UE Fluids Problem 2 Solution Spring '07

1) In general, the velocity seen by moving observer is $\vec{V}_{app} = \vec{V} - \vec{V}_{obs}$

In this case, $\vec{V}_{obs} = \dot{Z}\hat{k}$, so $\boxed{\vec{V}_{app}(x) = V_\infty \hat{\imath} + \left( V_\infty \alpha - \underset{\uparrow \text{ new}}{\dot{Z}} - \int_0^c \frac{\gamma \, d\xi}{2\pi(x-\xi)} \right) \hat{k}}$

2) From notes, $\hat{n} = -\frac{dZ}{dx}\hat{\imath} + \hat{k}$. This is just geometry, unchanged by motion.

So $\vec{V}_{app} \cdot \hat{n} = 0 \Rightarrow -V_\infty \frac{dZ}{dx} + V_\infty \alpha - \dot{Z} - \int_0^c \frac{\gamma \, d\xi}{2\pi(x-\xi)} = 0$

or $$\boxed{V_\infty \left( \alpha - \frac{dZ}{dx} \right) - \underset{\uparrow \text{ new}}{\dot{Z}} - \int_0^c \frac{\gamma \, d\xi}{2\pi(x-\xi)} = 0}$$

3) First define $\alpha_{app} = \alpha - \frac{\dot{Z}}{V_\infty}$, so $V_{app} \cdot \hat{n} = 0$ equation becomes

$$V_\infty \left( \alpha_{app} - \frac{dZ}{dx} \right) - \int_0^c \frac{\gamma \, d\xi}{2\pi(x-\xi)} = 0$$

This is exactly the same as for $\dot{Z} = 0$ standard case, with $\alpha$ replaced by $\alpha_{app}$.

Hence $c_\ell = 2\pi \alpha_{app} = 2\pi \left( \alpha - \dot{Z}/V_\infty \right)$

Now we have $\frac{\partial c_\ell}{\partial \alpha} = 2\pi$, and $\boxed{\frac{\partial c_\ell}{\partial \dot{Z}} = \frac{2\pi}{V_\infty}}$

QE Fluids  Problem 3+4 Solution  Spring '07

1) $$\frac{dz}{dx} = \begin{cases} 0.25\left(0.8 - 2\frac{x}{c}\right) = -0.05 + 0.25\cos\theta & \text{for } 0 < \frac{x}{c} < 0.4,\ 0 < \theta < 1.3694 \equiv \theta_c \\ 0.111\left(0.8 - 2\frac{x}{c}\right) = -0.0222 + 0.111\cos\theta & \text{for } 0.4 < \frac{x}{c} < 1,\ 1.3694 < \theta < \pi \end{cases}$$

Perform Fourier Analysis on $f = -\frac{dz}{dx}$, add $\alpha$ to $A_0$ separately.

$$A_0 - \alpha = \frac{1}{\pi}\int_0^{\pi} -\frac{dz}{dx}\,d\theta = \frac{-1}{\pi}\left\{\int_0^{\theta_c}(-0.05 + 0.25\cos\theta)\,d\theta + \int_{\theta_c}^{\pi}(-0.0222 + 0.111\cos\theta)\,d\theta\right\}$$

$$A_0 - \alpha = \frac{-1}{\pi}\left\{\left(-0.05\theta + 0.25\sin\theta\right)\Big|_0^{\theta_c} + \left(-0.0222\theta + 0.111\sin\theta\right)\Big|_{\theta_c}^{\pi}\right\} = -0.00903$$

$$\therefore\ A_0 = -0.00903 + \alpha \quad (\alpha \text{ in radians!})$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos\theta\,d\theta = \frac{2}{\pi}\left\{\int_0^{\theta_c}(-0.05 + 0.25\cos\theta)\cos\theta\,d\theta + \int_{\theta_c}^{\pi}(-0.0222 + 0.111\cos\theta)\cos\theta\,d\theta\right\}$$

