PAL
2/24/07

Unified Engineering Problem Set
Week 5 Spring, 2007
SOLUTIONS

M5.1 (a) There are three key sets of equations:

- Equilibrium Equations $\left(\frac{\partial \sigma_{mn}}{\partial x_n} + f_m = 0\right)$
  gives $\underline{\underline{3}}$ equations

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + f_1 = 0$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + f_2 = 0$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + f_3 = 0$$

These are based on the fundamental of $\boxed{\text{equilibrium}}$

- Strain - Displacement $\epsilon_{mn} = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right)$
  gives $\underline{\underline{6}}$ equations

$$\epsilon_{11} = \partial u_1 / \partial x_1$$

$$\epsilon_{22} = \partial u_2 / \partial x_2$$

$$\epsilon_{33} = \partial u_3 / \partial x_3$$

$$\epsilon_{21} = \epsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\epsilon_{31} = \epsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$$

$$\epsilon_{32} = \epsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$$

These are based on $\boxed{\text{geometrical relationships}}$ and have the key assumption that <u>strains are small</u> such that angular changes are small. This can be measured/expressed as: $\cos\theta \approx 1$; $\sin\theta \approx \theta$

• <u>Stress-Strain</u> $\sigma_{mn} = E_{mnpq}\,\epsilon_{pq}$

gives <u>6</u> equations

$$\sigma_{11} = E_{1111}\epsilon_{11} + E_{1122}\epsilon_{22} + E_{1133}\epsilon_{33} + 2E_{1123}\epsilon_{23} + 2E_{1113}\epsilon_{13} + 2E_{1112}\epsilon_{12}$$

$$\sigma_{22} = E_{1122}\epsilon_{11} + E_{2222}\epsilon_{22} + E_{2233}\epsilon_{33} + 2E_{2223}\epsilon_{23} + 2E_{2213}\epsilon_{13} + 2E_{2212}\epsilon_{12}$$

$$\sigma_{33} = E_{1133}\epsilon_{11} + E_{2233}\epsilon_{22} + E_{3333}\epsilon_{33} + 2E_{3323}\epsilon_{23} + 2E_{3313}\epsilon_{13} + 2E_{3312}\epsilon_{12}$$

$$\sigma_{23} = E_{1123}\epsilon_{11} + E_{2223}\epsilon_{22} + E_{3323}\epsilon_{33} + 2E_{2323}\epsilon_{23} + 2E_{1323}\epsilon_{13} + 2E_{1223}\epsilon_{12}$$

$$\sigma_{13} = E_{1113}\epsilon_{11} + E_{2213}\epsilon_{22} + E_{3313}\epsilon_{33} + 2E_{2313}\epsilon_{23} + 2E_{1313}\epsilon_{13} + 2E_{1213}\epsilon_{12}$$

$$\sigma_{12} = E_{1112}\epsilon_{11} + E_{2212}\epsilon_{22} + E_{3312}\epsilon_{33} + 2E_{2312}\epsilon_{23} + 2E_{1312}\epsilon_{13} + 2E_{1212}\epsilon_{12}$$

This is based only on <u>linear</u> <u>relationships</u> between stress and strain (and is $\boxed{\text{constitutive}}$)

(b) Compatibility equations come from geometrical restrictions as manifested in the strain-displacement equations. Displacements must be continuous functions of x_1, x_2, and x_3. Thus, with three such functions, the six strains cannot be independent. The compatibility equations relate the strain fields to be compatible with the continuity of the displacements.

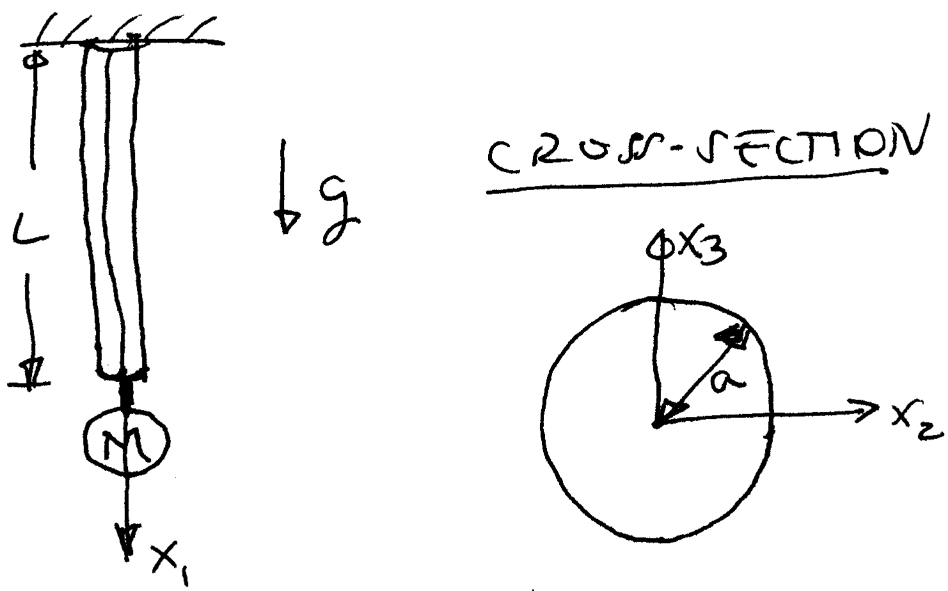
They are derived by using the strain-displacement equations, taking "cross" derivatives and equating these.

They express geometrical restrictions

(c) In using engineering equations, the form of the equations change (e.g. σ_x rather than σ_{11}), but the underlying fundamentals and associated assumptions stay the same and the equations represent the same thing. Only the notation changes.

One key change due to definition is that engineering shear strain is 2x tensorial shear strain, so this factor of 2 must be incorporated in all equations with engineering shear strains.

M 5.2



(a) Boundary Conditions:

@ $x_1 = 0$, rod is fixed to support ⇒ displacements are zero

$$\boxed{@\ x_1 = 0 : \quad u_1,\ u_2,\ u_3 = 0}$$

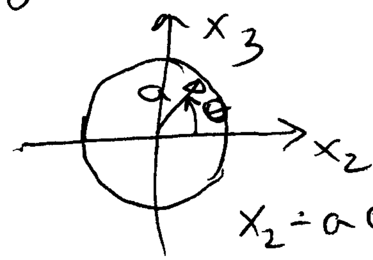
(NOTE: Could also express u_2 and u_3 in polar coordinates as $u_r, u_\theta = 0$)

@ $x_1 = L$, overall force of $F = Mg$ is applied over area $= \pi a^2$

So:

$$\boxed{\begin{array}{l} \sigma_{11} = F/A = \dfrac{Mg}{\pi a^2} \quad @\ x_1 = L \\ \sigma_{13},\ \sigma_{12} = 0 \end{array}}$$

Along all other surfaces, all stresses on surface are zero. Boundary surface is at



$x_2 = a\cos\theta$
$x_3 = a\sin\theta$
$0^\circ \leq \theta \leq 360^\circ$

So,

$$\boxed{\begin{array}{c} @\ x_2 = a\cos\theta,\ x_3 = a\sin\theta \\ 0^\circ \leq \theta \leq 360^\circ \\ \\ \sigma_{22},\ \sigma_{33},\ \sigma_{23} = 0 \end{array}}$$

(NOTE: Could also do in polar coordinates that @ $r = a$, $\sigma_{rr}, \sigma_{r\theta} = 0$)

(b) neglecting the mass of the rod $\Rightarrow$ no body forces

Apply the equilibrium equations. The only nonzero stress is σ_{11}, so:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \cancelto{=0}{f_1} = 0$$

integrating $\Rightarrow \sigma_{11} = \text{constant}$

Apply the B.C. @ $x_1 = L$: $\sigma_{11} = \dfrac{Mg}{\pi a^2}$

$$\Rightarrow \boxed{\begin{array}{l} \sigma_{11} = Mg/\pi a^2 \\ \text{all other stresses are zero everywhere} \end{array}}$$

To determine the strains, use the stress-strain relationships. For an isotropic material, need longitudinal (Young's) modulus, E, and Poisson's ratio, ν.

with σ_{11} the only nonzero stress:

$$\boxed{\epsilon_{12} = \epsilon_{13} = \epsilon_{23} = 0}$$ (no shear strains)

and: $\epsilon_{11} = \sigma_{11}/E$

$$\epsilon_{22} = -\frac{\nu}{E}\sigma_{11}$$

$$\epsilon_{33} = -\frac{\nu}{E}\sigma_{11}$$

$$\Rightarrow \boxed{\begin{aligned} \epsilon_{11} &= \frac{Mg}{E\pi a^2} \\ \epsilon_{22} = \epsilon_{33} &= -\frac{\nu Mg}{E\pi a^2} \end{aligned}}$$

(c) To find the displacements, apply the strain-displacement relations. The only primary consideration is u_1, so we use:

$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1} = \frac{Mg}{E\pi a^2}$$

integration gives:

$$u_1 = \frac{Mg}{E\pi a^2} x_1 + C$$

constant of integration since no variation in x_2 and x_3

To find the constant, apply the B.C.:

@ $x_1 = 0$, $u_1 = 0 \Rightarrow C = 0$

This results in:

$$\boxed{u_1 = \frac{Mg}{E\pi a^2} x_1}$$

By definition of the model, u_2 and u_3 are 0. However, note the slight inconsistency since ϵ_{22} and ϵ_{33} are nonzero. Using:

$$\epsilon_{22} = \frac{\partial u_2}{\partial x_2}, \quad \epsilon_{33} = \frac{\partial u_3}{\partial x_3}$$

and integrating gives:

$$\epsilon_{22} = -\frac{\nu Mg}{E\pi a^2} x_2 + C_2$$

$$\epsilon_{33} = -\frac{\nu Mg}{E\pi a^2} x_3 + C_3$$

Define the midpoint of the area as the point of zero displacement (NOTE: can define any point as reference point) to give $C_2 = 0$, $C_3 = 0$

(d) There are always inconsistencies in the model with regard to the u_2 and u_3 displacements. This is built into the model. One must use the St. Venant's principle in the vicinity of the attachment to the support.

"Away" from this region, the model is valid. "Near" the region, σ_{11} may vary with x_2 and x_3 and other stresses may be present.

(e) This is simply an extension of the model to allow area to vary with x_1. In the general case we replace the constant area we have ($A = \pi a^2$) with a general functional relationship to x_1: $A = A(x_1)$

Then use this in the equations used.

$$\sigma_{11} = \frac{Mg}{A(x_1)}$$

$\Rightarrow \sigma_{11}$ varies with x_1

similarly: $\epsilon_{11} = \frac{Mg}{EA(x_1)}$ $\Rightarrow$ and ϵ_{11} varies with x_1

and for ϵ_{22} and ϵ_{33}:

$$\epsilon_{22} = \epsilon_{33} = -\frac{\nu Mg}{EA(x_1)}$$

The expression for the displacement u_1 becomes more involved:

$$u_1 = \int \epsilon_{11}\, dx_1 = \int \frac{Mg}{EA(x_1)}\, dx_1$$

An expression for $A(x_1)$ is needed to be specific, but that is not necessary in order to explain this.

Similarly, u_2 and u_3 vary with x_1. This may make the inconsistencies more important. It will basically depend on the rate that the area varies with x_1 : $\partial A/\partial x_1$,

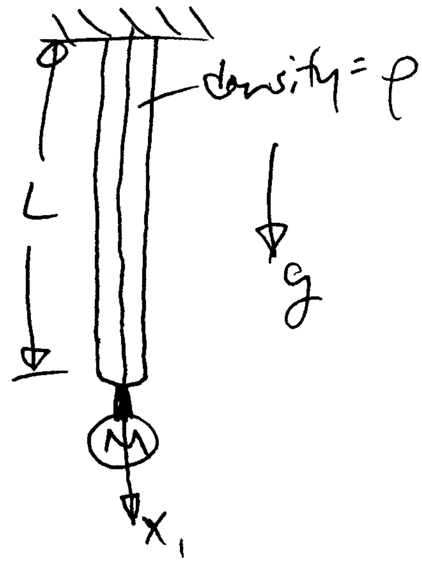
since area controls all the results... it is in the denominator for all the key items.

Finally, note that if σ_{11} is a function of x_1, then $\frac{\partial \sigma_{11}}{\partial x_1}$ is nonzero The first equilibrium equation is:

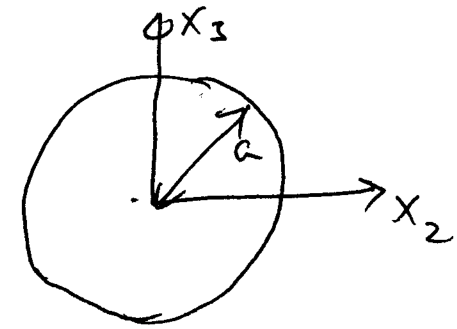
$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} = 0$$

since the first term is nonzero, one or both other terms <u>must</u> exist to satisfy equilibrium, so σ_{12} and/or σ_{13} <u>exist</u>! $\Rightarrow$ the model further falls apart .... not as applicable.

M 5.3



CROSS-SECTION



(a) The Boundary Conditions $\boxed{\text{do not change}}$ if the body force is taken into account. The body force, f_1, is internal and does not contribute to the surface/boundary conditions.

So repeat:

@ $x_1 = 0$: $u_1, u_2, u_3 = 0$

@ $x_1 = L$, $\sigma_{11} = \dfrac{Mg}{\pi a^2}$, $\sigma_{12} = \sigma_{13} = 0$

@ all surfaces; $\sigma_{33}, \sigma_{23}, \sigma_{22} = 0$
of
$x_2 = a\cos\theta$
$x_3 = a\sin\theta$

(b) We must now include the body force:

$$f_1 = \frac{\rho g \text{ Volume}}{\text{Volume}}$$

in the equilibrium equation. So:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \rho g = 0$$

Integrating this gives:

$$\sigma_{11} = -\rho g x_1 + C$$

Again, apply the B.C. @ $x_1 = L$, $\sigma_{11} = \frac{Mg}{\pi a^2}$

$$\Rightarrow \frac{Mg}{\pi a^2} = -\rho g L + C$$

$$\Rightarrow C = \frac{Mg}{\pi a^2} + \rho g L$$

This gives:

$$\boxed{\sigma_{11} = \frac{Mg}{\pi a^2} + \underbrace{\rho g (L - x_1)}_{\text{new term}}}$$

(c) The strains are related through the same equations as in the previous case (basic stress-strain equations do not change).

So we get:

$$\boxed{\begin{aligned}\epsilon_{11} &= \frac{Mg}{E\pi a^2} + \underbrace{\frac{\rho g}{E}(L-x_1)}_{\text{new term}} \\ \epsilon_{22} = \epsilon_{33} &= -\frac{\nu Mg}{E\pi a^2} - \underbrace{\frac{\nu\rho g}{E}(L-x_1)}_{\text{new term}}\end{aligned}}$$

(c) The displacement becomes more complicated as ϵ_{11} is a function of x_1. So:

$$u_1 = \int \left\{ \frac{Mg}{E\pi a^2} + \frac{\rho g}{E}(L-x_1) \right\} dx_1$$

$$\Rightarrow u_1 = \frac{Mg}{E\pi a^2} x_1 + \frac{\rho g}{E} L x_1 - \frac{\rho g}{2E} x_1^2 + C_1$$

Again, the B.C. gives $u_1 = 0$ @ $x_1 = 0 \Rightarrow C_1 = 0$

So: $$\boxed{u_1 = \frac{Mg}{E\pi a^2} x_1 + \underbrace{\frac{\rho g x_1}{E}\left(L - \frac{x_1}{2}\right)}_{\text{new term}}}$$

Again, the model says u_2 and u_3 are zero, though there is a slight inconsistency since ϵ_{22} and ϵ_{33} are nonzero.

(d) The same inconsistencies exist as in the previous case. However, their importance may be affected by the additional term and therefore the applicability of the model may be affected. This will depend on the specific values for the various parameters

(e) For all the key variables, there are two contributions. One from the end mass $= \frac{Mg}{\pi a^2}$ (same as before), and a new term due to the density of the rod $= \rho g (L - x_1)$.
The mass of the rod becomes important when it is of similar magnitude as that of the package mass M. To do a rough comparison, one can look at the maximum value for the rod contribution and compare it ~~to~~ the other:

$$\rho g L \quad vs. \quad \frac{Mg}{\pi a^2}$$

giving:

$$\boxed{\rho L \quad vs. \quad \frac{M}{\pi a^2}}$$

to ascertain the % contribution of the rod mass.

