Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 4

Name: ________________________

Due Date: 03/06/2007

	Time Spent (min)
F7&10	
F8	
F9	
S2	
S3	
S4	
Study Time	

Announcements:

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Fluids Problems F7+F10 (worth 2 Pset Q's)

The profile drag of any real airfoil increases as chord Reynolds number is reduced. At the low Reynolds numbers typical of small model aircraft ($Re < 50000$), a rough estimate of profile drag of an unstalled airfoil is

$$c_d \ = \ 5.0\, Re^{-1/2}$$

Airfoils in this flight regime are also limited to $c_\ell \leq 0.6$, otherwise they will stall.
Consider an elliptically-loaded wing with this airfoil, with a specified area S. This wing is to lift some specified aircraft weight W, at some atmospheric properties ρ, ν.

a) Determine the average wing chord $c_{\rm avg}$ as a function of the aspect ratio AR.

b) Determine average-chord Reynolds number Re as a function of AR and C_L.

c) Determine the total drag coefficient C_D as a function of AR and C_L.

d) Assuming that $C_L = 0.6$ (the maximum value just below stall), plot C_L/C_D over the range $AR = 10 \ldots 30$, for the following aircraft and atmospheric parameters:

$$S = 0.1\,\mathrm{m}^2 \qquad\qquad W = 10\,\mathrm{N}$$

$$\rho = 1.23\,\mathrm{kg/m}^3 \qquad\qquad \nu = 1.45 \times 10^{-5}\,\mathrm{m}^2/\mathrm{s}$$

and graphically estimate the maximum C_L/C_D value, and the corresponding "optimum" AR. Based on our finite-wing theory, wing performance as measured by C_L/C_D is expected to improve with increasing AR. So why does the performance start to decrease if AR is increased past the optimum value? Resolve this paradox.

e) Repeat part d) for a much lighter aircraft with $W = 1\,\mathrm{N}$. You may overlay the two plots.

f) The Albatross and the Arctic Tern have similar "design requirements", namely long-distance travel over the ocean, but the Arctic Tern is a very much smaller and lighter bird.
`http://en.wikipedia.org/wiki/Image:Royal_Albatross_near_Dunedin.jpg`
`http://en.wikipedia.org/wiki/Image:Havterne.jpg`
Use your results to postulate an evolutionary reason for their rather different aspect ratios.

A wing is to have an elliptic circulation distribution.

$$\Gamma(y) \;=\; \Gamma_0 \sqrt{1 - \left(\frac{2y}{b}\right)^2}$$

The planform is to be a straight taper, with root and tip chords defined in terms of the average chord c_{avg} and the *taper ratio* $r = c_t/c_r$.

$$c_r \;=\; c_{\mathrm{avg}} \frac{2}{1+r} \qquad\qquad c_t \;=\; c_{\mathrm{avg}} \frac{2r}{1+r}$$

a) Define the chord distribution $c(y)$ in terms of c_{avg} and r. Assuming $c_{\mathrm{avg}}/b = 0.1$, draw the planforms for $r = 0.8, 0.5, 0.2$.

b) Determine the spanwise $c_\ell(y)$ distribution, and plot for $r = 0.8, 0.5, 0.2$.
Also determine the overall C_L of the wings, and overlay as a horizontal line on the plot for reference.
Note: Only the shape of the $c_\ell(y)$ curves is of interest. All scaling constants like Γ_0, c_{avg}, etc. can be set to unity for plotting purposes.

c) Local stall occurs when the local c_ℓ at some spanwise location exceeds the $c_{\ell_{\max}}$ value for the airfoil. Which taper ratio appears to be most attractive for the purpose of giving the largest stall margin for the wing?

Consider the following circulation distribution:

$$\Gamma(y) \;=\; [0.05 - 0.02(2y/b)]\,\sqrt{1-(2y/b)^2}$$

For numerical and plotting simplicity, we will assume $\rho = 1$, $V_\infty = 1$, $b = 2$.

a) Perform a trigonometric substitution and thus determine the equivalent $\Gamma(\theta)$.

b) Determine a suitable number of A_n coefficients which represent the $\Gamma(\theta)$ you obtained.

c) Determine the lift L, induced drag D_i, and span efficiency e.

d) Determine the rolling moment M_{roll}, which is defined by

$$M_{\text{roll}} \;=\; \int_{-b/2}^{b/2} L'(y)\, y\, dy$$

Hint: Evaluate this in the θ coordinate.

e) Plot $\Gamma(y)$ and the downwash angle $\alpha_i(y)$.

Problem S2 (Signals and Systems)

A system has step response given by

$$g_s(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-t}, & t \geq 0 \end{cases}$$

Find and plot the response of the system to the input

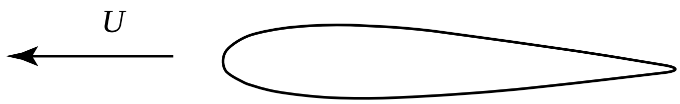
$$u(t) = \begin{cases} 0, & t < 0 \\ 1 - e^{-2t}, & t \geq 0 \end{cases}$$

using Duhamel's integral.

Problem S3 (Signals and Systems)

Note: Please do not use official or unofficial bibles for this problem.

An airfoil with chord c is moving at velocity U with zero angle of incidence through the air, as shown in the figure below:



The air is not motionless, but rather has variations in the vertical velocity, w. As the airfoil flies through this gust field, the leading edge of the airfoil "sees" a variation in the angle of attack. If w is small compared to U, then the angle of attack change seen by the airfoil is $\alpha = w/U$. Since the velocity profile varies in space, the angle of attack seen by the airfoil is a function of time, $\alpha(t)$.

One might expect that the lift coefficient of the airfoil is just

$$C_L(t) = 2\pi\alpha(t)$$

However, the airfoil does not respond instantaneously as the airfoil encounters the gust. If the airfoil encounters a "sharp-edged gust,"so that the apparent change in the angle of attack is a step function in time,

$$\alpha(t) = \alpha_0\sigma(t)$$

then the change in lift is given by

$$C_L(t) = 2\pi\alpha_0\psi(\bar{t})$$

where $\bar{t} = 2Ut/c$ is the dimensionless time. $\psi(\bar{t})$ is the *Küssner function*, and is the step response of the airfoil (neglecting multiplicative constants), if the input is considered to be the vertical gust at the leading edge as a function of time, and the output is considered to be the lift as a function of time. The Küssner function can be approximated as

$$\psi(\bar{t}) = \begin{cases} 0, & \bar{t} < 0 \\ 1 - \frac{1}{2}e^{-0.13\bar{t}} - \frac{1}{2}e^{-\bar{t}}, & \bar{t} \geq 0 \end{cases}$$

Assuming that the airfoil acts as an LTI system, determine and plot the lift coefficient, $C_L(t)$, and the gust velocity, $w(t)$, for the following conditions:

$$\begin{aligned} c &= 1 \text{ m} \\ U &= 1 \text{ m/s} \\ w(t) &= \begin{cases} 0 \text{ m/s}, & t < 0 \text{ s} \\ 0.1 \cdot (1 - e^{-2t}) \text{ m/s}, & t \geq 0 \text{ s} \end{cases} \end{aligned}$$

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Problem S4 (Signals and Systems)

Note: This problem is similar to one given a couple years ago. Please try to do this one without looking at bibles — the solution is instructive.

One of the benefits of the approach of using the superposition integral is that you don't have to guess the particular solution — it pops right out of the integral, automatically. In some cases, the particular solution can be hard to guess, but easy to find using the convolution integral. To see this, consider the system described by the differential equation

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = u(t)$$

1. Find the step response of the system.

2. Take the derivative of the step response to find the impulse response.

3. Now assume that the input is given by

$$u(t) = e^{-2t}\sigma(t)$$

   Before doing part (4), try to find the particular solution by the usual method, that is, by intelligent guessing. Be careful — it may not be what you expect!

4. Now find $y(t)$ using the superposition integral. Is the particular solution what you expected?