Using $\cos^2\theta = \frac{\cos 2\theta + 1}{2}$:

$$A_1 = \frac{2}{\pi}\left\{\left(-0.05\sin\theta + 0.25\left(\frac{\sin 2\theta}{4} + \frac{\theta}{2}\right)\right)\Big|_0^{\theta_c} + \left(-0.0222\sin\theta + 0.111\left(\frac{\sin 2\theta}{4} + \frac{\theta}{2}\right)\right)\Big|_{\theta_c}^{\pi}\right\}$$

$$A_1 = 0.1629$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{dz}{dx}\cos 2\theta\,d\theta = \frac{2}{\pi}\left\{\int_0^{\theta_c}(-0.05 + 0.25\cos\theta)\cos 2\theta\,d\theta + \int_{\theta_c}^{\pi}(-0.0222 + 0.111\cos\theta)\cos 2\theta\,d\theta\right\}$$

Using $\cos\theta\cos 2\theta = \frac{\cos 3\theta + \cos\theta}{2}$:

$$A_2 = \frac{2}{\pi}\left\{\left(-0.05\frac{\sin 2\theta}{2} + 0.25\left(\frac{\sin 3\theta}{6} + \frac{\sin\theta}{2}\right)\right)\Big|_0^{\theta_c} + \left(-0.0222\frac{\sin 2\theta}{2} + 0.111\left(\frac{\sin 3\theta}{6} + \frac{\sin\theta}{2}\right)\right)\Big|_{\theta_c}^{\pi}\right\}$$

$$A_2 = 0.02774$$

Note: Could have used numerical integration for $A_0, A_1, A_2$

a) $$C_\ell = \pi(2A_0 + A_1) = 2\pi\left(A_0 + \tfrac{1}{2}A_1\right) = 2\pi(\alpha + \underbrace{0.07242}_{-\alpha_{L=0}}) \Rightarrow \boxed{\alpha_{L=0} = -0.07242 = 4.15^\circ}$$

b) $$\boxed{C_\ell(\alpha = 3^\circ) = 2\pi\left(3\frac{\pi}{180} + 0.07242\right) = 0.7840}$$

cont'd

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2) $\boxed{C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = \pi(-0.03379) = -0.1062}$

$\boxed{x_{cp}/c = \frac{1}{4} - \frac{C_{m_{c/4}}}{C_\ell} = \frac{1}{4} - \frac{-0.1062}{0.7840} = 0.3854}$

3)

| | XFOIL | | | TAT |
|---|---|---|---|---|
| | 4412 | 4407 | 4402 | 44xx |
| $\alpha_{L=0}$ | $-4.21°$ | $-4.19°$ | $-4.12°$ | $-4.15°$ |
| $C_\ell(\alpha=3°)$ | 0.8715 | 0.8357 | 0.7949 | 0.784 |

TAT appears to be quite accurate at predicting $\alpha_{L=0}$ for all three

TAT underpredicts $C_\ell$ by $100\%\left(\frac{0.784}{0.8715} - 1\right) = -10\%$ for the 4412

$100\%\left(\frac{0.784}{0.8357} - 1\right) = -6.2\%$ for the 4407

$100\%\left(\frac{0.784}{0.7949} - 1\right) = -1.4\%$ for the 4402

TAT clearly gets more accurate as the airfoil thickness decreases. This is expected, since one main assumption of TAT is that the airfoil is thin. Another assumption is that the camber is small, but this does not appear to cause significant errors here.

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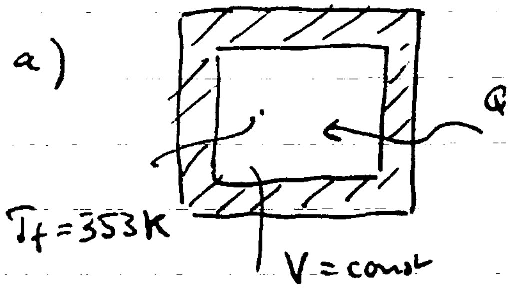
Pr. $T_3$ concepts : 1st and 2nd laws, state changes, entropy