(f) The operative equation from part (b) is:

$$\sigma_{11} = \frac{Mg}{\pi a^2} + \rho g (L - x_1)$$

Failure occurs when this reaches σ_{ult}. It will be maximum at $x_1 = 0$, so:

$$\sigma_{ult} = \frac{Mg}{\pi a^2} + \rho g L$$

Manipulating this to get an expression for the package mass that is to be maximized:

$$\frac{Mg}{\pi a^2} = \sigma_{ult} - \rho g L$$

$$\Rightarrow M = \pi a^2 \left(\frac{\sigma_{ult}}{g} - \rho L \right)$$

πa^2 is a constant for all cases, so there are two possible figures of merit. First, if the radius is a design variable:

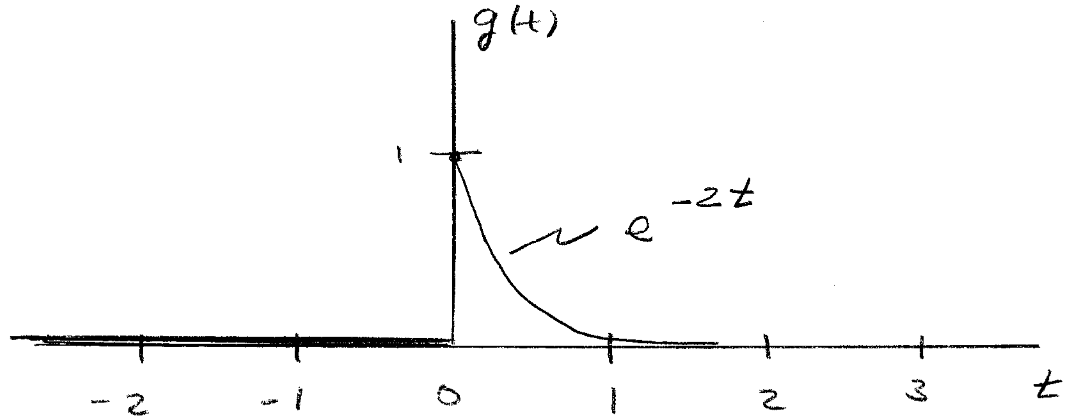
$$\boxed{M = \pi a^2 \left(\frac{\sigma_{ult}}{g} - \rho L \right)}$$

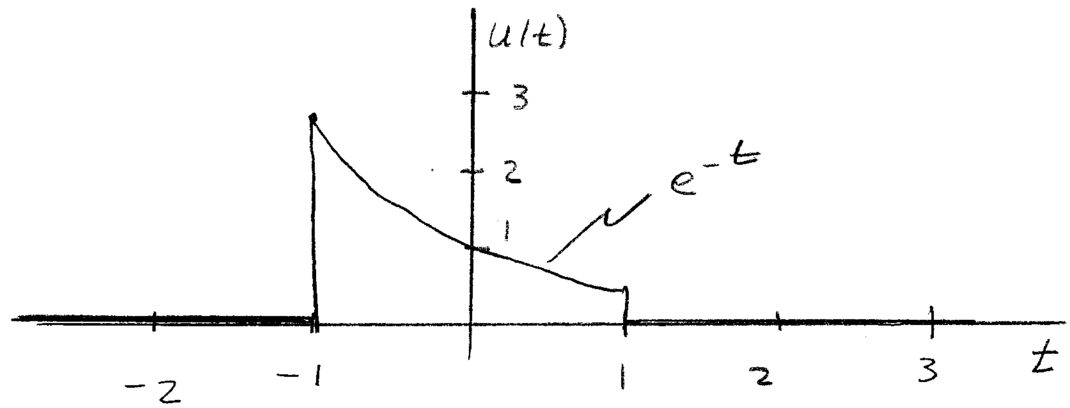
or if it is not a design parameter:

$$\boxed{M \propto \frac{\sigma_{ult}}{g} - \rho L}$$

proportional to

1.





2. To convolve, flip g(t), slide left and right, multiply by u(t), and find the area

There will be 3 important ranges:

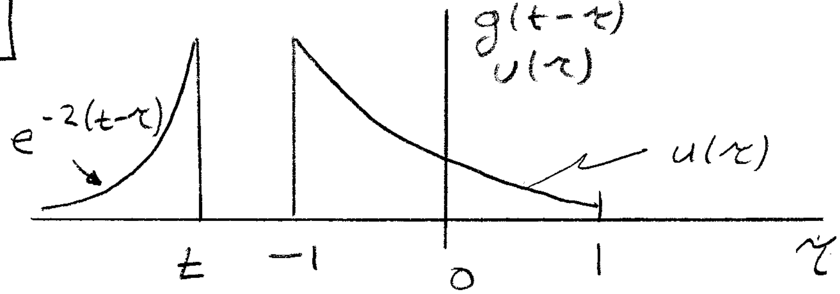
$t < -1$ no overlap

$-1 < t < 1$ overlap from -1 to t

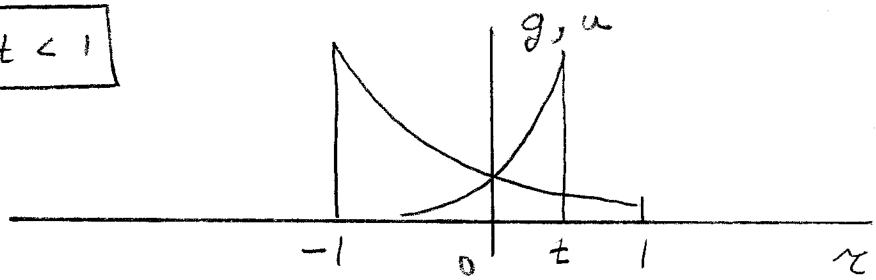
$t > 1$ overlap from -1 to 1

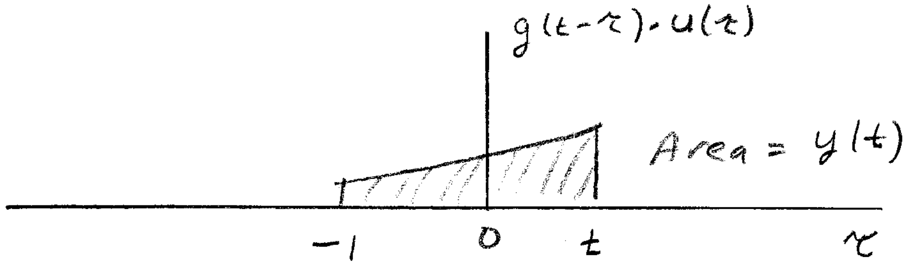
These are illustrated below:

$\boxed{t < -1}$

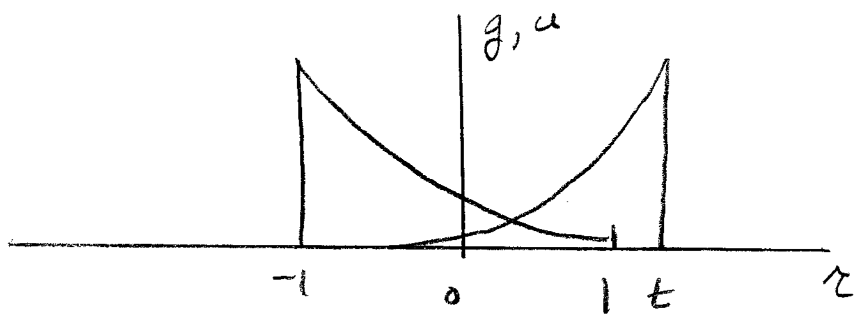


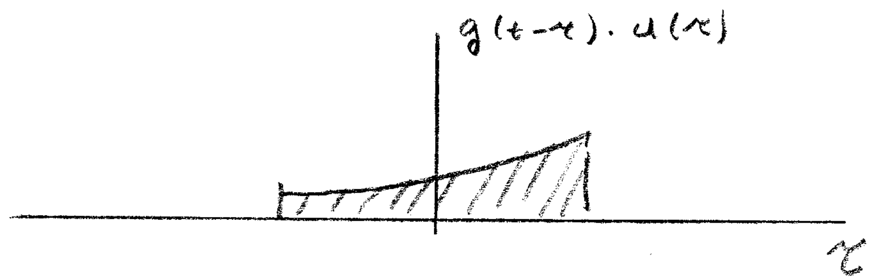
$\boxed{-1 < t < 1}$



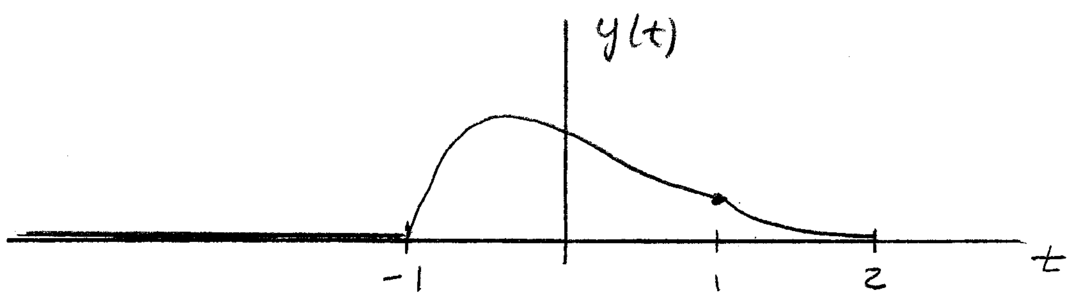


$\boxed{t > 1}$





So the convolution will look like:



3. The convolution is:

$\underline{t < -1}$ $\quad y(t) = 0$

$\underline{-1 < t < 1}$ $\quad y(t) = \int_{-1}^{t} e^{-2(t-\tau)} e^{-\tau} d\tau$

$$= e^{-2t} \int_{-1}^{t} e^{\tau} d\tau$$

$$= e^{-2t} \left. e^{\tau} \right|_{t=-1}^{t}$$

$$= e^{-2t} \left(e^{t} - e^{-1} \right)$$

$$= e^{-t} - e^{-2t-1}$$

$\underline{t > 1}$ $\quad y(t) = \int_{-1}^{1} e^{-2(t-\tau)} e^{-\tau}$

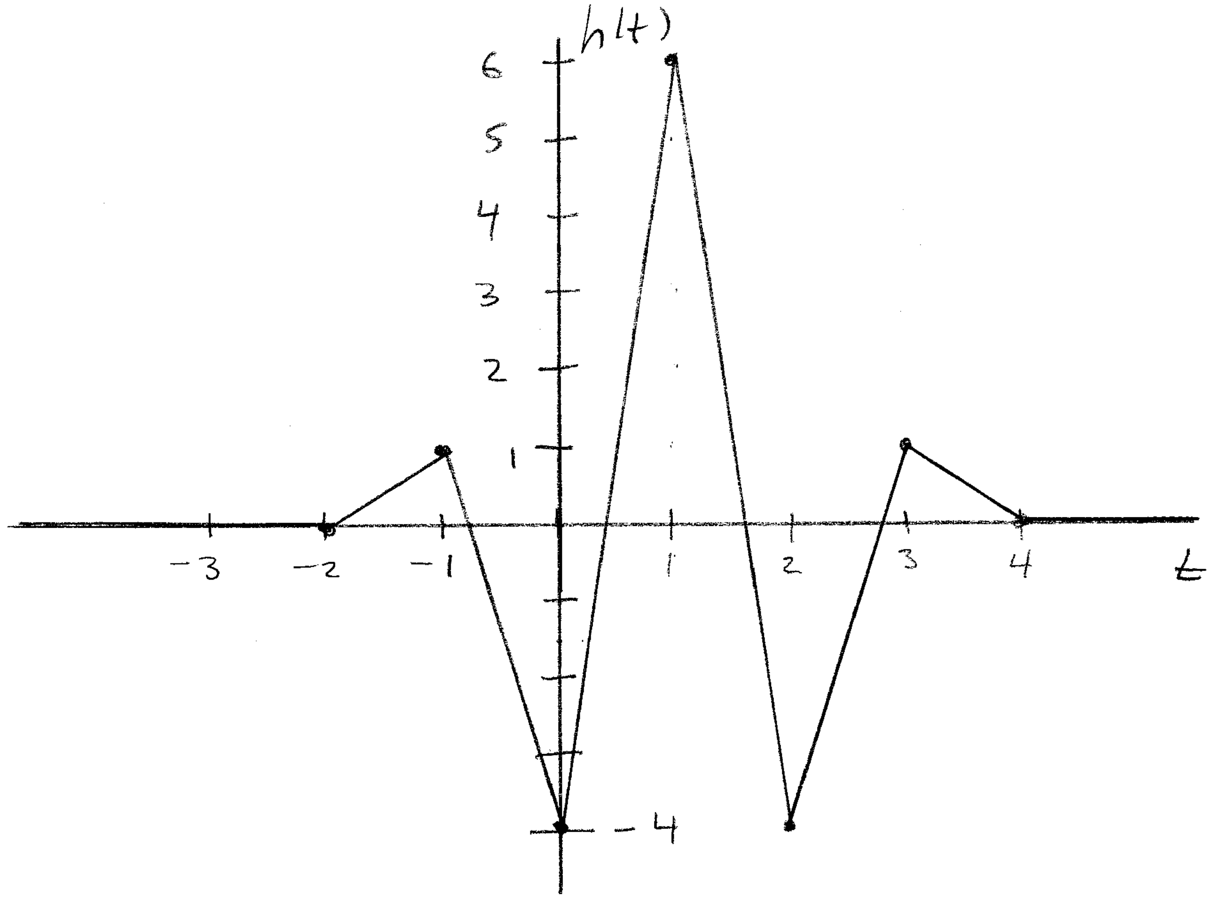
$$= e^{-2t} \left. e^{\tau} \right|_{t=-1}^{1}$$

$$= e^{-2t} \left(e^{1} - e^{-1} \right)$$

So,

$$y(t) = \begin{cases} 0, & t > -1 \\ e^{-t} - e^{-2t-1}, & -1 < t < 1 \\ e^{-2t}\left(e^{1} - e^{-1}\right), & t > 1 \end{cases}$$

4. This part is very much like S6. See S6 solution for general approach. The result is


h(t)
6
5
4
3
2
1
-4
-3
-2
-1
1
2
3
4
t

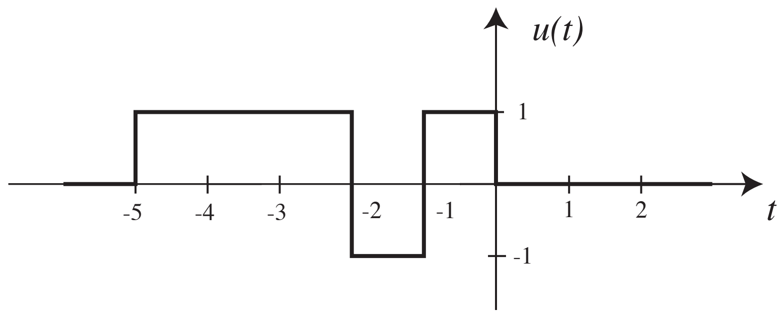
Unified Engineering II **Spring 2007**

Problem S6 (Signals and Systems) SOLUTION

1. For the signal $u(t)$ shown below, and $g(t) = u(-t)$, find the convolution

$$y(t) = g(t) * u(t)$$

using the flip and slide method.



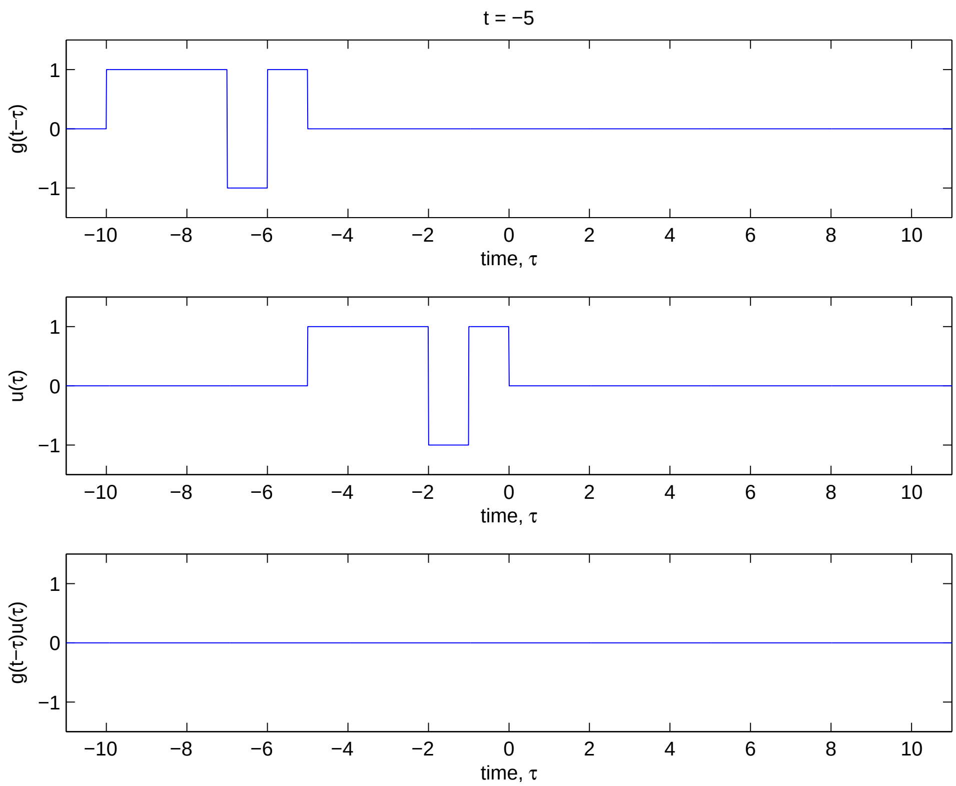
Solution. For each time t, draw $g(t - \tau)$, $u(\tau)$, and the product $g(t - \tau)u(\tau)$. The value of $y(t)$ is just the area under the curve of $g(t - \tau)u(\tau)$. I wrote a little Matlab script to automate this:

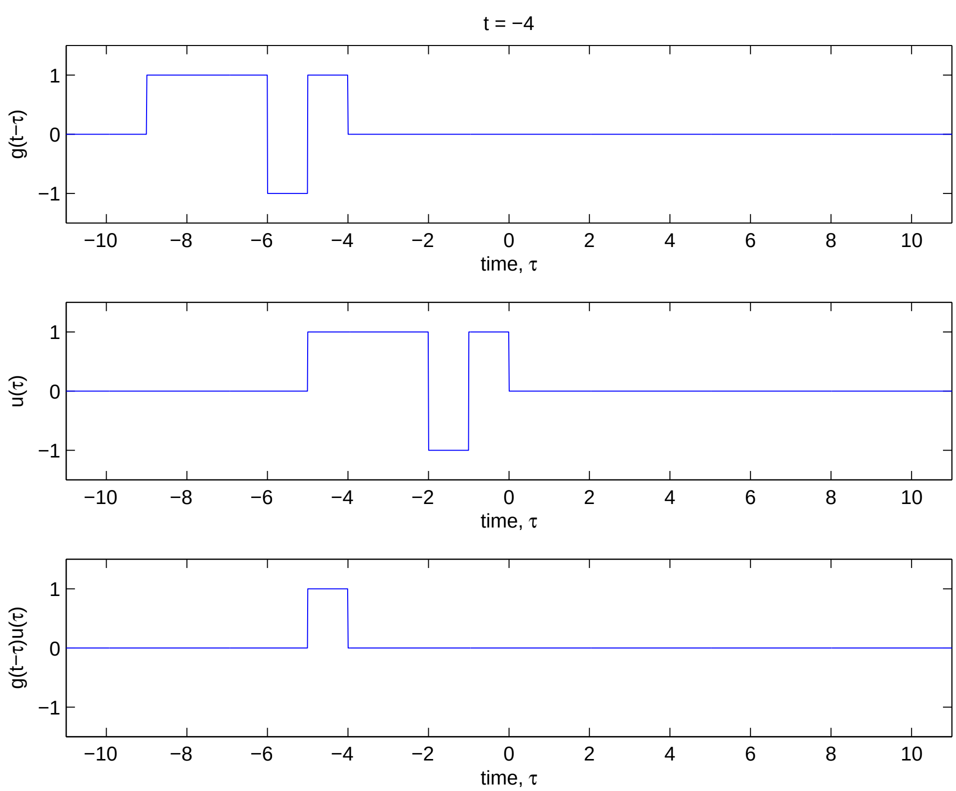
```
% Define implicit function
u = @(t) (t>=-5)-2*(t>=-2)+2*(t>=-1)-(t>=0);
g = @(t) u(-t);
dt = 0.01;
tau = -12+dt/2:dt:12;
n=0;
for t = -5:5

    n = n+1;
    figure(n);

    subplot(311)
    plot(tau,g(t-tau))
    axis([-11,11,-1.5,1.5])
    xlabel('time, t')
    ylabel('g(t-\tau)')
    subplot(312)
    plot(tau,u(tau))
    axis([-11,11,-1.5,1.5])
    xlabel('time, t')
    ylabel('u(\tau)')
    subplot(313)
    plot(tau,g(t-tau).*u(tau))
    axis([-11,11,-1.5,1.5])
```

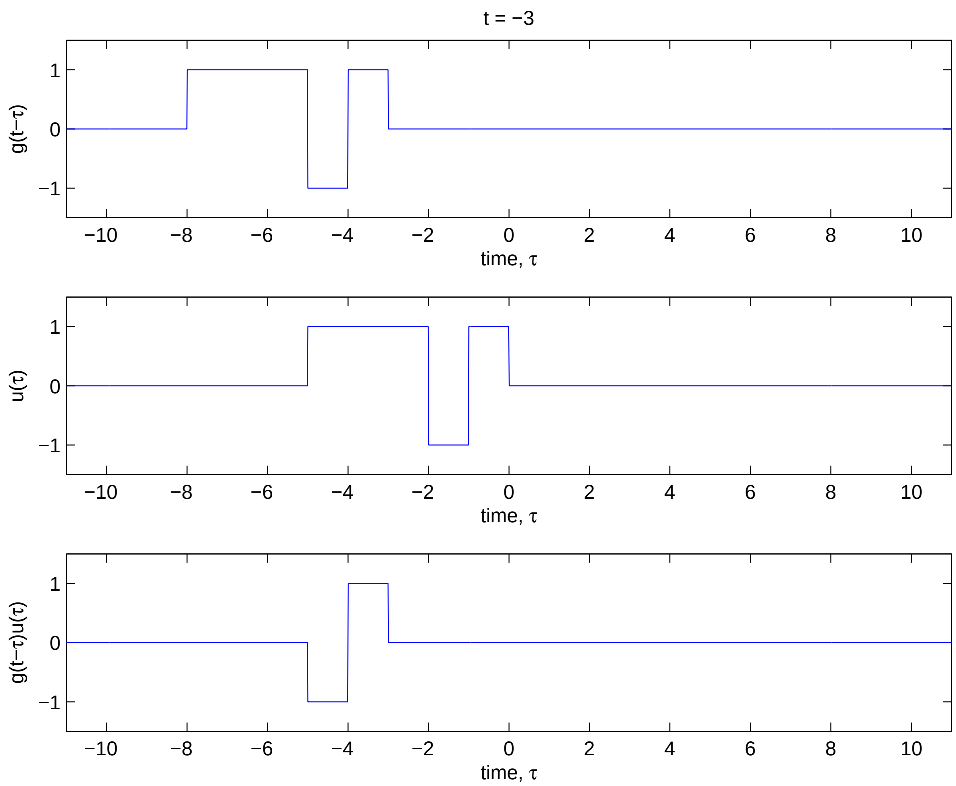
```
    xlabel('time, t')
    ylabel('g(t-\tau)u(\tau)')

    drawnow
    print -depsc ['figure' num2str(n) '.eps']

    % the convolution integral

    area = sum(g(t-tau).*u(tau))*dt;
    disp(sprintf('y(%d) = %f',t,area))

end
```

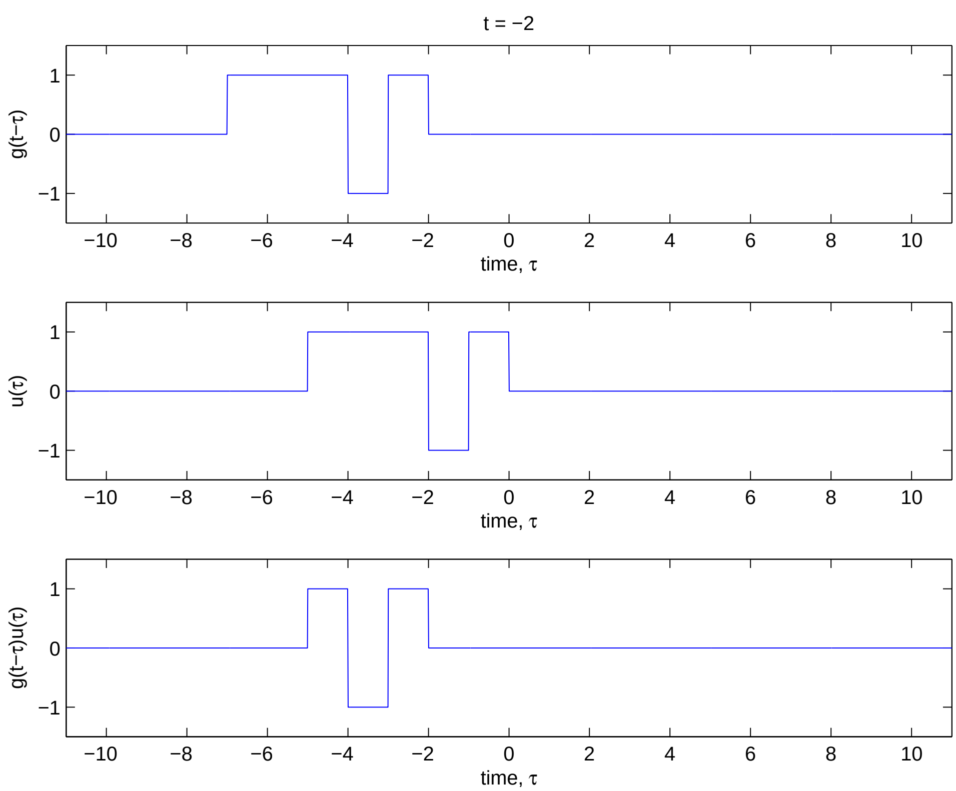
The result is printed below; the plots follow.

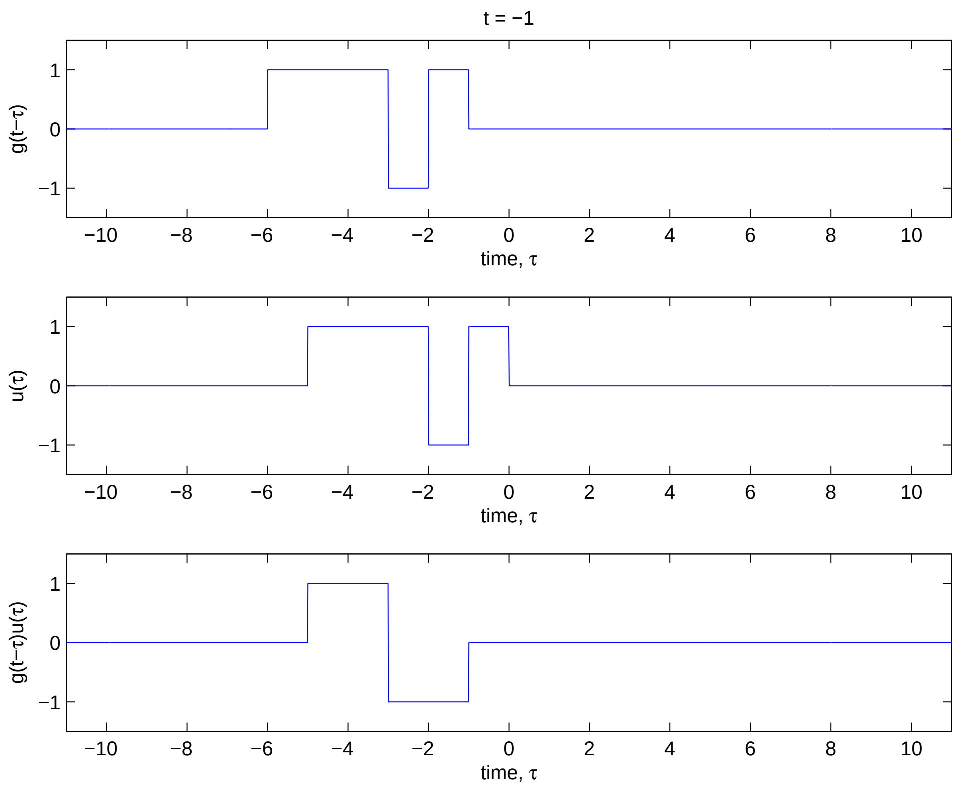
```
y(-5) = 0.000000
y(-4) = 1.000000
y(-3) = 0.000000
y(-2) = 1.000000
y(-1) = 0.000000
y(0) = 5.000000
y(1) = 0.000000
y(2) = 1.000000
y(3) = 0.000000
y(4) = 1.000000
y(5) = 0.000000
```

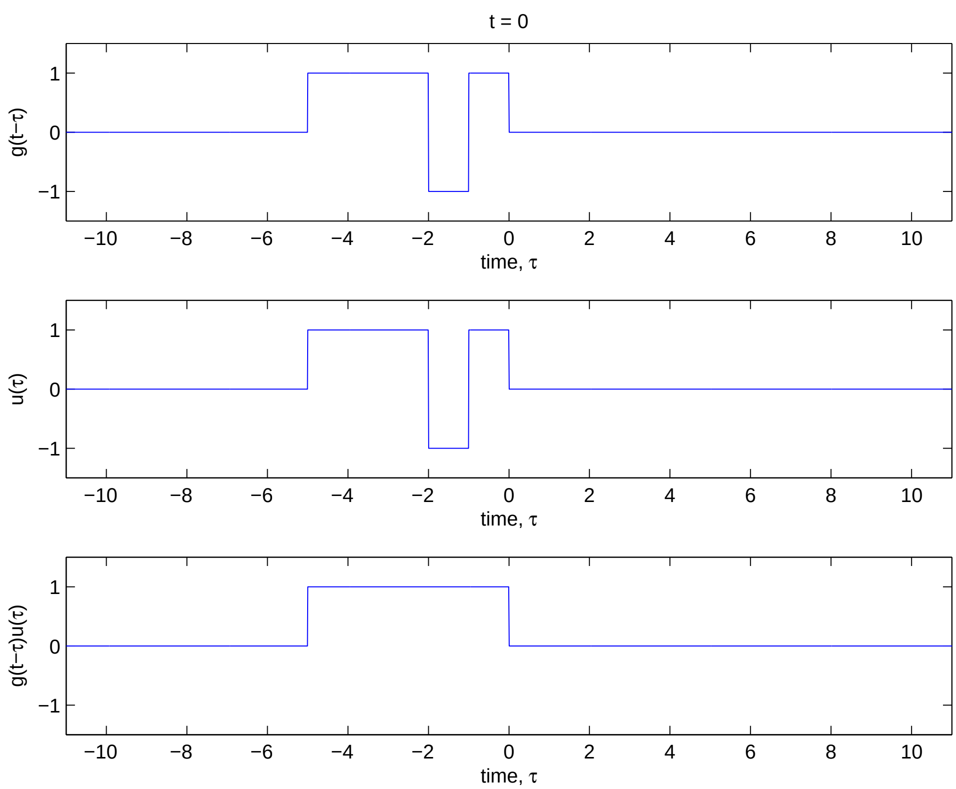

t = −5
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ

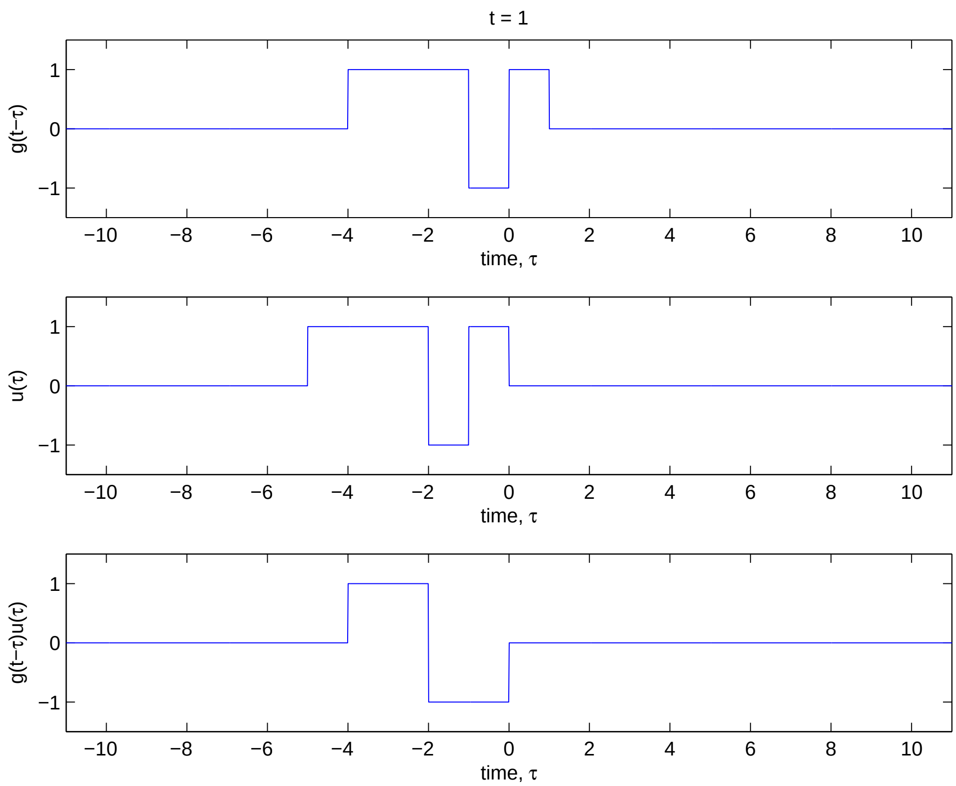

t = −4
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ

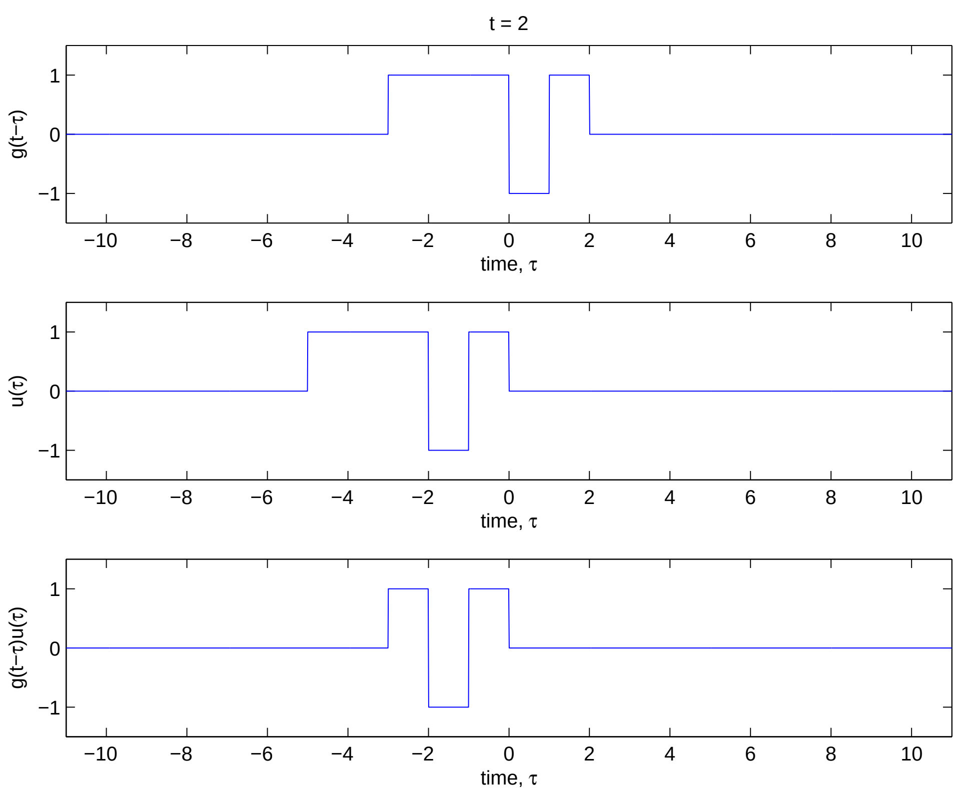

t = −3
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10
−8
−6
−4
−2
0
2
4
6
8
10
time, τ


t = −2
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10
−8
−6
−4
−2
0
2
4
6
8
10
time, τ

