Problem T7

Concepts: entropy change; 1st and 2nd laws

Initially: \( T_{A_i} = 300 \, \text{K} \)
\( T_{B_i} = 100 \, \text{K} \)
\( T_{C_i} = 300 \, \text{K} \)

Final state: \( T_f = 150 \, \text{K} \) \((A, B)\)

All heat exchange in quasi-equilibrium manner.

a) 1st law: \( du = -dQ \)
\[ Q_A = -\int dQ = C (T_{A_i} - T_f) \]
\[ Q_A = 150 \, \text{MJ} \]

b) \( du = +dQ_{net} \)
\[ Q_{B_{net}} = \int C dT = C (T_f - T_{B_i}) \]
\[ Q_{B_{net}} = 50 \, \text{MJ} \]

c) system: H.E. plus H.P. \( 0 = Q_A - Q_{B_{net}} - Q_C \) (1st law)
\[ Q_C = C (T_{cf} - T_{ci}) \]
\[ Q_C = Q_A - Q_{B_{net}} \]

Find \( T_{cf} = T_{ci} + \frac{Q_A - Q_{B_{net}}}{C} \)
\[ T_{cf} = 400 \, \text{K} \]

d) \( ds = \frac{dQ}{T} \) (quasi-equil.)
\[ \Delta S_A = C \ln \left( \frac{T_f}{T_{A_i}} \right) = -693.2 \, \text{kJ/K} \]
\[ \Delta S_B = C \ln \left( \frac{T_f}{T_{B_i}} \right) = +405.5 \, \text{kJ/K} \]
\[ \Delta S_C = C \ln \left( \frac{T_f}{T_{C_i}} \right) = +287.7 \, \text{kJ/K} \]

e) \( \Delta S_{total} = \Delta S_A + \Delta S_B + \Delta S_C + \Delta S_{H.P.} + \Delta S_{H.E.} \)
\[ \Delta S_{total} = C \ln \left( \frac{T_f^2 \cdot T_{C_f}}{T_{A_i} \cdot T_{B_i} \cdot T_{C_i}} \right) \]
\[ \Delta S_{total} = C \ln \left( \frac{150^2 \cdot 100}{300^2 \cdot 100} \right) = C \ln (1) \]
\[ \Delta S_{total} = 0 \] (quasi-equil.)
Problem T8

ideal gas

\( T_1 = 300 \text{ K} \)  \( \rightarrow \)  \( c_2 = 16 \text{ m/s} \)

\( c_1 = 150 \text{ m/s} \)

adiabatic

Concepts: 1st law (CV form, entrop, generation), T-s diagrams

a) \( \frac{T_{t1}}{T_1} = 1 + \frac{c_1^2}{2cp} \)

\( T_{t1} = 311.25 \text{ K} \)

b) 1st law \( 0 = h_{t1} - h_{t2} \) \( \rightarrow \) \( T_{t2} = T_{t1} \)

\( T_{t2} = 311.25 \text{ K} \)

c) Gibbs

\( Tds = dh - vdp \)

\( ds = cp d\frac{T}{T} \)

\( \Delta S_{12} = \int_{T_1}^{T_2} \frac{c_2^2}{2cp} d\frac{T}{T} \)

\( T_2 - \frac{c_2^2}{2cp} = 311.25 \text{ K} \) \( \rightarrow \)

\( \Delta S_{12} = 3.665 \text{ J/kg K} \)

d) \( p = \frac{3RT}{V} \) so

\( \frac{S_2}{S_1} = \frac{T_1}{T_2} \)

\( S_2 = 0.964 \)

e) \( \Delta S_{\text{total}} = \Delta S_{\text{gen}} = \Delta S_{12} > 0 \) irreversible

(\( Tds = dh \) - no heat transfer)

f) \( T \)

\( 1 \)

\( 2 \)

\( T_{t1} = T_{t2} \)

\( c_1^2 \)

\( \frac{c_2^2}{2cp} \)

\( T_2 \)

\( T_{t1} = T_{t2} \)

\( 1 \)

\( 2 \)

\( s \)

\( p_1 = p_2 \)

\( T_1 \)

\( T_2 \)

g) \( \frac{P_{2S}}{P_1} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{\gamma - 1}} \) \( \frac{P_{2S}}{P_1} = 1.137 \)

\( T_2 = T_{t2} \)

\( T_{t1} = T_{t2} \)

\( \Delta S_{12} = \int_{T_1}^{T_2} \frac{c_2^2}{2cp} d\frac{T}{T} \)
Problem T9

\[ p_4 = p_1 \]
\[ T_4 \]
\[ \eta_{ad} = 0.7 \]
\[ P_3, T_3 \]
\[ w_s \]

\[ v_1 = 7.5 \text{ m}^3/\text{min} \]
\[ p_1 = 1.0135 \text{ bar} \]
\[ T_1 = 288 \text{ K} \]
\[ \eta_{ad} = 0.8 \]
\[ \frac{p_2}{p_1} = \frac{p_3}{p_4} = 1.5 = \eta \]

a) \[ h \]
\[ s \]

\[ w_s = h_2 - h_1 = c_p T_1 \left( \frac{\Pi}{T} \right) - 1 \frac{1}{\eta_{ad}} \]

\[ w_s = h_3 - h_4 = c_p T_3 \left( 1 - \frac{1}{\Pi} \right) \eta \frac{T}{T_3} \]

\[ T_3 = T_1 \frac{\Pi - 1}{1 - \frac{1}{\Pi}} \frac{1}{\eta_{ad}} \eta \frac{T}{T_3} = 577.5 \text{ K} \]

\[ T_{4s} = T_3 \left( \frac{1}{\Pi} \right) \frac{T_3}{T} \]

b) \[ h_3 - h_4 = w_s = h_2 - h_1 \]

\[ T_4 = T_3 - \eta_{ad} T_3 \left( 1 - \frac{1}{\Pi} \right) \]

\[ T_4 = 533.2 \text{ K} \]

c) \[ P = \frac{81 v_1 c_p T_1 \left( \Pi \right)}{1} \frac{1}{\eta_{ad}} \]

\[ P = \frac{p_1}{R} \frac{v_1 c_p \left( \Pi \right)}{1} \frac{1}{\eta_{ad}} \]

\[ P = 6.8 \text{ kW} \]

\[ s = \frac{p_1}{RT_1} \]
a) C.V. Analysis. \( F_2 = L \)

\[ \oint p \, dA = 0 \quad \text{since} \quad p = constant \]

\[ \oint (\nabla \cdot \mathbf{u}) \, w \, dA = \int_1 + \int_2 + \int_3 + \int_4 \]

\[ \int_1 = 0 \quad \text{since} \quad w = 0 \]

\[ \int_2 = 0 \quad \text{since} \quad \mathbf{V} \cdot \mathbf{u} = 0 \]

\[ \int_3 = 0 \quad \text{since} \quad \mathbf{V} \cdot \mathbf{u} = 0 \]

\[ + \int_4 = \dot{m} \, w_{in} + \dot{m}_{rest} \cdot 0 = \dot{m} \, w_{in} = \rho V A w_{in} \]

\[ \rho V A w_{in} + \dot{L} = 0 \quad \Rightarrow \quad w_{in} = \frac{-\dot{L}}{\rho V A} \]

b) \( w_i = \frac{w_{in}}{2} = \frac{-\dot{L}}{2 \rho V A}, \quad \alpha_i := \arctan \left( \frac{-w_i}{V} \right) = \frac{-w_i}{V} = \frac{L}{2 \rho V^2 A} \)

\[ D_i = \alpha_i \cdot L = \frac{L^2}{2 \rho V^2 A} \]

c) Pick \( A \) so that our model matches the correct \( D_i : \)

\[ \frac{L^2}{2 \rho V^2 A} = \frac{L^2}{\frac{1}{2} \rho V^2 \pi b^2 e} \quad \Rightarrow \quad A = \frac{\pi}{4} b^2 e \]

Area \( A \) is that of a circular cylinder of diameter \( b \).
UE Fluids  PS 6 Solution

a) \( L = \int_{b/2}^{b} \rho V_0 \Gamma \, dy = \frac{1}{2} \rho V_0 \Gamma \) graphically:

b) \( \gamma = \begin{cases} \frac{2\Gamma_0}{b} & , \ y > 0 \\ -\frac{2\Gamma_0}{b} & , \ y < 0 \end{cases} \)

c) \( W(y_0) = \int_{b/2}^{b} \frac{-y(y) \, dy}{b} \frac{y-y_0}{b} \)

\[ W(y_0) = \frac{-\Gamma_0}{2\pi b} \int_{b/2}^{b} \frac{dy}{y-y_0} + \frac{\Gamma_0}{2\pi b} \int_{0}^{b/2} \frac{dy}{y-y_0} \]

\[ = \frac{-\Gamma_0}{2\pi b} \left( \ln \frac{|y-y_0|}{b/2-y_0} \right)_{b/2}^{b} + \frac{\Gamma_0}{2\pi b} \left( \ln \frac{|y-y_0|}{b/2+y_0} \right)_{0}^{b/2} \]

\[ W(y_0) = \frac{\Gamma_0}{2\pi b} \left[ \ln \left| \frac{y_0}{b/2-y_0} \right| + \ln \left| \frac{y_0}{b/2+y_0} \right| \right] \]

d) \( \frac{\partial \gamma}{\partial y} = -\frac{W(y)}{V} \)

\[ D_i = \int_{b/2}^{b} \rho V_i \Gamma \, dy = \int_{b/2}^{b} \frac{-\rho W(y) \Gamma(y)}{b} \]

Integrand is \( \frac{dD_i}{dy} = -\rho W(y) \Gamma(y) \)

\[ \frac{dD_i}{dy} = \begin{cases} \frac{\rho \Gamma_0^2}{2\pi b} \left[ \ln \frac{|y|}{b/2-y} + \ln \frac{|y|}{b/2+y} \right] \left( 1 - \frac{2y}{b} \right), \ y > 0 \\ \frac{\rho \Gamma_0^2}{2\pi b} \left[ \ln \frac{|y|}{b/2-y} + \ln \frac{|y|}{b/2+y} \right] \left( 1 + \frac{2y}{b} \right), \ y < 0 \end{cases} \]
1. Find and plot the step response of the system

\[
\begin{align*}
\text{R} & \quad i_a \\
\text{L} & \quad y(t) \\
u(t) & \quad - \\
\end{align*}
\]

where \( R = 1 \, \Omega \), and \( L = 2 \, \text{H} \).

**Solution:** Use the loop method to solve. Define the single loop current to be \( i_a \). The sum of the voltage drops around the loop is

\[
0 = -u(t) + R i_a + L \frac{di_a}{dt}
\]

Plugging in numbers, and putting inhomogenous terms on the right, we have

\[
2 \frac{di_a}{dt} + i_a = u
\]

Simplify by dividing by 2 to obtain

\[
\frac{di_a}{dt} + \frac{1}{2} i_a = \frac{1}{2} u
\]

We want to find the step response, so assume that the input is \( u(t) = \sigma(t) \). Then for \( t \geq 0 \),

\[
\frac{di_a}{dt} + \frac{1}{2} i_a = \frac{1}{2}
\]

To find the response, must find the particular solution and the homogenous solution.

First, find the particular solution. Since the input is a constant, guess that the solution is a constant, \( i_p(t) = c \). Since the time derivative of a constant is zero, the differential equation becomes

\[
\frac{1}{2} c = \frac{1}{2}
\]

and so \( i_p(t) = c = 1 \).

Next, find the homogenous solution. The homogeneous equation is

\[
\frac{di_h}{dt} + \frac{1}{2} i_h = 0
\]
which has solution

\[ i_h(t) = Ae^{-t/2} \]

for some constant \( A \).

The total solution is

\[ i_a(t) = i_p(t) + i_h(t) = 1 + Ae^{-t/2} \]

In order to satisfy the initial condition that \( i_a(0) = 0 \), we must have that \( A = -1 \).

Therefore,

\[ i_a(t) = 1 - e^{-t/2} \]

for \( t \geq 0 \), and zero otherwise. The output \( y(t) \) is given by \( y = u - i_a R \). Therefore, the step response is

\[ g_s(t) = (1 - (1 - e^{-t/2})) \sigma(t) \]
\[ = e^{-t/2} \sigma(t) \]

See the plot below:

![Plot of step response](image)

2. For the input signal

\[ u(t) = \begin{cases} 
0, & t < 0 \\
2, & 0 \leq t < 2 \\
-1, & t \geq 2 
\end{cases} \]

find and plot the output \( y(t) \), using superposition.

**Solution:** The input can be written as a sum of steps, as

\[ u(t) = 2\sigma(t) - 3\sigma(t - 2) \]
Therefore, by linearity and time invariance, the output can be written as

\[ y(t) = 2g_s(t) - 3g_s(t - 2) \]

Plugging in, we have

\[ y(t) = 2e^{-t/2} \sigma(t) - 3e^{-(t-2)/2} \sigma(t - 2) \]

This may be simplified by breaking the result down into regions:

\[ y(t) = \begin{cases} 
0, & t < 0 \\
2e^{-t/2}, & 0 \leq t < 2 \\
2e^{-t/2} - 3e^{-t/2+1}, & t \geq 2 
\end{cases} \]

See the plot below: