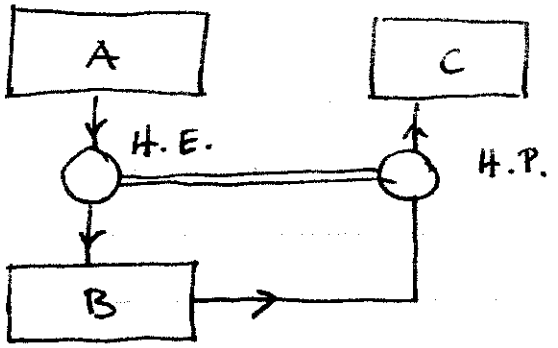
Spring 2007, ZSP

Problem T7



Concepts: entropy changes, 1st and 2nd laws

Initially: $T_{A_i} = 300K$

$T_{B_i} = 100K$

$T_{C_i} = 300K$

Final state: $T_f = 150K$ (A, B)

all heat exchange in quasi-equilibrium manner

a) 1st law: $dU = -dQ$ $\qquad Q_A = -\int_{T_{A_i}}^{T_f} C\,dT = C(T_{A_i} - T_f)$

$\underline{Q_A = 150\ MJ}$

b) $dU = +dQ_{net}$, $\qquad Q_{B_{net}} = \int_{T_{B_i}}^{T_f} C\,dT = C(T_f - T_{B_i})$

$\underline{Q_{B_{net}} = 50\ MJ}$

c) system: H.E. plus H.P. $\qquad 0 = Q_A - Q_{Bnet} - Q_C$ (1st law)

$Q_C = C(T_{cf} - T_{c_i})$ $\qquad Q_c = Q_A - Q_{Bnet}$

Find $T_{cf} = T_{c_i} + \frac{Q_A - Q_{Bnet}}{C}$ $\qquad \underline{T_{cf} = 400K}$

d) $dS = \frac{dQ}{T}$ (quasi-equil.!) $\qquad \Delta S_A = C \ln\left(\frac{T_f}{T_{Ai}}\right) = \underline{-693.2\ kJ/K}$

$\Delta S_B = C \ln\left(\frac{T_f}{T_{Bi}}\right) = \underline{+405.5\ kJ/K}$

$\Delta S_C = C \ln\left(\frac{T_f}{T_{c_i}}\right) = \underline{+287.7\ kJ/K}$

e) $\Delta S_{total} = \Delta S_A + \Delta S_B + \Delta S_c + \cancel{\Delta S_{H.P.}}^{0} + \cancel{\Delta S_{H.E.}}^{0}$, $\qquad \Delta S_{total} = C \ln \frac{T_f^2 \cdot T_{fc}}{T_{A_i} \cdot T_{B_i} \cdot T_{C_i}}$

$\Delta S_{total} = C \ln \frac{150^2 \cdot 400}{300^2 \cdot 100} = C \ln(1)$ $\qquad \Delta S_{total} = 0$! (quasi-equil.)

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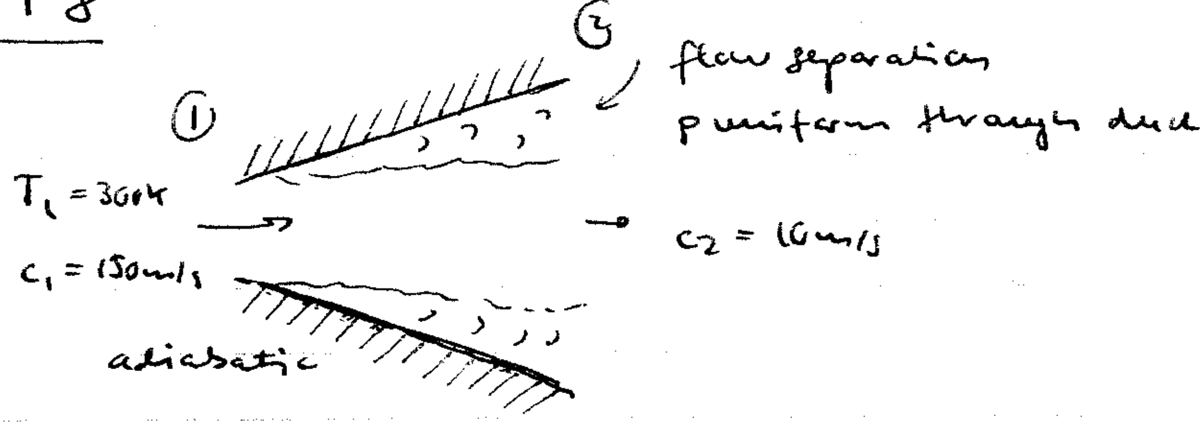
Problem T8

ideal gas



concepts: 1st law CV form, entropy generation, T-s diagrams

a) $T_{t_1} = T_1 + \frac{c_1^2}{2c_p}$ $\qquad$ $\underline{T_{t_1} = 311.25 K}$

b) 1st law $\quad 0 = h_{t_1} - h_{t_2} \longrightarrow T_{t_2} = T_{t_1}$ $\qquad$ $\underline{T_{t_2} = 311.25 K}$

