Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 6

Name: ________________________

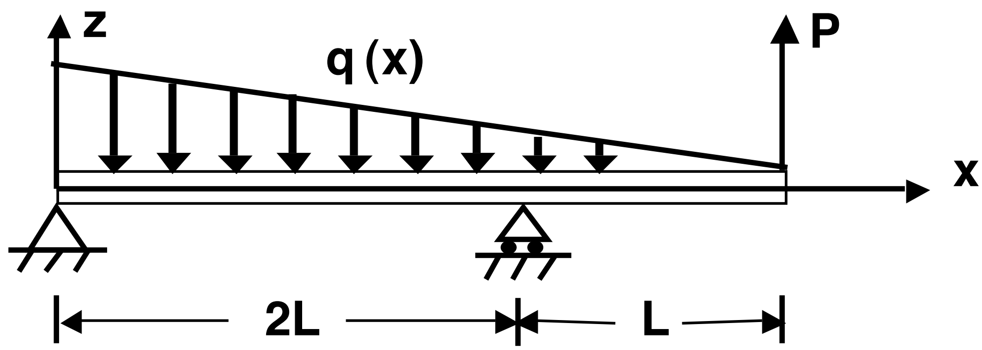
Due Date: 03/20/2007

	Time Spent (min)
M6.1	
M6.2	
M6.3	
S8	
S9	
S10	
Study Time	

Announcements:

Unified Engineering Problem Set **Lectures: M4, M5, M6**
Week 6 Spring, 2007 **Units: M4.3**

M6.1 ***(10 points)*** A beam of total length 3L is pinned at one end and has a roller support at the two-thirds point (2L). The beam is loaded by a linearly-varying distributed downward loading of magnitude q(x) that tapers to zero at the tip, and by a concentrated upward load of magnitude P at the beam tip. The load, P, is equal in magnitude to the integrated load due to the linearly varying distributed loading, q(x).

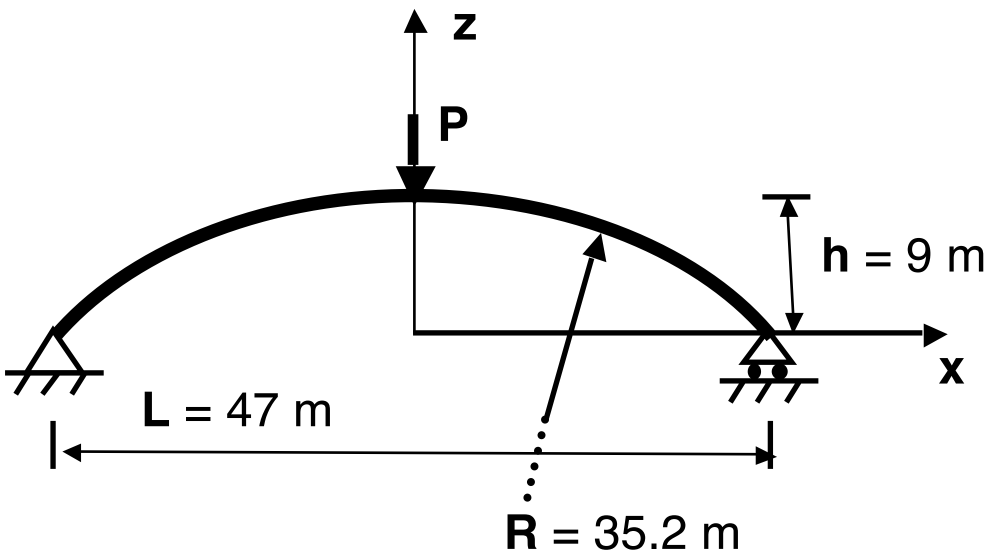


(a) Determine the reactions for this structural configuration.

(b) Using the relationships between loading, shear, and moment, determine the loading, shear, and moment distributions. Draw these diagrams.

(c) Check the obtained values for these parameters at the mid-point between the two beam supports (x = L).

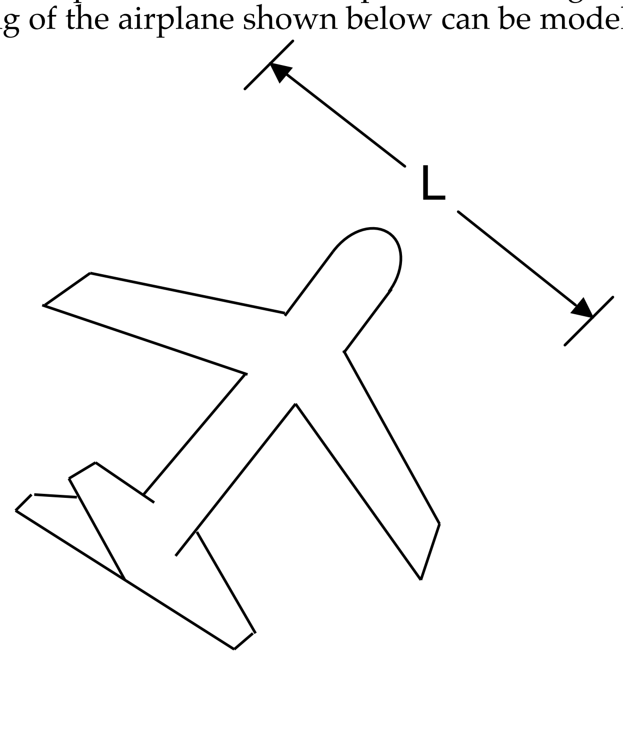
M6.2 ***(10 points)*** We will model the primary arched beam of the front façade of the Kresge auditorium as an arc piece of a circular geometry with a radius of 35.2 meters. One end is supported by a pinned support (the point near McCormick) and the other by a roller support (the point near the Student Center). The distance between the two points is 47.0 meters. The distance of the beam from the ground at its peak is 9.0 meters. Consider the case of a concentrated load, P, applied downward at the peak.



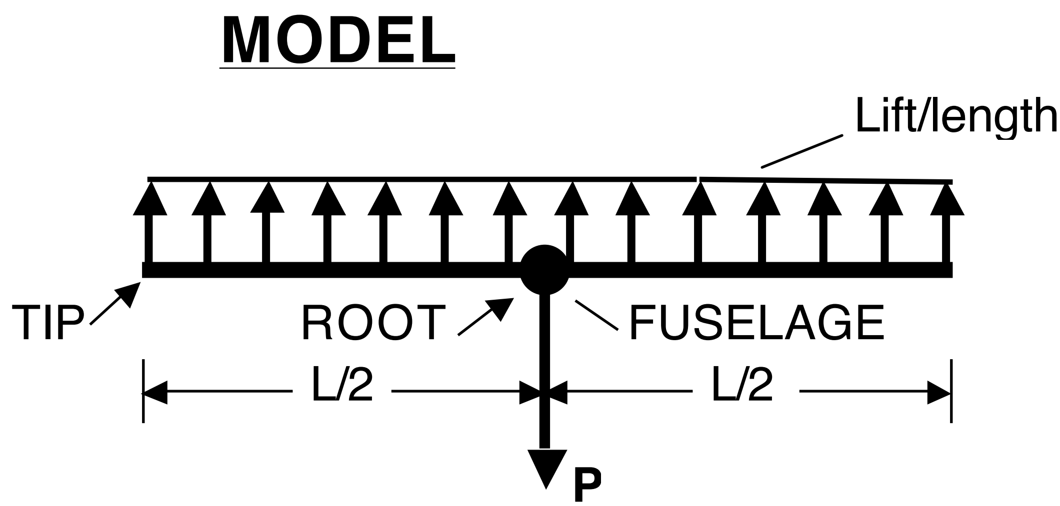
(a) Determine the reactions for this structural configuration.

(b) Determine the axial force, shear force, and bending moment along the beam as functions of the distance from the origin along the x-axis. Draw these as functions of x.
(NOTE: The axial and shear force resultants at any point act **parallel** and **perpendicular** to the tangent line of a beam at that point. This is because the "virtual cut" is perpendicular to the tangent line.)

M6.3 ***(10 points)*** Now that we have begun to learn about beams, we can start out on simple models of airplanes. Let's first explore how wings carry load in level flight. The wing of the airplane shown below can be modeled as a beam of



total span L which has no supports. The beam has a concentrated load (the weight of the *fuselage*[1], its contents, and the *empennage*[2]) P at its center and a distributed load (the lift of the wing minus its weight) along its span. You learn from Fluids that this distribution is often modeled as varying, with different possible variations, along the span. For simplicity in an initial model, let us assume that we can model the distribution as being a distributed load of constant magnitude. We also ignore the weight of the wing at this point. The model is shown here.



(a) Determine the reactions for this structural configuration.

(b) Determine the axial force, shear force, and bending moment as functions of the distance from the *root*[3] of the wing.

(c) Using common sense arguments and the results from part (b), describe where it is likely that the wing is most highly loaded.

(for thought) We now increase the wingspan by 10% (i.e. by a factor of 1.1) while keeping the total lift and the center load P the same. What is the effect on the shear and bending moment at the root? (**NOTE:** You should be able to answer this by inspection.)

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1 The *fuselage* is that part of the airplane between the wings where the passengers and/or freight are carried.
2 The *empennage* is more commonly known as the tail.
3 The *root* of the wing is the location where the wing is joined to the fuselage.

Problem S8 (Signals and Systems)

For each of the functions below, find the Laplace transform of the function, as well as the region of convergence. Do not use any outside reference (table of transforms or the Unified bible) to find the answer. You may use a table of Laplace transforms to *check* your work. However, show the derivation of the result, *i.e.*, work out the Laplace integral. Make sure that you include the region of convergence.

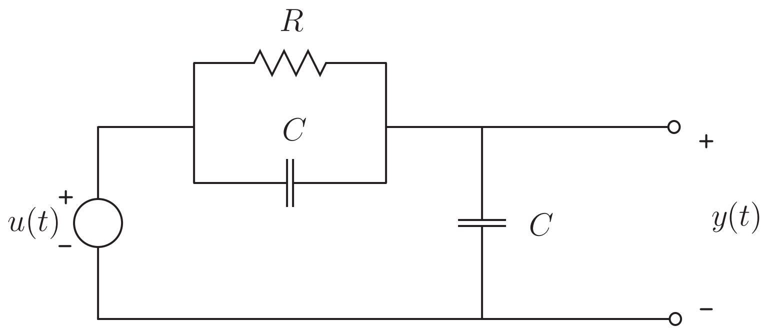
1. $g(t) = \begin{cases} te^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

2. $g(t) = \begin{cases} t^2e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

3. $g(t) = \begin{cases} t^ne^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$, where n is a positive integer.

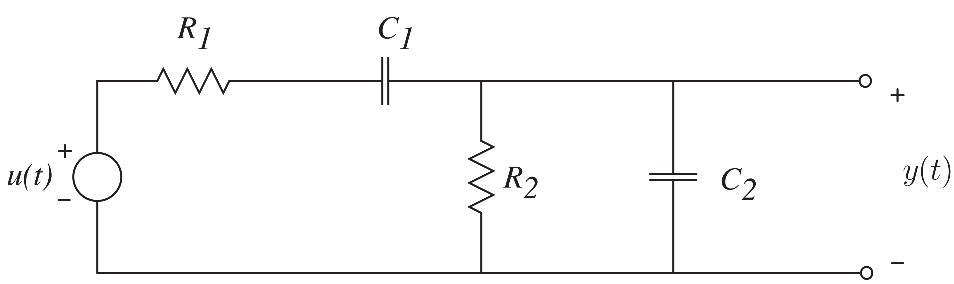
Problem S9 (Signals and Systems)

1. Find and plot the step response of the circuit below, using Laplace transform and impedance techniques.



Assume that $R = 2\ \Omega$, and $C = 0.1\,\mathrm{F}$. Note: This is a problem that's tricky to do using ordinary differential equation methods — Can you see why?

2. Find and plot the step response of the circuit below, using Laplace transform techniques.



Assume that $R_1 = R_2 = 2\,\Omega$, $C_1 = 0.2\,\mathrm{F}$, and $C_2 = 0.3\,\mathrm{F}$.

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Problem S10 (Signals and Systems)

This problem provides lots of practice using partial fraction expansions to determine inverse Laplace transforms. Please use the coverup method — it really is superior to other methods, and more reliable. Also, please check your answer, that is, verify that your expansion really is equivalent to the $G(s)$ given. For each of the following Laplace transforms, find the inverse Laplace transform.

1. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad \text{Re}[s] > 2$

2. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad \text{Re}[s] > -1$

3. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad \text{Re}[s] > -1$

4. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad \text{Re}[s] > 0$

5. $G(s) = \dfrac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)}, \quad \text{Re}[s] > 0$