Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 3

Name: ________________________

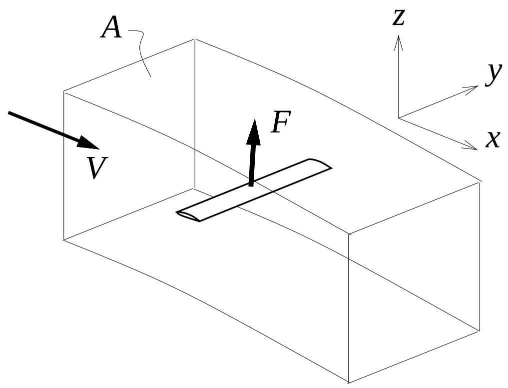
Due Date: 02/27/2007

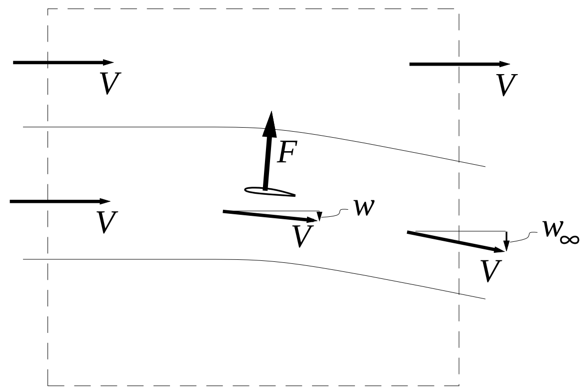
	Time Spent (min)
F5	
F6	
T7	
T8	
T9	
S1	
Study Time	

Announcements:

Unified Engineering Spring 2007
Fluids Problem 5

A somewhat crude but simple way to approximate the downwash velocity behind a lifting wing is to assume that the wing acts only on a streamtube of air of some area A. Since the wing has a lift force F on it, it will push down on the streamtube with a force $-F$, and thus impart it with a downward vertical velocity w_∞. This is analogous to a knife deflecting a faucet stream, but sideways.





a) Apply the z-momentum integral equation

$$\oiint \rho \left(\vec{V}\cdot\hat{n}\right) w \, dA \;+\; \oiint p \, n_z \, dA \;+\; F_z \;=\; 0$$

to the large dashed control volume shown, in order to determine w_∞ in terms of A, L, ρ, V. Assume $w_\infty \ll V$, which is true for almost all wings.
Note: In this model, only the mass flow in the streamtube is deflected, and the rest stays along the freestream direction. Also, the pressure far from the wing is constant at some p_∞.

b) The pressure field of the wing which deflects the streamtube is roughly symmetric fore/aft. Hence, about half of the deflection will occur in front of the wing, and the rest will occur behind the wing. So the downwash velocity at the wing itself will be about $w = w_\infty/2$. Determine the induced angle α_i, and the induced drag D_i on the wing. Use the $w_\infty \ll V$ assumption to simplify your results as much as possible.

c) The actual induced drag on a lifting wing is known to be

$$D_i = \frac{L^2}{\frac{1}{2}\rho V^2 \, \pi b^2 e}$$

where $e \simeq 1$ is the *span efficiency* (we will derive this expression later). Propose an estimate for the streamtube area A which the wing effectively acts on.

A wing has the following piecewise-linear circulation distribution:

$$\Gamma(y) \; = \; \begin{cases} \Gamma_0(1 - 2y/b) & , \quad y > 0 \\ \Gamma_0(1 + 2y/b) & , \quad y < 0 \end{cases}$$

a) The wing is operating at velocity V_∞ and air density ρ. Determine the lift L of this wing.

b) Determine the strength $\gamma(y)$ of the vortex sheet shed by this wing.

c) Determine the downwash velocity $w(y)$ produced by the $\gamma(y)$ distribution. Sketch $\Gamma(y)$, $\gamma(y)$, and $w(y)$. You may assume $\Gamma_0 = 1$ and $b = 2$ for plotting purposes.

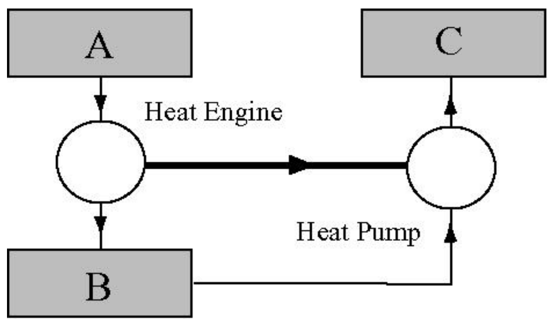
d) Write down the spanwise integral for the the induced drag D_i of this wing (but don't bother to evaluate it — this would normally be done numerically). Plot the integrand versus y to see how D_i is distributed along the span for this particular $\Gamma(y)$.