Ideal gas with $c_v = 5/2\, R$, $c_p = 7/2\, R$, $m = 1$ kg

Initial state : $p_i = 1$ bar, $T_i = 293$ K $\qquad R = 287$ J/kg K

a)




rigid container $\rightarrow$ $\Delta U = Q$, $W = 0$
no work

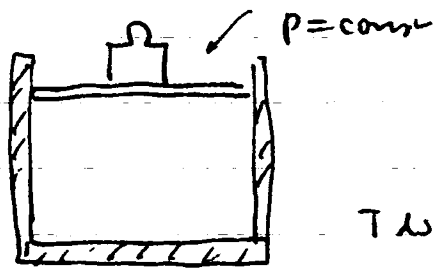
and $\Delta U = m c_v \Delta T \quad \left( c_v = \frac{\partial u}{\partial T}\Big|_v \right)$

$\Delta H = m c_p \Delta T \quad \left( c_p = \frac{\partial h}{\partial T}\Big|_p \right)$

$T ds = du + p\,\underbrace{dv}_{0} \rightarrow \Delta S = m c_v \ln\left(\frac{T_f}{T_i}\right)$

so
$\Delta U = 1 \cdot 5/2 \cdot R \cdot 60 = 43.05$ kJ $\qquad Q = \Delta U = 43.05$ kJ $\qquad W = 0$
$\Delta H = 1 \cdot 7/2\, R \cdot 60 = 60.27$ kJ $\qquad \Delta S = 133.7$ J/K

b)




same change in temp. $\rightarrow$ $\Delta U = 43.05$ kJ
$\Delta H = 60.27$ kJ

$T ds = dh - v dp \rightarrow \Delta S = m c_p \ln\left(\frac{T_f}{T_i}\right) = 187.1$ J/K

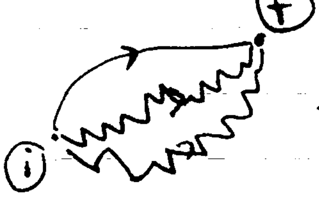
rev. work : $W = \int p dV = p(V_f - V_i) = R(T_f - T_i) = \underline{17.22 \text{ kJ}}$

$Q = \Delta U + W \Rightarrow \underline{60.27 \text{ kJ}}$ (check $Q = \Delta H$ ✓)

c)


U, H and S are state variables
$\rightarrow$ $\Delta U$, $\Delta H$, $\Delta S$ are the <u>same</u> for any process between (i) and (f)
heat and work are path dependent
$\rightarrow$ Q and W are <u>not the same</u>

d)




same answer as c) since processes are between the same states independent of irreversibility / reversibility

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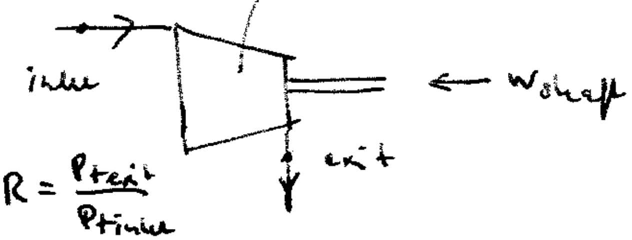
## Problem T4

isothermal compressor (ideal)




$PR = \dfrac{P_{t\,exit}}{P_{t\,inlet}}$

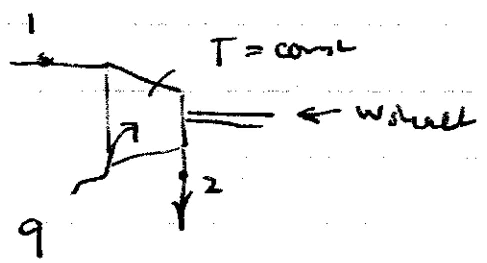
Concepts: Gibbs, entropy changes for ideal gas

a) Gibbs $\quad T\,ds = dh - v\,dp \quad$, isothermal ideal gas $\rightarrow dh = c_p dT$
$\quad dT = 0 \rightarrow dh = 0$

$$ds = -\frac{vP}{T}\frac{dp}{P}$$

$$\underline{\Delta s = -R \ln(PR)}$$ note: entropy of the fluid decreases

b)