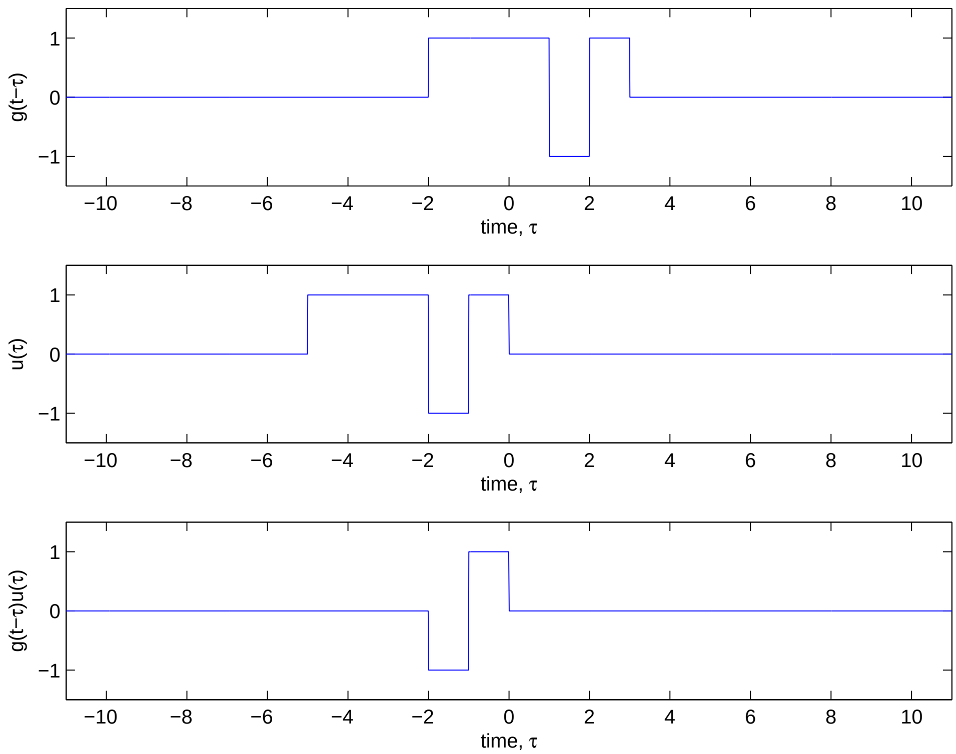

t = −1
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ


t = 0
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ

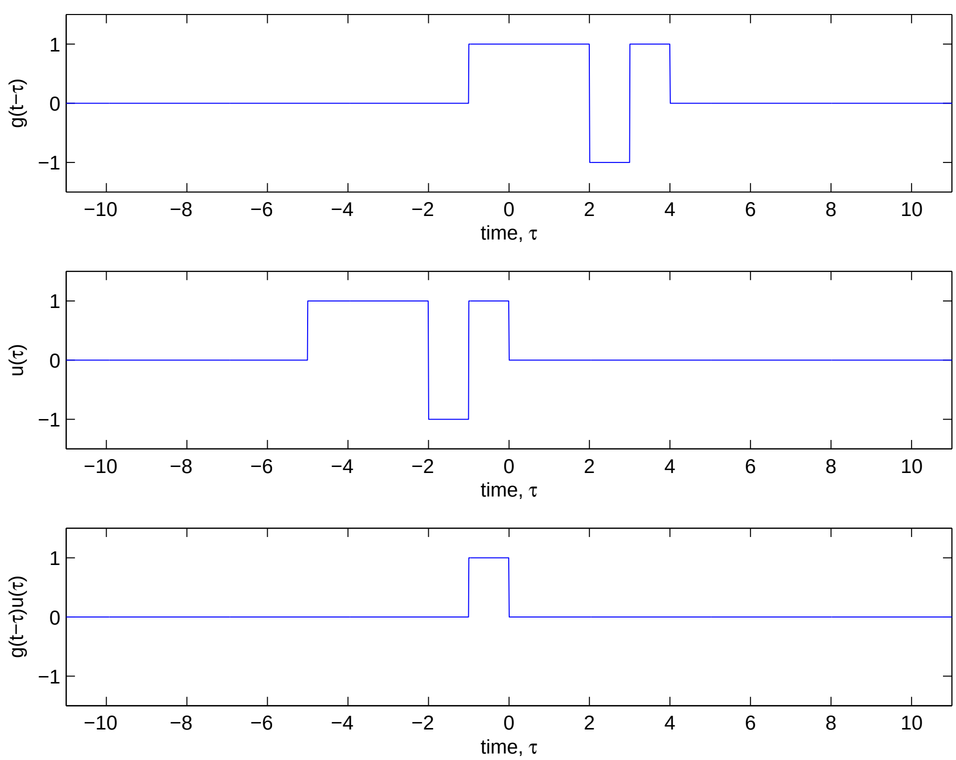

t = 1
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ

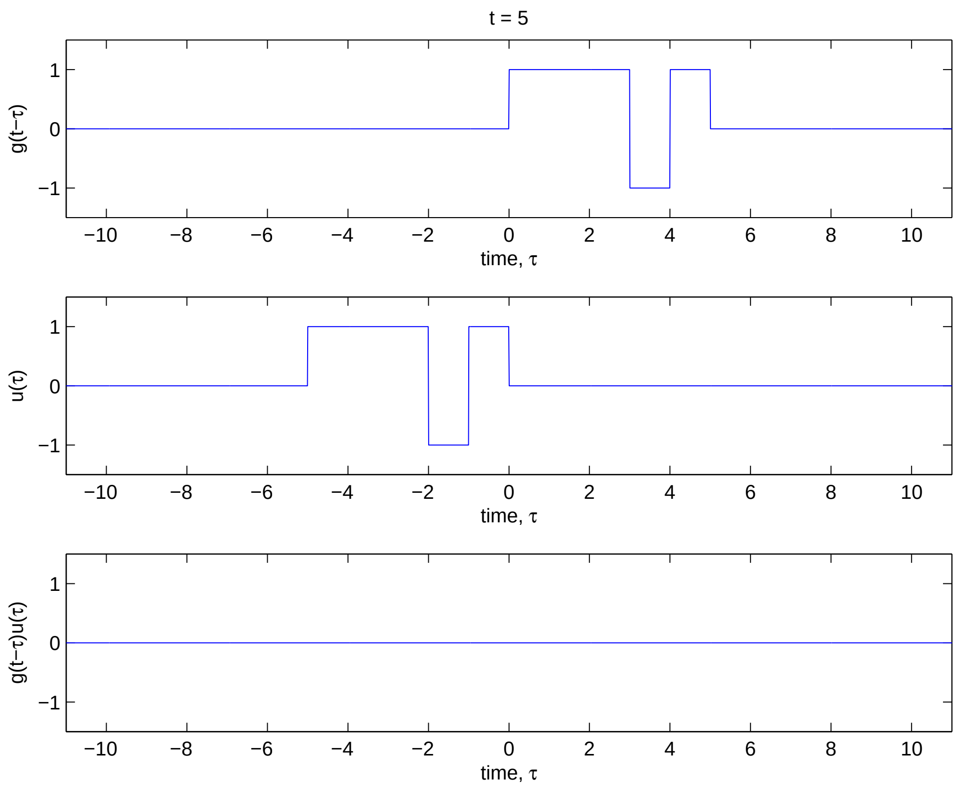

t = 2
g(t−τ)
u(τ)
g(t−τ)u(τ)
1
0
−1
−10 −8 −6 −4 −2 0 2 4 6 8 10
time, τ

t = 3

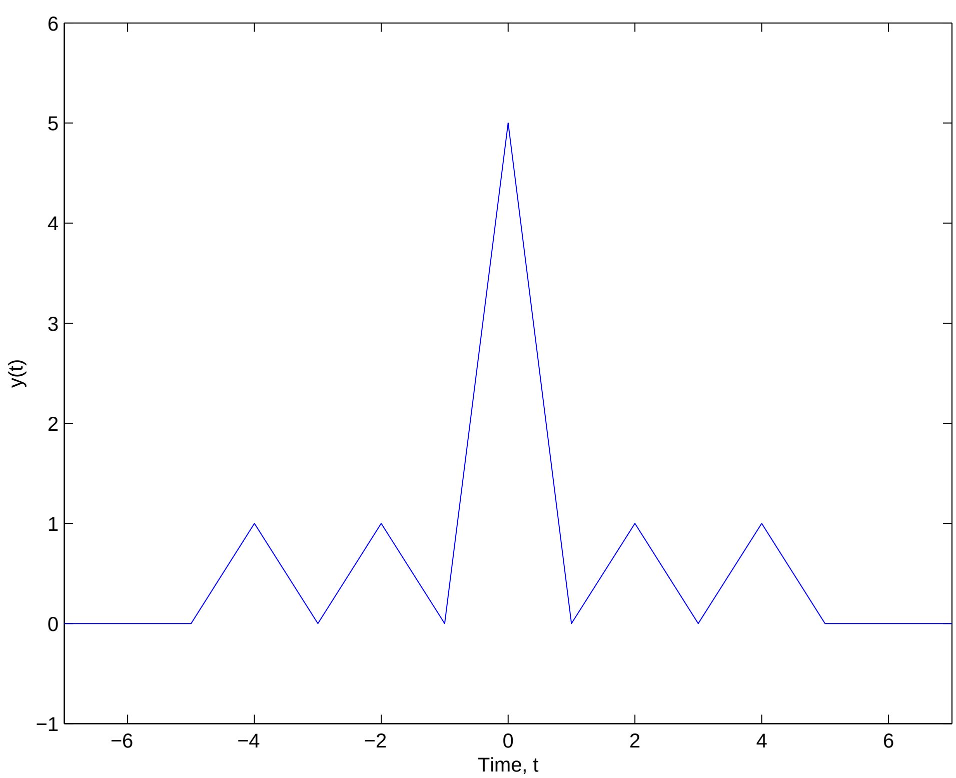


t = 4





Because u and g are piecewise cosntant, y will be pirecwise linear and continuous. Therefore, $y(t)$ is as shown below:



2. Using the results of part (1) and the LTI properties of the system G, what is the convolution
$$y(t) = g(t) * u(t - T)$$

   Solution. Time invariance implies that the solution is just the solution of part (1) shifted to the right by T.

3. What feature of $y(t)$ would you use to identify the time T?

   Solution. For the unshifted $y(t)$, the peak of the function is quite pronounced at $t = 0$. For the shifted function, the peak will be quite pronounced at $t = T$. therefore, we whould use the peak of the signal $y(t)$ to indicate the time delay T.

4. Can you explain why the right impulse response for G is a signal that has the same shape as $u(t)$, but time reversed?

   Solution. To make $g(t - \tau)$ and $u(\tau)$ line up exactly when $t = 0$, we must have therefroe that $g(-\tau) = u(\tau)$. This ensures that the highest peak in the convolution will occur when $t = 0$ (or $t = T$ in the time-shifted case).

Unified Engineering II **Spring 2007**

Problem S7 Solution (Signals and Systems)

1. The convolution is given by

$$y(t) = g(t) * u(t) = \int_{-\infty}^{\infty} g(t-\tau)u(\tau)\,d\tau$$

Note that $u(\tau)$ is nonzero only for $-3 \le \tau \le 0$, and $g(t-\tau)$ is nonzero only for $0 \le t-\tau \le 3$, that is, for $-3+t \le \tau \le t$. So there are four distinct regimes:

(a) $t < -3$
(b) $-3 \le t \le 0$
(c) $0 \le t \le 3$
(d) $t > 3$

For cases (a) and (d), there is no overlap between $g(t-\tau)$ and $u(\tau)$, so $y(t) = 0$. For case (b), the overlap is for $-3 \le \tau \le t$. So

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} g(t-\tau)u(\tau)\,d\tau \\ &= \int_{-3}^{t} \sin(-2\pi(t-\tau))\sin(2\pi\tau)\,d\tau \end{aligned}$$

At this point, we have to do a little trig:

$$\begin{aligned} \sin(-2\pi(t-\tau))\sin(2\pi\tau) &= \sin(2\pi(\tau-t))\sin(2\pi\tau) \\ &= [\sin(2\pi\tau)\cos(2\pi t) - \cos(2\pi\tau)\sin(2\pi t)]\sin(2\pi\tau) \\ &= \cos(2\pi t)\sin^2(2\pi\tau) - \sin(2\pi t)\cos(2\pi\tau)\sin(2\pi\tau) \\ &= \cos(2\pi t)\frac{1-\cos(4\pi\tau)}{2} - \sin(2\pi t)\frac{\sin(4\pi\tau)}{2} \end{aligned}$$

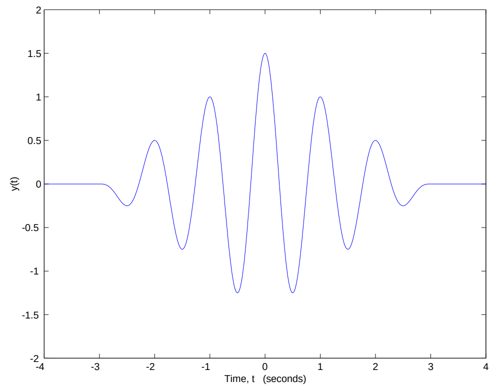
So the integral is given by

$$\begin{aligned} y(t) &= \int_{-3}^{t} \frac{\cos(2\pi t)}{2}\,d\tau - \int_{-3}^{t} \frac{\cos(2\pi t)}{2}\cos(4\pi\tau)\,d\tau - \int_{-3}^{t} \frac{\sin(2\pi t)}{2}\sin(4\pi\tau)\,d\tau \\ &= \frac{\cos(2\pi t)}{2}(t+3) - \frac{\cos(2\pi t)}{8\pi}\sin(4\pi\tau)\Big|_{\tau=-3}^{t} + \frac{\sin(2\pi t)}{8\pi}\cos(4\pi\tau)\Big|_{\tau=-3}^{t} \\ &= \frac{\cos(2\pi t)}{2}(t+3) - \frac{\cos(2\pi t)}{8\pi}\sin(4\pi t) + \frac{\sin(2\pi t)}{8\pi}[\cos(4\pi t) - 1] \end{aligned}$$

(As often happens with problems involving trig functions, there are other equivalent expressions.) For case (c), the region of integration is $-3+t \le \tau \le 0$. So

$$\begin{aligned} y(t) &= \int_{-3+t}^{0} \frac{\cos(2\pi t)}{2}\,d\tau - \int_{-3+t}^{0} \frac{\cos(2\pi t)}{2}\cos(4\pi\tau)\,d\tau - \int_{-3+t}^{0} \frac{\sin(2\pi t)}{2}\sin(4\pi\tau)\,d\tau \\ &= \frac{\cos(2\pi t)}{2}(3-t) - \frac{\cos(2\pi t)}{8\pi}\sin(4\pi\tau)\Big|_{\tau=-3+t}^{0} + \frac{\sin(2\pi t)}{8\pi}\cos(4\pi\tau)\Big|_{\tau=-3+t}^{0} \\ &= \frac{\cos(2\pi t)}{2}(3-t) + \frac{\cos(2\pi t)}{8\pi}\sin(4\pi t) - \frac{\sin(2\pi t)}{8\pi}[\cos(4\pi t) - 1] \end{aligned}$$

2. $y(t)$ is plotted below.



3. The maximum value of $y(t-T)$ occurs at time T. So I would use this center peak to identify the delay time T.

4. The adjacent peaks are nearly as tall as the center peak, so if noise were added to the signal, the tallest peak might not be the center peak, so we might use the wrong peak to determine the delay time.

5. The chirp signal of Problem S6 produces an ambiguity function with only one prominent peak. Therefore, the addition of noise should not make it difficult to accurately determine the delay time.