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T7

Three identical finite bodies of constant heat capacity are initially at temperatures T_A = 300 K, T_B = 100 K, T_C = 300 K. No heat or work is supplied from outside. A heat engine is run between bodies A and B which drives a heat pump between bodies B and C. The purpose of the heat engine / heat pump system is to raise the temperature of body C. The heat engine is run until bodies A and B come to the same final temperature, T_f = 150 K.



Assume that the thermal heat capacity of each of the bodies is 1000 kJ/K and that the heat exchange process takes place in a series of quasi-equilibrium states.

a) How much heat is transferred from body A?

b) What is the ***net*** heat transferred to body B?

c) What is the final temperature of body C?

d) What is the change in entropy of each body? Indicate the change with proper sign.

e) What is the change in entropy of the overall system consisting of the three bodies, the heat engine and the heat pump? Is the overall process reversible? Why or why not?

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T8

Air flows in steady-state in an insulated duct, in which the static pressure is uniform. The velocity at the inlet is 150 m / s. The duct area increase between inlet and exit is large enough such that the velocity at the exit is 10 m / s. The static temperature at inlet is 300K. The specific heats of air can be taken as c_p = 1000 J / kg-K, c_v = 714 J / kg-K.

a) What is the stagnation temperature at inlet?

b) What is the stagnation temperature at exit?

c) What is the entropy change (per kg) from inlet to exit?

d) What is the ratio of densities at inlet and exit?

e) Is the flow process reversible? Explain in quantitative terms why or why not.

f) Represent the various states in the process in a *T-s* diagram. Indicate static and stagnation states (both temperature and pressure) at inlet and exit. Show the constant static pressure line.

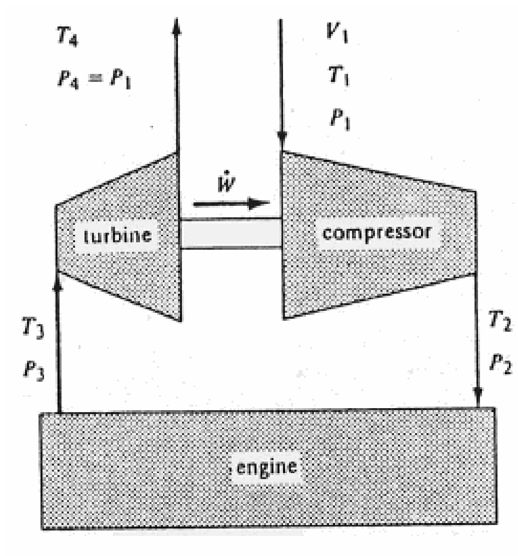
g) If the flow were reversible, what would be the static pressure ratio between the duct exit and the duct inlet assuming that the inlet and exit velocities are the same as before?

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T9

The compressor of a turbo-charger shown below has an inlet flow rate of 7.5 m^3/minute of air as measured at the inlet condition. This condition is P_1 = 1.013 bar and T_1 = 288 K. The compressor has a pressure ratio of 1.5 and an adiabatic efficiency of 80%. The compressor is driven directly by an exhaust turbine with an adiabatic efficiency of 70%. The turbo-charger and engine are arranged so that both the turbine and the compressor operate under conditions of steady flow. The turbine pressure ratio is the same as that for the compressor and the mass of engine fuel may be neglected. Assume air and exhaust gas are a perfect gas with ? = 1.4 and R = 287 J/kg-K.



a) Sketch the compression in the compressor and the expansion in the turbine in an *h-s* diagram. Indicate the shaft work and how the inlet and exit pressures of the turbine relate to the ones of the compressor.

b) Calculate the inlet temperature to the turbine.

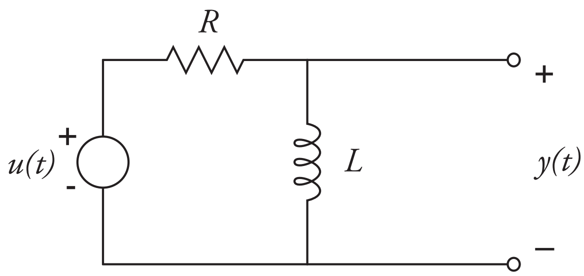
c) Calculate the turbine exit temperature.

d) Calculate the power produced by the turbine.

Unified Engineering II **Spring 2007**

Problem S1 (Signals and Systems)

1. Find and plot the step response of the system



where $R = 1\ \Omega$, and $L = 2$ H.

2. For the input signal

$$u(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t < 2 \\ -1, & t \geq 2 \end{cases}$$

find and plot the output $y(t)$, using superposition.