1st law: $\underbrace{\Delta u}_{0} = +q - \underbrace{\left(-W_{shaft} + \cancel{p_2}\overbrace{V_2}^{RT} - \cancel{p_1}\overbrace{V_1}^{RT}\right)}_{\int p\,dv}$

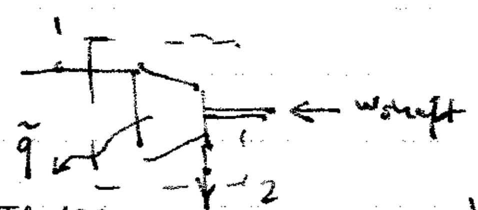
$\rightarrow \quad q = -W_{shaft}$

also $\quad q = \int_1^2 p\,dV = RT \ln\left(\frac{V_2}{V_1}\right) = RT\left(\frac{P_1}{P_2}\right)$

so $\quad \underline{q} = -RT \ln\left(\frac{P_2}{P_1}\right) = \underline{-RT \ln(PR)}$ $\quad$ heat must be rejected!

[OR: rev. $T\,ds = dq \quad q = T\Delta s$ ; $\underline{q = -RT\ln(PR)}$ ]

c) CV form of 1st law (for a change, since used CM above!)




$\tilde{q} = +RT\ln(PR)$

$$0 = -\tilde{q} + W_{shaft} + h_1 - h_2 \quad , \quad h_1 = h_2$$

$$\underline{W_{shaft} = RT\ln(PR)}$$ [OR see CM approach in b)]

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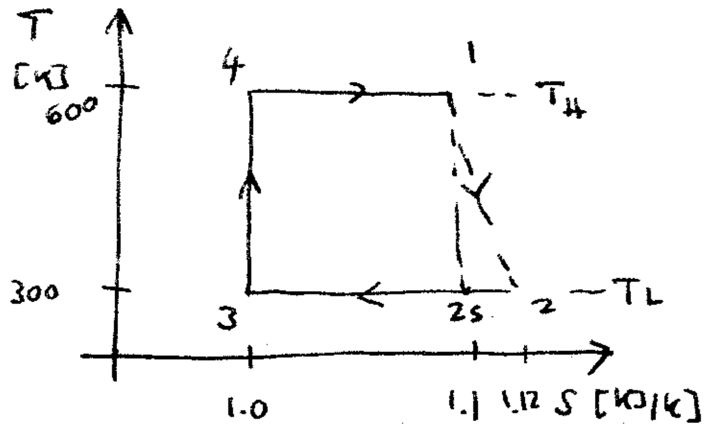
## Problem T5

Concepts: T–s diagram, internally irrev. processes, Gibbs (1st + 2nd law)




heat exchange reversible along $\underbrace{4 \to 1}_{q \text{ absorbed}}$ and $\underbrace{2 \to 3}_{q \text{ rejected}}$

a) net work $\neq \oint T\,ds$ because irreversible process $1 \to 2$!

1st law: $0 = q_A + q_R - W_{net}$

$$q_A = T_H(s_1 - s_4) = 60 \text{ kJ} \quad , \quad q_R = T_L(s_3 - s_2) = -36 \text{ kJ}$$

so $W_{net} = 60 - 36 = \underline{24 \text{ kJ}}$

b) definition $\eta_{th} = \dfrac{W_{net}}{q_A} = \dfrac{24}{60} = \underline{0.4}$

c) no irrev: have state 2s and Carnot cycle!

$q_A$ unaltered but $\tilde{q}_R = T_L(S_3 - S_{2s}) = \underline{-30 \text{ kJ}}$

d) $q_A = 60 \text{ kJ}$

e) $W_C = q_A + \tilde{q}_R = \underline{30 \text{ kJ}}$

f) $\eta_C = \dfrac{W_C}{q_A} = \underline{0.5}$ $\left[ \text{OR} \quad \eta_C = 1 - \dfrac{T_L}{T_H} = \underline{0.5} \right]$

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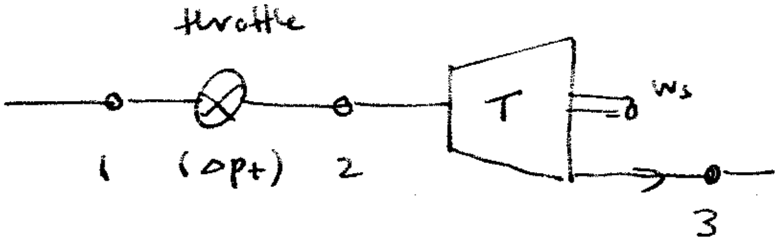
## Problem T6

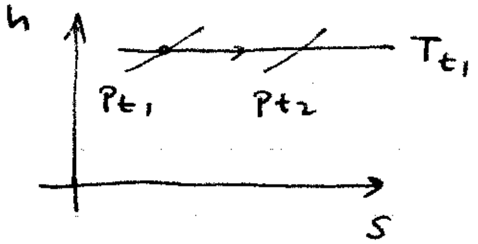
Concepts: throttling process (free expansion!) 1st law and 2nd law




$P_{t_1} = 10\,\text{bar}$, $T_{t_1} = 600\text{C}$

$P_{t_3} = 1\,\text{bar}$

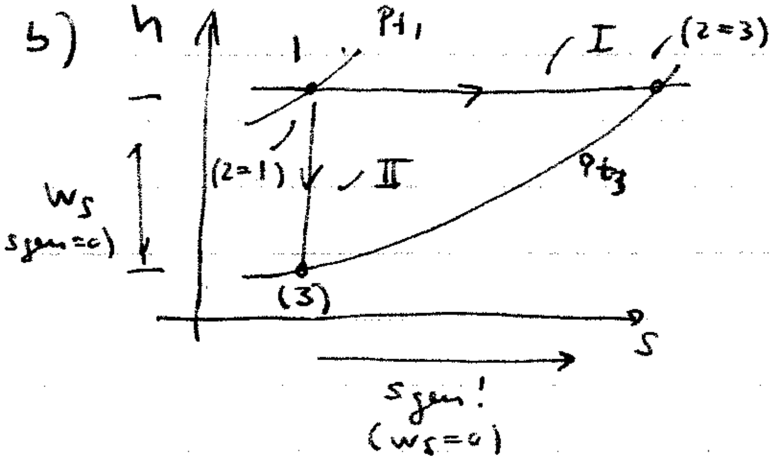
a)




no useful work done, no heat transfer (adiabatic) $\rightarrow h_{t_1} - h_{t_2} = 0$


(1st law CV form) but dissipation!

b)




I: $P_{t_2} = P_{t_3}$ no turbine work at all

$\rightarrow$ lost all work could have extracted in turbine

II: $P_{t_1} = P_{t_2}$ no throttling

$\rightarrow$ extract all possible work

c) II is isentropic expansion $\quad w_s = c_p (T_{t1} - T_{t3})$

$$T_{t3} = T_{t1}\left(\frac{P_{t3}}{P_{t1}}\right)^{\frac{\gamma-1}{\gamma}} \qquad w_s = c_p T_{t_1}\left(1 - \left(\frac{P_{t3}}{P_{t_1}}\right)^{\frac{\gamma-1}{\gamma}}\right) = \underline{420.8\ \text{kJ/kg}}$$

d) dissipation in throttle: $\quad T\,ds = dh - v\,dp$

$$T_t\,ds = dh_t - v_t\,dp_t \quad , \; dh_t = 0$$

$$ds = -\frac{v_t p_t}{T_t}\frac{dp_t}{p_t} = -R\frac{dp_t}{p_t} \qquad \Delta s_{1\to2} = -R\ln\left(\frac{P_{t_2}}{P_{t_1}}\right)$$

$P_{t_2} = P_{t3}$ so $\quad \Delta s_{1\to2} = R\ln\left(\frac{P_{t1}}{P_{t3}}\right) = \underline{660.8\ \text{J/kg K}}$

($\Delta s_{1\to2} = \Delta s_{total}$ irreversible process)