Paris Law"

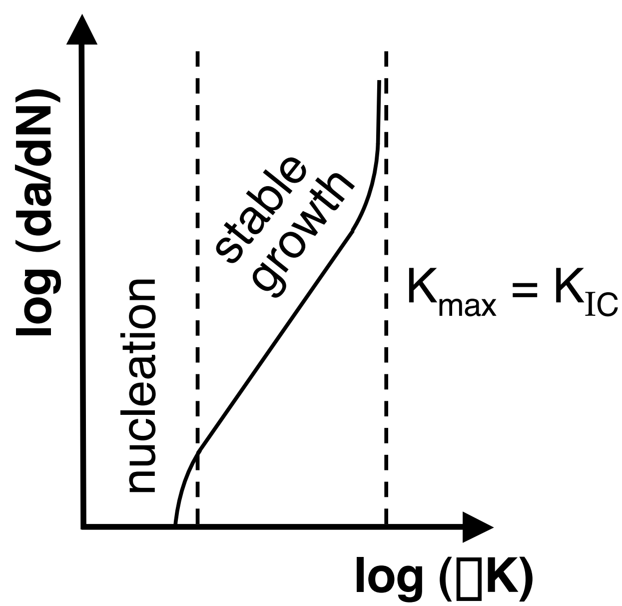
$$\frac{da}{dN} = A(\Delta K)^m$$

where ΔK is change in stress intensity factor:

$$\Delta K = \Delta\sigma\sqrt{\pi a}$$

This is generally plotted on a log-log scale to get fits to determine A and n

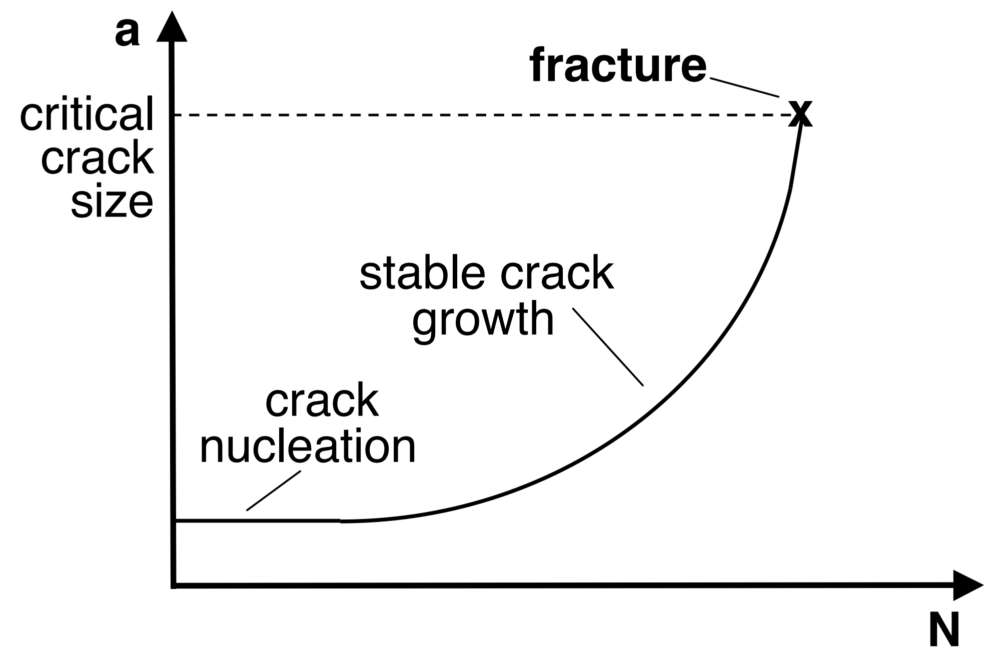
Figure M5.4-5 **Typical crack growth plot**



This consists of three areas/phases:

- nucleation (crack is forming as initial crack is not macroscopic)
- stable growth region (governed by law)
- fast fracture (maximum stress intensity approaches critical stress intensity)

Plot this with regard to cycles:



start depends on initial a

--> **Design** for fatigue (cyclic loading)

Just as there are two ways that fatigue is characterized, there are two ways to design for fatigue/cyclic loading tied to the principle associated with the characterization

A. Safe-Life Design

- Assumes that initial part is perfect
- Life determined by time to initiate and propagate damage
- Based on S-N curves and "Miner's Rule"
- Uses life (scatter) factor of 4

--> Basics of Miner's Rule

- for a given stress cycle, damage equals 1 (D = 1) at failure cycle (N_f)
- if N cycles occur at this stress cycle, damage caused is ratio:

$$D = \frac{N}{N_f}$$

- Damage can be added (for different types of stress cycles)

$$\text{total damage} = \sum_{i}^{\#\text{ cycle types}} \left(\frac{N}{N_f} \right)_i$$

- When sum of damage equals 1, failure occurs
- Divide by 4 to get "safe life"
- Retire part when it reaches "safe life"

B. Damage Tolerant Design

- Assumes that cracks are present
- Uses inspection (visual, non-destructive) to determine maximum initial crack size
- Based on crack growth modes/laws to determine growth for sets of cyclic load
- Specify next inspection and maximum crack there should be
- Maintain experted crack size below critical crack size
- (often) use factor of 2 concerning number of cycles

Additional Items

- Corrosion and the environment
- Wear

Unit 5.4 (New) Nomenclature

a -- (half) crack length
G_c -- energy per unit area of new crack
Hz -- Hertz (frequency)
K_I -- stress intensity factor in mode I
K_{Ic} -- critical stress intensity factor in mode I (a.k.a. “fracture toughness”)
K_T -- stress concentration
N -- number of stress cycle
N_f -- number of stress cycles to failure
R -- stress ratio
U -- internal elastic strain energy
W -- external work
$\Delta\sigma$ -- change in cyclic stress
Y -- geometrical factor-in fracture mechanics equation
σ_a -- cyclic stress amplitude
σ_{end} -- endurance limit stress
σ_m -- mean stress
σ_{max} -- maximum cyclic stress
σ_{min} -- minimum cyclic stress