Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 7

Name: ________________________

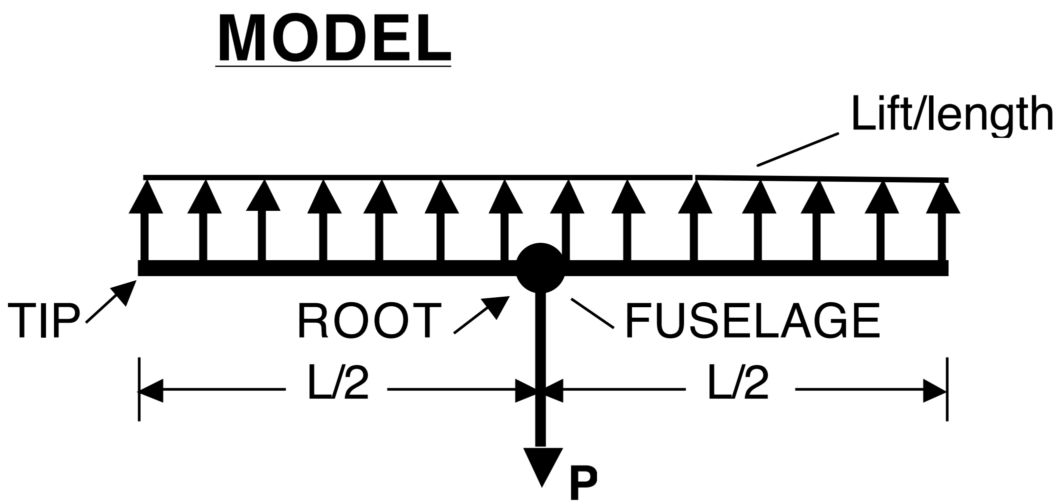
Due Date: 04/03/2007

	Time Spent (min)
M7.1	
M7.2	
M7.3	
F11-13	
Study Time	

Announcements: Have a good spring break

Unified Engineering Problem Set **Lectures: M7, M8, M9, M10**
Week 7 Spring, 2007 **Units: M4.4, M4.5**

M7.1 ***(10 points)*** Let's continue exploring our simple model of how wings carry load in level flight. We can expand this to consider stress distributions and deflections. The model of the load configuration is again shown below. Use the results from the solution for the problem set for Week #6 for the axial force, shear force, and bending moment as appropriate. Assume that the wing has constant cross-sectional properties of I and A and is made of an isotropic material with a modulus of E and Poisson's ratio ν.



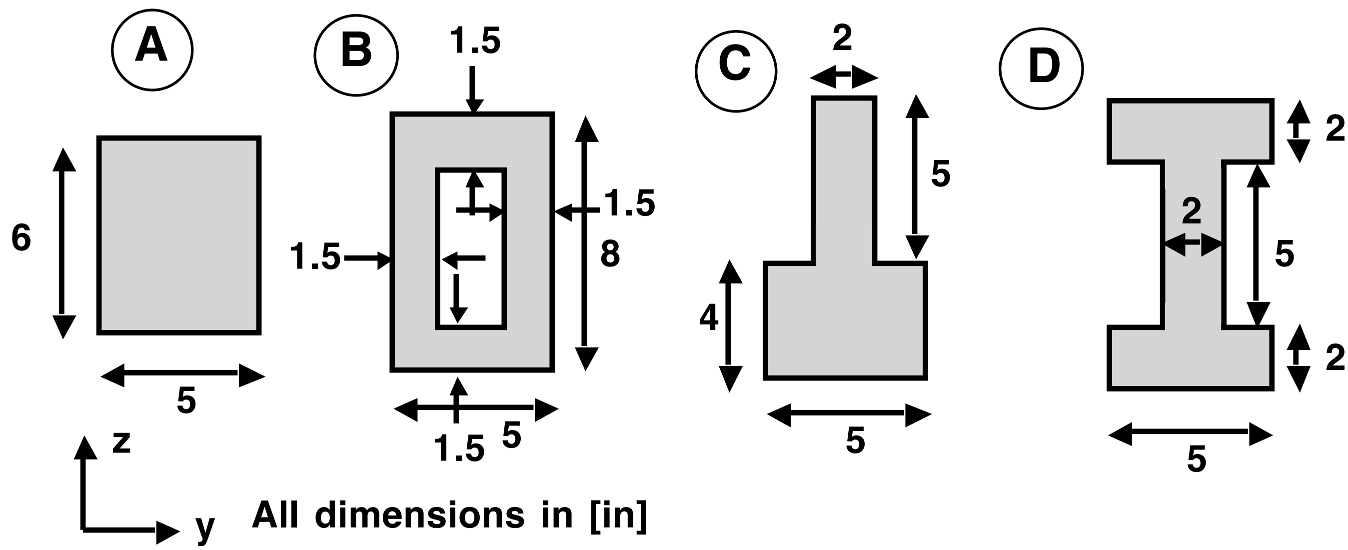
(a) Determine and sketch the distribution of the axial stress, σ_{xx}, along the wing; and find the location of the maximum value along the wing.

(b) Determine and sketch the distribution of the shear stress, σ_{xz}, along the wing; and find the location of the maximum value along the wing.

(c) Determine and sketch the deflection of the wing, w; and find the location of the maximum value along the wing.

(for thought) As in the problem from last week, we now increase the wingspan by 10% (i.e. by a factor of 1.1) while keeping the total lift and the center load P the same. What is the effect on the maximum values of the axial and shear stresses and their location? What is the effect on the maximum value of the deflection w and its location? Compare the slope of the wing, dw/dx, at this location (the slope is known as the "*dihedral angle*").

M7.2 ***(15 points)*** You are asked to evaluate different designs of a 10-foot long statically determinate cantilevered beam to be made out of titanium. The beam is loaded by a downward tip load of 1000 pounds. Four different cross-sections are under consideration: a solid rectangle, a rectangular tube, a T-beam, and an I-beam. In each case, the cross-sectional area of the beam is the same: 30 in^2. The dimensions of each of the cross-sections are given in the accompanying figure.



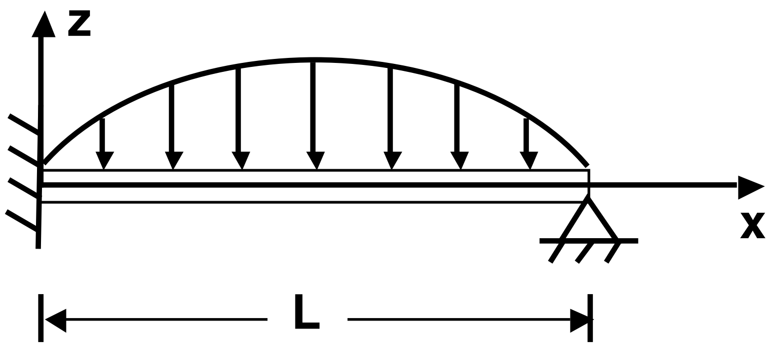
(a) Determine the cross-section which will give the smallest deflection of the beam.

(b) Determine the cross-section which will have the smallest value of the maximum magnitude of the axial stress σ_{xx} and find the location in the cross-sectional plane.

(c) Determine the cross-section which will have the smallest value of the maximum magnitude of the shear stress τ_{xz} and find the location in the cross-sectional plane.

(d) Comment on the possible beam selections.

M7.3 ***(15 points)*** A beam of length L is clamped at one end and pinned at the other. The beam has a constant cross-section with area A and moment of inertia I, and is made of a material with modulus E and Poisson's ratio □. The beam is loaded by a quadratically-varying downward load of intensity equal to zero at each end and a maximum intensity of p_o.



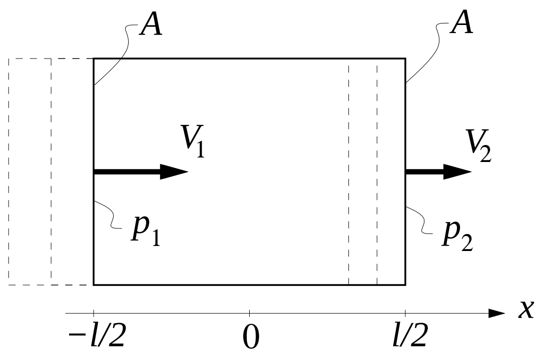
(a) Determine the maximum deflection of this beam and its location.

(b) Determine the maximum axial stress magnitude, $□_{xx}$, and its location in the x-direction.

(c) Determine the maximum shear stress magnitude, $□_{xz}$, and its location in the x-direction.

Unified Engineering Spring 2007
Fluids Problems F11+F12+F13 (worth 3 Pset Q's)

A box-shaped Lagrangian Control Volume of cross-sectional area A is moving as shown. The velocity and pressure are uniform over each face 1 and 2, but their values are slightly different between faces 1 and 2. The instantaneous locations of faces 1 and 2 are at $x = \pm\ell/2$.



a) Determine the volume rate of change $d\mathcal{V}/dt$ of the CV. Also determine the velocity V_{cg} of the CV's center of gravity, assuming it has a uniform density ρ inside.

b) Evaluate the overall pressure-work integral for this CV.

$$\dot{W} \;=\; \oiint -p\,\hat{n}\cdot\vec{V}\,dA$$

c) Rewrite your result from b) in terms of only the average and difference quantities:

$$p_{\text{avg}} \;=\; \tfrac{1}{2}(p_1 + p_2) \qquad\qquad \Delta p \;=\; p_1 - p_2$$

$$V_{\text{avg}} \;=\; \tfrac{1}{2}(V_1 + V_2) \qquad\qquad \Delta V \;=\; V_1 - V_2$$

Simplify the result as much as possible, into only two terms.

d) Determine the kinetic energy of the gas inside the CV, assuming V varies linearly in x.

$$E_{\text{kin}} \;=\; \iiint \frac{1}{2}\rho V^2 \, d\mathcal{V}$$

Introduce the mass m inside the CV to simplify your result. Hint: This computation is much simpler if you write $V(x)$ in terms of V_{avg} and ΔV, rather than V_1 and V_2.

e) The First Law for a Lagrangian CV has no energy flux terms $\dot{E}_{\text{in}}$ and $\dot{E}_{\text{out}}$, since this type of CV is by definition impermeable. For the adiabatic case, where $\dot{Q} = 0$, the First Law for this CV then has the form

$$\frac{dE_{\text{int}}}{dt} + \frac{dE_{\text{kin}}}{dt} \;=\; \dot{W}$$

where the total energy E has been split up into its internal and kinetic energy parts. Which of the two terms of your c) result appears to give dE_{int}/dt, and which one appears to give dE_{kin}/dt? You must explain your reasoning. Hint: Consider the overall force on the CV, and also use your previous results as appropriate.

f) Determine the temperature rate of change dT/dt of the gas inside the CV. Assume the gas has some uniform density ρ and some specific heat c_v.