UNIFIED HANDOUT

MATERIALS AND STRUCTURES - #M-10

Spring, 2007

Concept Review Sheet

for Unified Q4M: Units M4.1-4.5

OVERALL ASPECTS re/ SOLVING STRUCTURAL PROBLEMS

- The 15 equations of elasticity are always applicable.
- Modeling assumptions of three types are made for the structural member: on geometry, on the loading/stress state, on the displacement/strain state.
- All surfaces have boundary conditions either on stresses or displacements.
- There are basically two approaches to solving structural problems as modeled: exact/analytical and numerical techniques.
- St. Venant's Principle tells us that "far away" from boundaries, the general solution holds.

THE ROD/BAR

- A rod is long compared to its other two dimensions (L >> b,h).
- The load applied is along the axial direction (long dimension).
- There are no variations in the directions perpendicular to the long dimension.
- The cross-section deforms uniformly.
- The pertinent internal resultant is the axial load, F.
- The axial resultant load at any location along the rod is equipollent to the axial stress acting and integrated over the "cut" surface.
- The pertinent structural stiffness is the modulus of the material along the long dimension times the cross-sectional area, EA.
- There is a slight inconsistency in the model when applying the equations of elasticity with regard to the displacements and strains in the directions perpendicular to the long direction.

THE BEAM

- A beam is long compared to its other two dimensions (L >> b,h).
- The load applied is perpendicular to the long dimension.
- The two pertinent internal resultants are the Shear, S, and the Moment, M.
- The change in the Shear resultant is equal to the applied loading.
- The change in the Moment resultant is equal to the Shear resultant.
- The only significant stresses have components in the direction along the long dimension.
- The centroid is the center of area.

- Plane sections remain plane and perpendicular to the midplane (centroid line) after bending displacement (*Bernouilli-Euler Hypothesis*).
- All displacements and deformations can be defined through the displacement of the midplane/centroid line, w.
- The shear resultant at any location along the beam is equipollent to the shear stress acting and integrated over the "cut" surface.
- The moment resultant at any location along the beam is equipollent to the moment caused by the axial stress acting and integrated over the "cut" surface.
- The area (second) moment of inertia, I, defines the geometric contribution of the structural configuration to the resistance to bending deformation.
- The extensional modulus defines the material contribution of the structural configuration to the resistance to bending deformation.
- The total bending stiffness is EI.
- Maximum magnitude of axial stress occurs at a point farthest from the centerline/centroid of the beam.
- Shear stress is related the the (first) moment of area about the centerline/centroid, Q.
- Maximum magnitude of shear stress occurs at a point in the cross-section where Q divided by the width is maximized.
- There is no axial stress or strain due to bending at the centerline/centroid.
- Stress is compressive (negative) on the top surface and tensile (positive) on the bottom surface when a beam bends up.
- Stress is tensile (positive) on the top surface and compressive (negative) on the bottom surface when a beam bends down.
- The moment of inertia of a body about any axis is the moment of inertia of the body about its centroid plus its area times the square of the distance from its centroid to the axis (*Parallel Axis Theorem*).
- The efficiency of a cross-section in carrying bending load is measured by the ratio of moment of inertia to area.

GENERAL ITEMS WITH REGARD TO "1-D" STRUCTURAL MODELS

- The assumptions of the model result in limitations for application of the models.
- The internal resultant(s) can be determined by applying equilibrium for a virtual "cut" cross-section.
- The pertinent deformation is related to loading divided by a structural stiffness which has a material parameter and a geometrical parameter of the cross-section.
- The pertinent stress is related to loading times a cross-sectional distance divided by a geometrical parameter of the cross-section.
- The use of these models involves six steps: 1. Drawing the Free Body diagram; 2. Determining the reactions; 3. Determining the internal stress resultant(s); 4. Determining the major deflection parameter as related to the pertinent structural stiffness; 5. Determining the pertinent stresses; 6. Determining related strains and deflections/deformations as needed.
- The six parts can be solved sequentially for a statically determinate case or simultaneously for a statically indeterminate case.
- The solutions are linear and superposition can be used.
- The models can be "extended" slightly to allow for variations in dimensions and directions not strictly allowed.
- The key in the use of all models is **consistency**.