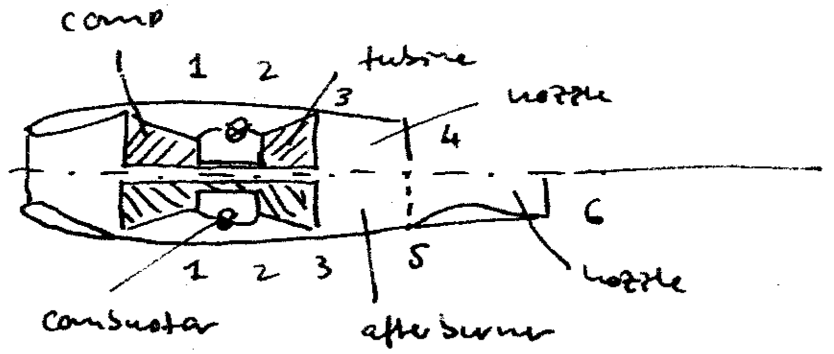
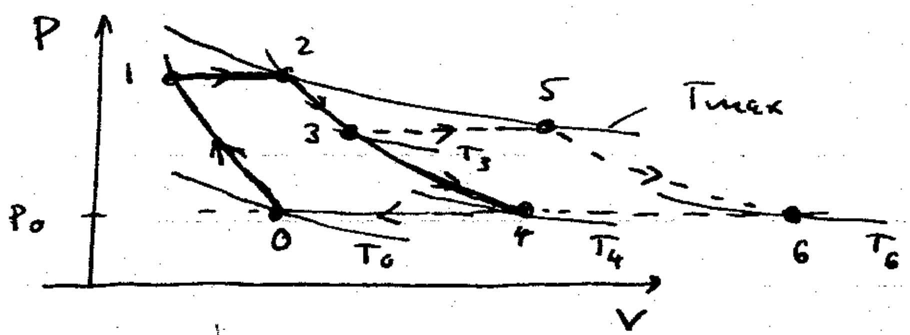
Spring 2007 ZSP

# Problem T1




Concepts: 1st law
ad. rev. exp. / compr
thermal efficiency

a)




$$\frac{T_1}{T_0} = 2$$

$$\frac{T_{max}}{T_0} = 10$$

b) $\eta_{th} = \frac{w_{net}}{q_A} = 1 - \frac{T_0}{T_1}$ * ; $\underline{\eta_{th} = 1/2}$

c) $w_{net} = q_A \cdot \eta_{th} = c_p(T_2 - T_1)\cdot \eta_{th} = c_p T_0 \left( \frac{T_{max}}{T_0} - \frac{T_1}{T_0} \right) \cdot \frac{1}{2} = \underline{4 c_p T_0}$

d) see dashed lines on fig a) note $T_5 = T_2 = T_{max}$

e) shaft work balance: $w_T = w_C$ ; $c_p(T_2 - T_3) = c_p(T_1 - T_0)$

$T_3 = T_2 - T_1 + T_0 = T_{max} - T_1 + T_0$ ; $\underline{T_3 = 9 T_0}$

f) $\frac{P_6}{P_5} = \frac{P_4}{P_3} \xrightarrow[\text{process}]{\text{ad-rev}} \frac{T_6}{T_5} = \frac{T_4}{T_3}$ , $T_6 = T_{max} \frac{T_4}{T_2} \cdot \frac{T_2}{T_3}$

also $\frac{T_4}{T_2} = \frac{T_0}{T_1}$ (ad. rev. process) so $T_6 = \frac{T_{max}}{T_0} \frac{T_0}{T_1} \cdot \frac{T_{max}}{9 T_0} \cdot T_0 = \underline{\frac{50}{9} T_0}$

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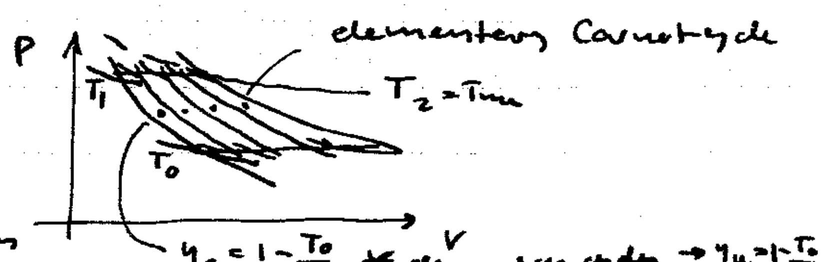
*: Brayton cycle (ideal) thermal efficiency
→ use elementary Carnot (c) cycles to find $\eta_{th}$ Brayton



Problem $T_2$ Spring 2007, ZSP

concepts: 1st and 2nd law of thermo (Gibbs)

a) $T_1 \rightarrow T_2$ show that $\Delta S_{12}|_{p=const} > \Delta S_{12}|_{v=const}$

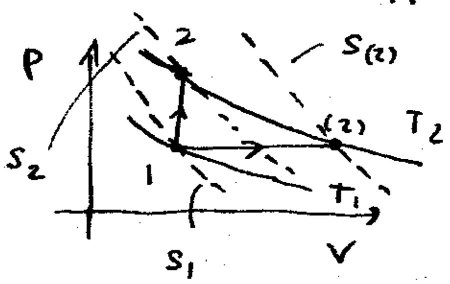
Gibbs: $T\,ds = du + p\,dv$
$= dh - v\,dp$ (entropy is a state var.!)

$p = const$: $dp = 0$
$\hookrightarrow T\,ds = dh = c_p\,dT$ ; $ds = c_p \frac{dT}{T}$

$v = const$: $dv = 0$
$\hookrightarrow T\,ds = du = c_v\,dT$ ; $ds = c_v \frac{dT}{T}$

$$\frac{\Delta S_{12}|_p}{\Delta S_{12}|_v} = \frac{c_p \ln\left(\frac{T_2}{T_1}\right)}{c_v \ln\left(\frac{T_2}{T_1}\right)} = \frac{c_p}{c_v} \rightarrow \frac{\Delta S_{12}|_p}{\Delta S_{12}|_v} = \gamma > 1 \text{ q.e.d.}$$

b)




$ds = 0$ : $c_v\,dT = -p\,dv$
(-----) $c_p\,dT = v\,dp$

so $\frac{c_p}{c_v} = -\frac{v}{dv}\frac{dp}{p}$ ; $\frac{dp}{p} = -\gamma \frac{dv}{v} \rightarrow pv^\gamma = const$

c) $p_1 \rightarrow p_2$ show $\Delta S_{12}|_{T=const} = (1-\gamma)\,\Delta S_{12}|_{v=const}$

$T = const$: $dT = 0 \rightarrow T\,ds = -v\,dp$ ; $ds = -\frac{vp}{T}\frac{dp}{p} = -R\frac{dp}{p}$

$v = const$: $dv = 0 \rightarrow T\,ds = c_v\,dT$

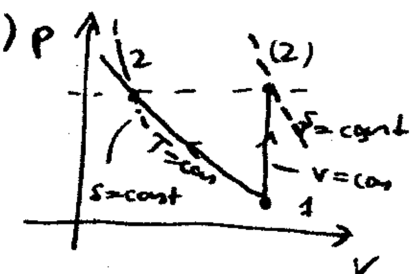
$\boxed{\text{equ. of state}}$

$pv = RT \rightarrow \frac{dp}{p} + \cancel{\frac{dv}{v}}^{0} = \frac{dT}{T}$

$\Bigg\}\; ds = c_v \frac{dT}{T} = c_v \frac{dp}{p}$

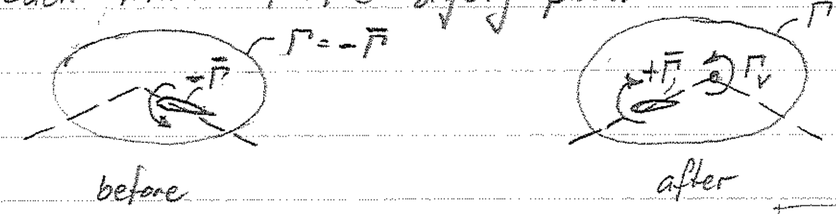
$$\frac{\Delta S_{12}|_T}{\Delta S_{12}|_v} = \frac{-R}{c_v} = \frac{-(c_p - c_v)}{c_v} = 1 - \gamma \text{ q.e.d.}$$

d)



UE Fluids    Problem 1 Solution    Spring '07

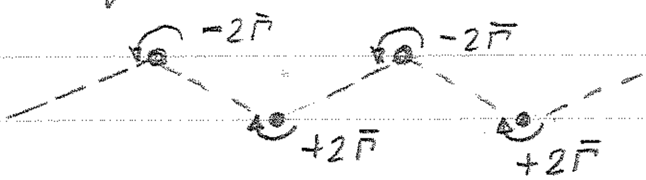
1) A vortex will be shed whenever $\Gamma(t)$ changes. This occurs at each kink of the zigzag path.



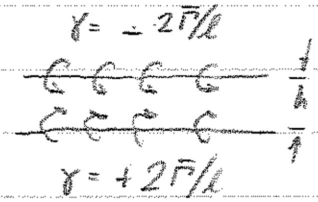

Must have $+\bar{\Gamma} + \Gamma_v = -\bar{\Gamma}$

$\Rightarrow \boxed{\Gamma_v = -2\bar{\Gamma}}$

Mirror image occurs on bottom kinks, with $\boxed{\Gamma_v = +2\bar{\Gamma}}$

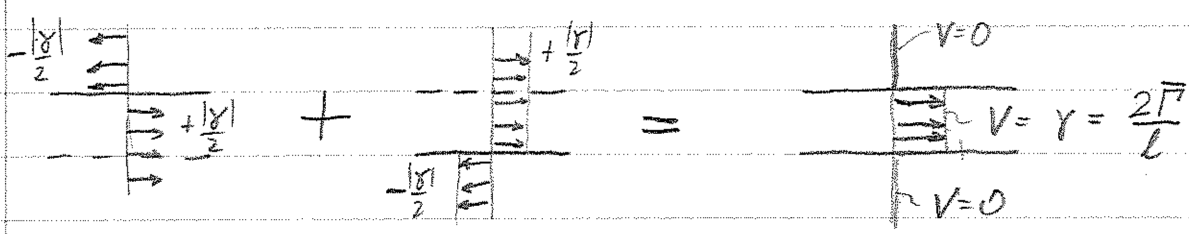



2) Averaged sheet strength is $\gamma = \dfrac{\Gamma_v}{\ell} = \pm \dfrac{2\bar{\Gamma}}{\ell}$

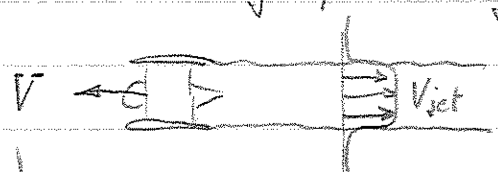



The sheet spacing is $h$, same as the vortices

3) Superimpose the velocities of the two sheets.




The velocity between the sheets looks like the jet from an engine, seen by stationary observer:




In the frame of the dolphin (airplane), the jet is just a velocity excess: