Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 5

Name: ________________________

Due Date: 03/13/2007

	Time Spent (min)
M5.1	
M5.2	
M5.3	
S5	
S6	
S7	
Study Time	

Announcements:

Unified Engineering Problem Set **Lectures: M1, M2, M3**
Week 5 Spring, 2007 **Units: M4.1, M4.2**

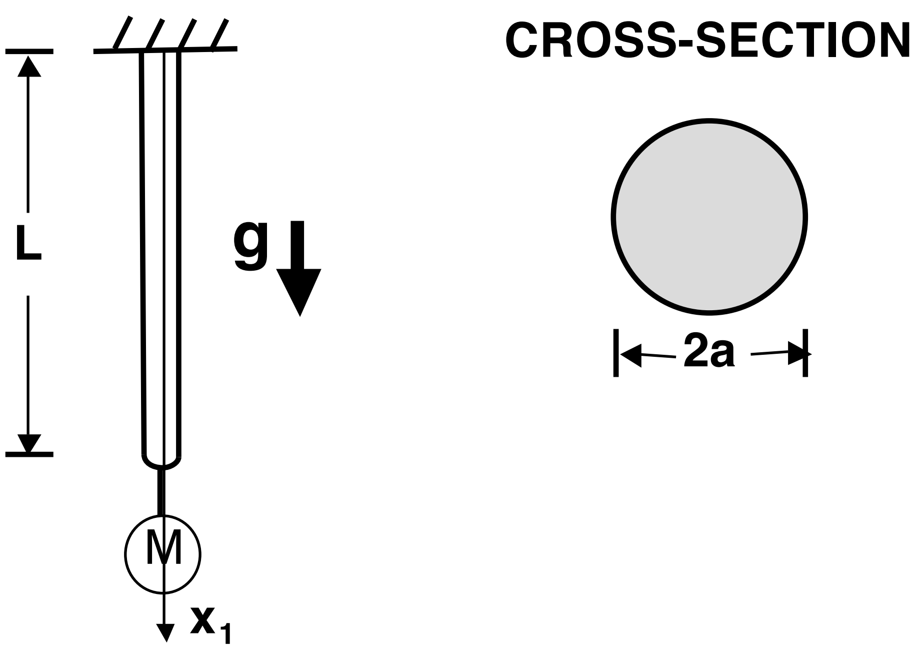
M5.1 ***(5 points)*** This problem serves as a mechanism to review the key concepts and governing equations in general elasticity.

(a) Write out fully, in tensor notation, the governing *independent* equations of elasticity for a solid body. Identify the key assumptions associated with each set of these equations and the underlying fundamental(s) upon which these are based.

(b) There are also several other equations, known as "compatibility equations". Describe what they are and from where they come. Why aren't these also independent equations?

(c) If engineering notation is used, do these equations change? How? Indicate this either through careful and complete description or by writing out the equations that change. In either case, highlight any key differences.

M5.2 ***(15 points)*** A rod with a circular cross-section is suspended from an overhead support as indicated in the accompanying figure. The rod is of length L, and has a radius equal to a. The rod is made of an isotropic material with a longitudinal modulus of E, Poisson's ratio of □, shear modulus of G, and density of □. A large package of mass M hangs at the end of the rod. The entire arrangement is subjected to a gravity field of value g.



For this problem, ignore the effects of the mass of the rod and only consider the effects of the package hanging at the end.

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

(e) If the radius of the rod varies along the length of the rod, can the rod model still be used? Why or why not? Be sure to explain clearly using equations if/as needed. If you need a particular case, use the radius varying from a at the upper support to a/2 at the rod tip ($x_1 = L$).

M5.3 ***(10 points)*** Consider the same configuration as in problem M5.2, but now give consideration to the effects of the mass of the rod. Repeat parts (a) through (d) including the effects of the mass of the rod using the results from the previous problem as reference.

(a) What are the boundary conditions for this configuration?

(b) Determine the stress and strain states throughout the rod.

(c) Determine the displacements throughout the rod.

(d) Comment on the applicability of the rod model for this configuration.

(e) Now make an overall comparison of the solutions for the two cases (i.e., including and ignoring the mass of the rod) and make comments. In particular, answer the question as to when the mass of the rod becomes important.

(f) If failure occurs when the stress in the rod reaches a material ultimate value, σ_{ult}, determine the pertinent figure of merit for this configuration if the objective is to carry the maximum package mass without failure. (See Unit M3.1 for review of "figure of merit".)

Problem S5 (Signals and Systems)

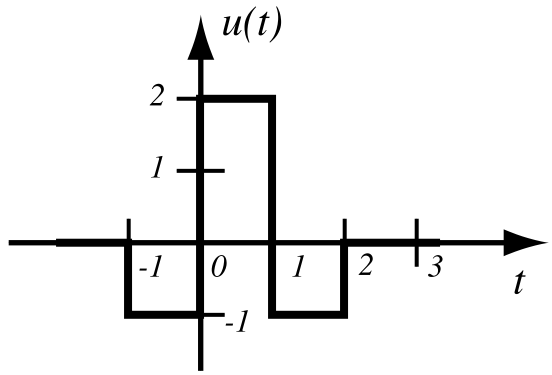
Consider the signals

$$g(t) = \begin{cases} 0, & t < 0 \\ e^{-2t}, & t \geq 0 \end{cases}$$

and

$$u(t) = \begin{cases} 0, & t < -1 \\ e^{-t}, & -1 \leq t \leq 1 \\ 0, & t > 1 \end{cases}$$

1. Sketch the two signals. Your sketch should be fairly accurate, as this will aid in the rest of the problem.
2. Using graphical convolution techniques, sketch the signal $y(t) = g(t) * u(t)$. Make sure to label important features on the graph, such as points in time where the graph of $y(t)$ changes character, the value of $y(t)$ at those breakpoints, etc. The goal of this part is not to calculate $y(t)$ exactly, but to rather to use graphical convolution to determine (approximately) what the final solution should look like. It's always a good thing to know the answer before you have to calculate it!
3. Now evaluate the convolution exactly. You should be able to use your results in part 2 above to help identify the limits of convolution.
4. A different signal $u(t)$ is shown in the graph below. Sketch, as accurately as possible, the signal $h(t) = u(t) * u(t)$. Explain your reasoning.



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Problem S6 (Signals and Systems)

Note: This problem is similar to one given a couple years ago. Please try to do this one without looking at bibles — the solution is instructive. This problem shows, in a somewhat simplified way, how a radar system determines the distance to an aircraft. More basically, it will give you practice doing convolution using the "flip and slide" method.

The radar sends out a signal, $u(t)$, that reflects off the aircraft and returns to the radar system. The time it takes the signal to return is twice the distance to the aircraft, divided by the speed of light. The received signal is $u(t-T)$, where T is the round trip travel time of the signal. It may not seem obvious, but it's not easy to determine T by measuring the returned signal. In principle, one could simply display the returned signal on an oscilloscope and measure the time of some key feature of the signal, such as the leading edge of the signal, or the peak of the signal, or whatever. However, the signal received by the radar is relatively weak, because most of the energy radiated by the radar does not strike the aircraft, and most of the reflected energy is not received by the radar. Often the noise in the received signal is as large or larger than the signal itself.

Instead, a better approach is to try to identify the whole received signal instead of a single key feature. How can we do this? Convolution! The received signal is run through an LTI system G with impulse response $g(t)$. The impulse response is designed to accentuate the signal, so that one can measure a key feature of the output signal, $y(t)$, which is relatively insensitive to noise. Often, the best choice for $g(t)$ is a time-reversed version of $u(t)$. That is,

$$g(t) = u(-t) \tag{1}$$

This is called a "matched filter."

1. For the signal $u(t)$ shown below, and $g(t) = u(-t)$, find the convolution

   $$y(t) = g(t) * u(t)$$

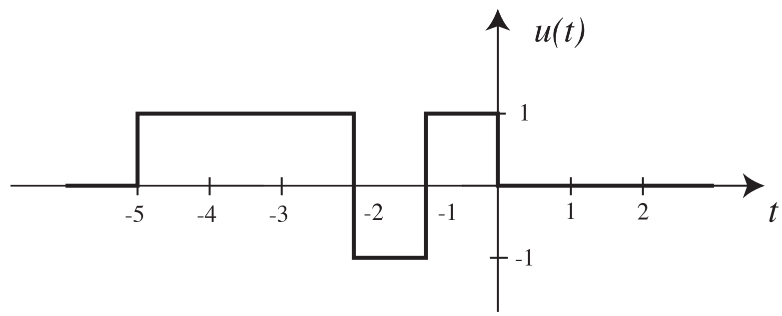
   using the flip and slide method.

2. Using the results of part (1) and the LTI properties of the system G, what is the convolution

   $$y(t) = g(t) * u(t-T)$$

3. What feature of $y(t)$ would you use to identify the time T?

4. Can you explain why the right impulse response for G is a signal that has the same shape as $u(t)$, but time reversed?


u(t)
1
-1
-5
-4
-3
-2
-1
1
2
t

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Problem S7 (Signals and Systems)

This problem shows why a radar system sends out a chirp, which has a broad range of frquencies in the signal, and *not* a short sinusoidal pulse, which is at a single frequency. To see why a sinusoidal pulse doesn't work well, let's try a radar signal

$$u(t) = \begin{cases} \sin(2\pi t), & -3 \le t \le 0 \\ 0, & \text{otherwise} \end{cases}$$

The matched filter for this pulse has impulse response

$$g(t) = u(-t) = \begin{cases} \sin(-2\pi t), & 0 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$$

The radar sends out a signal, $u(t)$, that reflects off the aircraft and returns to the radar system. The time it takes the signal to return is twice the distance to the aircraft, divided by the speed of light. The received signal is $u(t-T)$, where T is the round trip travel time of the signal. For the purposes of this problem, we can ignore the time delay, T, and just look at how the matched filter response to $u(t)$.

1. Find the convolution
$$y(t) = g(t) * u(t)$$
You will find it helpful to use the flip and slide method to set up the integral. The integral can be evaluated relatively easily in closed form, if you set up the integral properly.

2. Plot $y(t)$.

3. $y(t)$ as plotted above is the signal that results when the round-trip time of the pulse is zero. When the delay time is greater, of course, the signal that results is $y(t-T)$, which is just $y(t)$ shifted right by T. What feature of $y(t-T)$ would you use to identify the time T?

4. Explain why it might be difficult to determine T from a returned radar pulse, especially if there is additional noise added to the signal.

5. The signal $y(t)$ as plotted in Part 2 is called the *ambiguity function*, because it helps determine how ambiguous the delay time T is in the presence of noise. Explain why the ambiguity function corresponding to the chirp signal of Problem S6 is better than the ambiguity function in this problem.

In practice, the chirp signals sent out by a radar pulse are longer than a few cycles of Problem S6 or this problem. Nevertheless, these problems show that in order to have low ambiguity, the radar signal needs to have a well-designed shape.