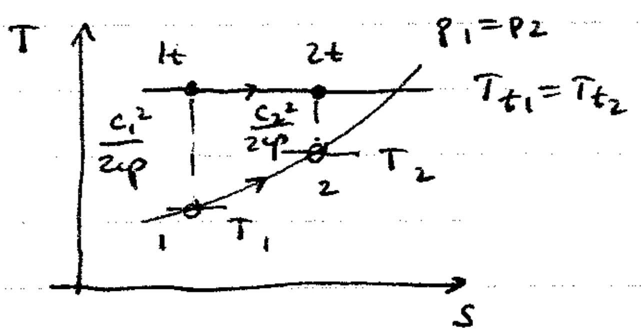
c) Gibbs $\quad T ds = dh - v \cancel{dp}^{\,0}$; $\quad ds = c_p \frac{dT}{T}$ $\qquad \Delta s_{12} = c_p \ln \frac{T_2}{T_1}$

$T_2 = T_{t_2} - \frac{c_2^2}{2c_p} = 311.2 K \longrightarrow$ $\underline{\Delta s_{12} = 36.65\ J/kgK}$

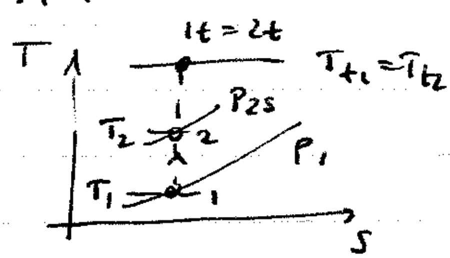
d) $p = \rho R T$ $\quad$ so $\quad \frac{\rho_2}{\rho_1} = \frac{T_1}{T_2}$, $\quad \underline{\frac{\rho_2}{\rho_1} = 0.964}$
$p = \text{const}$

e) $\Delta s_{total} = \Delta s_{gen} = \Delta s_{1-2} > 0$ $\qquad$ <u>irreversible</u>
($T ds = dq$ – no heat transfer)

f)



if reversible



g) $\frac{p_{2s}}{p_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$, same T_2 since same $\frac{c_2^2}{2c_p}$, $\quad \underline{\frac{p_{2s}}{p_1} = 1.137}$
$(T_{t_2} = T_{t_{2s}} = T_{t_1})$

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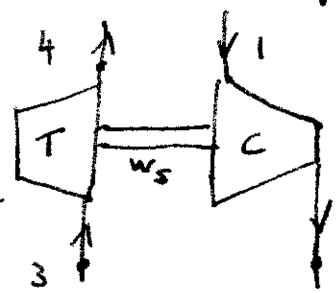
Problem T9

$P_4 = P_1$

T_4

$\eta_{ad}^{T} = 0.7$

P_3, T_3



$\dot{V}_1 = 7.5\,m^3/min$

$P_1 = 1.013$ bar

$T_1 = 288\,K$

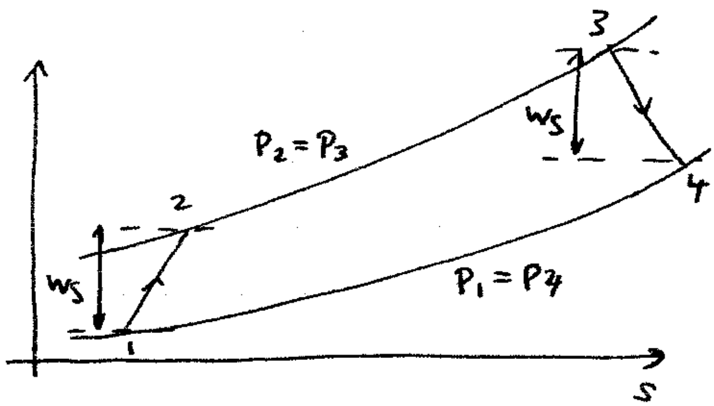
$\eta_{ad}^{c} = 0.8$

P_2, T_2

Concepts: 1st law (CV)
shaft power balance
adiabatic eff.
h-s diagrams

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = 1.5 = \Pi$$

a)



shaft power balance

$h_3 - h_4 = w_s = h_2 - h_1$

b) $$w_s = h_2 - h_1 = c_p T_1 \left(\Pi^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{ad}^{c}}$$ since $\eta_{ad}^{c} = \dfrac{h_{2s} - h_1}{h_2 - h_1}$

$T_{2s} = T_1 \, \Pi^{\gamma-1/\gamma}$

$$w_s = h_3 - h_4 = c_p T_3 \left(1 - \frac{1}{\Pi^{\frac{\gamma-1}{\gamma}}} \right) \cdot \eta_{ad}^{T}$$ since $\eta_{ad}^{T} = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$

$T_{4s} = T_3 \left(\frac{1}{\Pi} \right)^{\frac{\gamma-1}{\gamma}}$

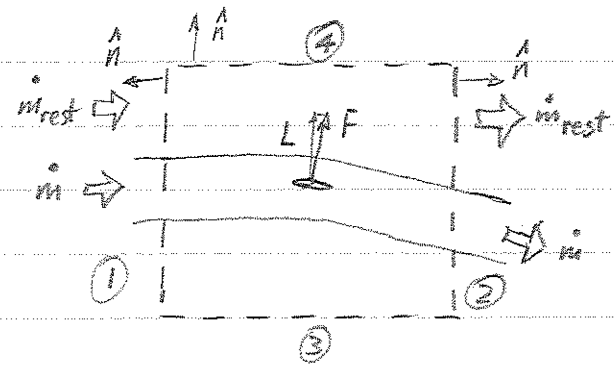
$$T_3 = T_1 \frac{\Pi^{\frac{\gamma-1}{\gamma}} - 1}{1 - \Pi^{\frac{1-\gamma}{\gamma}}} \cdot \frac{1}{\eta_{ad}^{c}\,\eta_{ad}^{T}} = \underline{577.5\,K}$$

c) $$T_4 = T_3 - \eta_{ad}^{T} T_3 \left(1 - \Pi^{\frac{1-\gamma}{\gamma}} \right)$$ $\underline{T_4 = 533.2\,K}$

d) $$P = \rho_1 \dot{V}_1 c_p T_1 \left(\Pi^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{ad}^{c}}$$

$$P = \frac{P_1}{R} \dot{V}_1 c_p \left(\Pi^{\frac{\gamma-1}{\gamma}} - 1 \right) \frac{1}{\eta_{ad}^{c}}$$ $\underline{P = 6.8\,kW}$

since $\rho_1 = \dfrac{P_1}{R T_1}$



a) C.V. Analysis, $F_z = L$

$\oint p\hat{n}\,dA = 0$ since $p = p_\infty = \text{constant}$

$\oint \rho(\vec{V}\cdot\hat{n})\,w\,dA = \iint_1 + \iint_2 + \iint_3 + \iint_4$

$\iint_1 = 0$ since $w = 0$

$\iint_2 = 0$ since $\vec{V}\cdot\hat{n} = 0$

$\iint_3 = 0$ since $\vec{V}\cdot\hat{n} = 0$

$+\ \iint_4 = \dot{m}\,w_\infty + \dot{m}_{rest}\cdot 0 = \dot{m}\,w_\infty = \rho V A w_\infty$

$$\rho V A w_\infty + L = 0 \implies \boxed{w_\infty = \frac{-L}{\rho V A}}$$

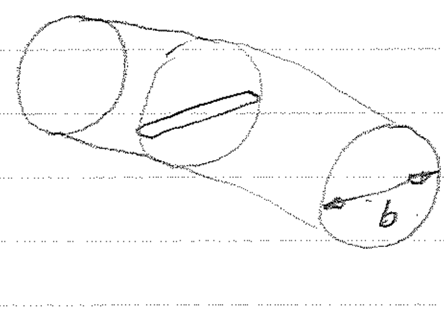
b) $w_i = \dfrac{w_\infty}{2} = \dfrac{-L}{2\rho V A}$, $\boxed{\alpha_i = \arctan\left(\frac{-w_i}{V}\right) \approx \frac{-w_i}{V} = \frac{L}{2\rho V^2 A}}$

$$\boxed{D_i = \alpha_i L = \frac{L^2}{2\rho V^2 A}}$$

c) Pick A so that our model matches the correct D_i:

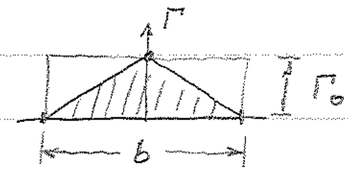
$$\frac{L^2}{2\rho V^2 A} = \frac{L^2}{\frac{1}{2}\rho V^2 \pi b^2 e} \implies \boxed{A = \frac{\pi}{4} b^2 e}$$

Area A is that of a circular cylinder of diameter $b\sqrt{e} \approx b$

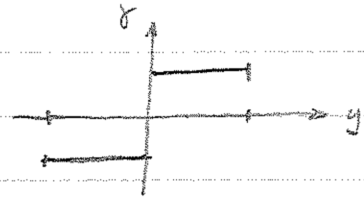


UE Fluids PS6 Solution S'07

a) $L = \int_{-b/2}^{b/2} \rho V_\infty \Gamma \, dy = \frac{1}{2} \rho V_\infty \Gamma_o$ graphically:



b) $\gamma = -\frac{d\Gamma}{dy} = \begin{cases} +2\Gamma_o/b \,, & y>0 \\ -2\Gamma_o/b \,, & y<0 \end{cases}$

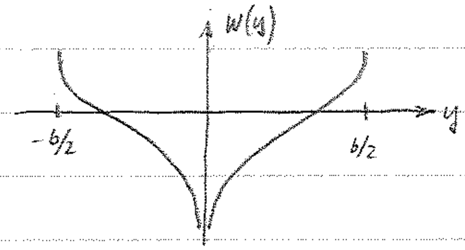


c) $w_i(y_o) = \int_{-b/2}^{b/2} \frac{-\gamma(y)\,dy}{4\pi(y-y_o)}$

$$w_i(y_o) = \frac{-\Gamma_o}{2\pi b}\int_0^{b/2} \frac{dy}{y-y_o} + \frac{\Gamma}{2\pi b}\int_{-b/2}^{0} \frac{dy}{y-y_o}$$

$$= \frac{-\Gamma_o}{2\pi b}\left(\ln|y-y_o|\right)\Big|_0^{b/2} + \frac{\Gamma_o}{2\pi b}\left(\ln|y-y_o|\right)\Big|_{-b/2}^{0}$$

$$w_i(y_o) = \frac{\Gamma_o}{2\pi b}\left[\ln\frac{|y_o|}{|b/2-y_o|} + \ln\frac{|y_o|}{|b/2+y_o|}\right]$$

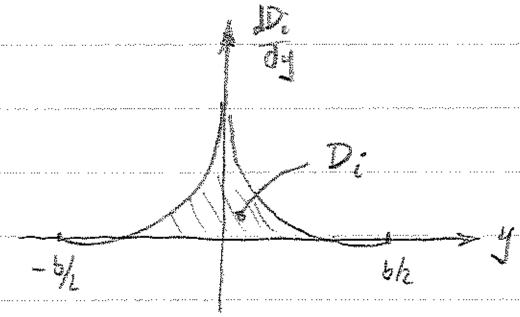


d) $\alpha_i(y) = \frac{-w(y)}{V}$

$$D_i = \int L' \alpha_i \, dy = \int \rho V \Gamma \alpha_i \, dy = \int_{-b/2}^{b/2} -\rho w_i \Gamma \, dy$$

Integrand is $\frac{dD_i}{dy} = -\rho w_i(y)\Gamma(y)$

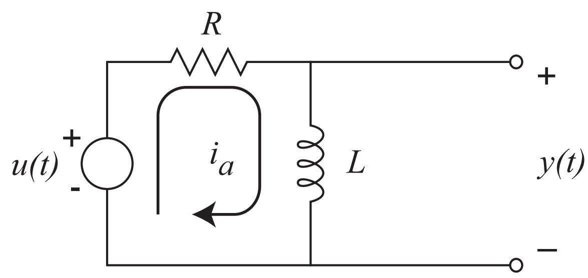
$$\frac{dD_i}{dy} = \begin{cases} \frac{\rho\Gamma_o^2}{2\pi b}\left[\ln\frac{|y|}{|b/2-y|} + \ln\frac{|y|}{|b/2+y|}\right]\left(1-\frac{2y}{b}\right), & y>0 \\ \frac{\rho\Gamma_o^2}{2\pi b}\left[\ln\frac{|y|}{|b/2-y|} + \ln\frac{|y|}{|b/2+y|}\right]\left(1+\frac{2y}{b}\right), & y<0 \end{cases}$$



Unified Engineering II **Spring 2007**

Problem S1 (Signals and Systems) SOLUTION

1. Find and plot the step response of the system



where $R = 1\ \Omega$, and $L = 2$ H.

Solution: Use the loop method to solve. Define the single loop current to be i_a. The sum of the voltage drops around the loop is

$$0 = -u(t) + Ri_a + L\frac{di_a}{dt}$$

Plugging in numbers, and putting inhomogenous terms on the right, we have

$$2\frac{di_a}{dt} + i_a = u$$

Simplify by dividing by 2 to obtain

$$\frac{di_a}{dt} + \frac{1}{2}i_a = \frac{1}{2}u$$

We want to find the step response, so assume that the input is $u(t) = \sigma(t)$. Then for $t \geq 0$,

$$\frac{di_a}{dt} + \frac{1}{2}i_a = \frac{1}{2}$$

To find the response, must find the particular solution and the homogenous solution.

First, find the particular solution. Since the input is a constant, guess that the solution is a constant, $i_p(t) = c$. Since the time derivative of a constant is zero, the differential equation becomes

$$\frac{1}{2}c = \frac{1}{2}$$

and so $i_p(t) = c = 1$.

Next, find the homogenous solution. The homogeneous equation is

$$\frac{di_h}{dt} + \frac{1}{2}i_h = 0$$

which has solution

$$i_h(t) = Ae^{-t/2}$$

for some constant A.

The total solution is

$$i_a(t) = i_p(t) + i_h(t) = 1 + Ae^{-t/2}$$

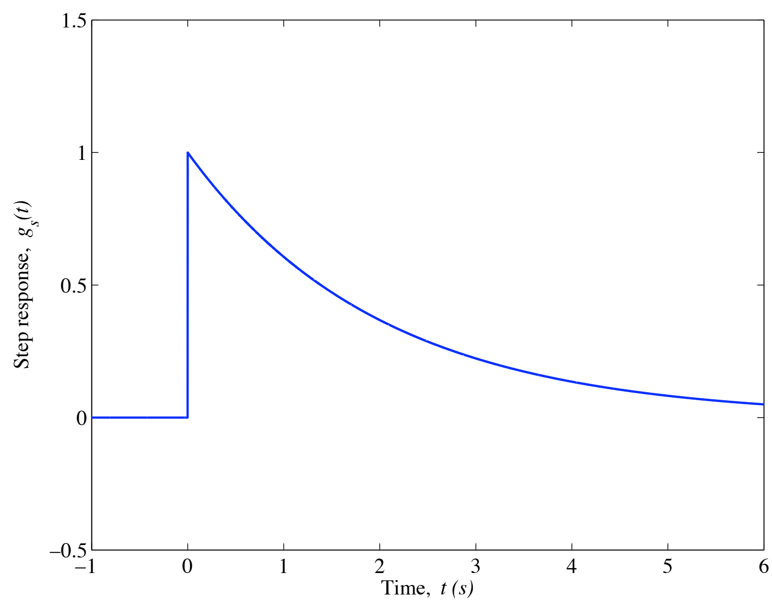
In order to satisfy the initial condition that $i_a(0) = 0$, we must have that $A = -1$. Therefore,

$$i_a(t) = 1 - e^{-t/2}$$

for $t \geq 0$, and zero otherwise. The output $y(t)$ is given by $y = u - i_a R$. Therefore, the step response is

$$\begin{aligned} g_s(t) &= \left(1 - \left(1 - e^{-t/2}\right)\right)\sigma(t) \\ &= e^{-t/2}\sigma(t) \end{aligned}$$

See the plot below:



2. For the input signal

$$u(t) = \begin{cases} 0, & t < 0 \\ 2, & 0 \leq t < 2 \\ -1, & t \geq 2 \end{cases}$$

find and plot the output $y(t)$, using superposition.

Solution: The input can be written as a sum of steps, as

$$u(t) = 2\sigma(t) - 3\sigma(t-2)$$

Therefore, by linearity and time invariance, the output can be written as

$$y(t) = 2g_s(t) - 3g_s(t-2)$$

Plugging in, we have

$$y(t) = 2e^{-t/2}\sigma(t) - 3e^{-(t-2)/2}\sigma(t-2)$$

This may be simplified by breaking the result down into regions:

$$y(t) = \begin{cases} 0, & t < 0 \\ 2e^{-t/2}, & 0 \le t < 2 \\ 2e^{-t/2} - 3e^{-t/2+1}, & t \ge 2 \end{cases}$$

See the plot below: