Massachusetts Institute of Technology
Department of Aeronautics and Astronautics
Cambridge, MA 02139

16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 8

Name: ______________________

Due Date: 04/10/2007

	Time Spent (min)
T10	
T11	
F14	
F15/16	
Study Time	

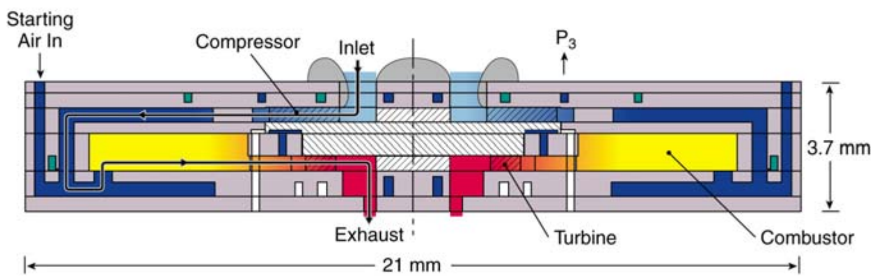
Announcements:

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T10

The Gas Turbine Laboratory in the Department is working on a breakthrough technology for thermal engines, called "Micro-Engines", which was invented by Professor Epstein, Director of the Laboratory. These are small (10mm in diameter or less) complete gas turbine engines made using micro-fabrication technology which has been developed in the electronics industry. The first use of these will be as a source of power in small packages (the power and energy densities are more than an order of magnitude larger than current batteries) but one can also think of many propulsion uses for them.



A key feature of a successful micro-engine is a high enough thermal efficiency. This problem addresses the component technology needed to achieve this. The peak temperature in the cycle is limited by the material used and is 1500K. Suppose the cycle pressure ratio is 3:1 and the turbine has an adiabatic efficiency of 0.7. The inlet conditions can be assumed 300 K and 1 bar. Assume air to be a perfect gas with $\gamma = 1.4$ and R = 287 J/kg K.

a) How does the thermodynamic cycle efficiency vary with compressor efficiency? (*A plot is wanted here)*

b) At what value of compressor efficiency does the cycle produce positive work?

c) If the inlet of the micro-engine has a diameter of 2 mm, and the inlet Mach number is 0.3, what sort of power can be delivered (via a micro power turbine downstream of the micro-engine) if the compressor efficiency is 0.55?

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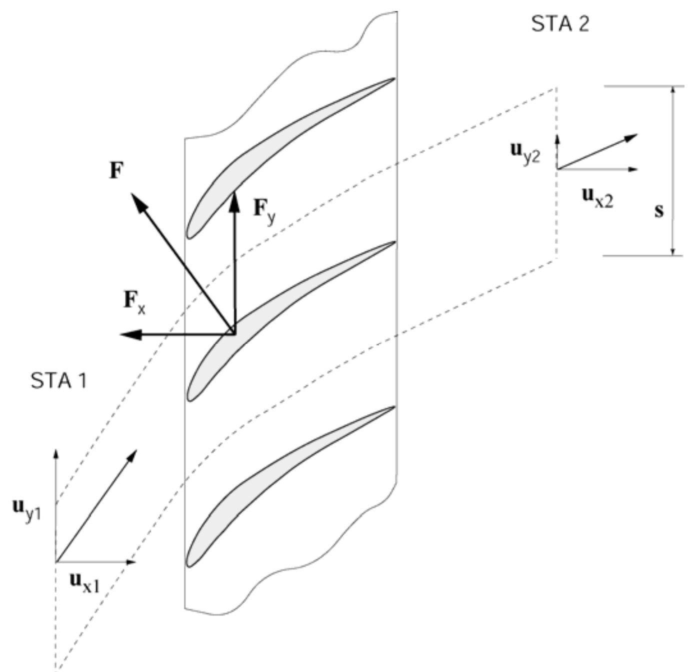
Problem T11

We are interested in the blade force of a row of turbomachinery airfoils in steady flow as shown below. The flow can be treated as incompressible and inviscid. Consider the control volume indicated by the dashed lines. The bounding surfaces are two streamlines a distance apart equal to the blade spacing, *s*, and two vertical lines parallel to the plane of the blade row which are far upstream and far downstream respectively. The depth of all faces of the control surfaces can be taken as unity.



a) Using the integral momentum theorem in the x-direction, what forces and fluxes need to be accounted for? Find the blade force F_x. Express your answer in terms of quantities given in the above figure and simplify your result.

b) Similarly, find the blade force in the y-direction, F_y and discuss the relevant terms in the integral momentum theorem.

c) Express the blade force components F_x and F_y in terms of the blade circulation Γ and find the magnitude of the total blade force F in terms of Γ and a vector mean velocity u_m. Interpret your result.

d) The limiting case of large blade spacing *s* is the isolated airfoil. Discuss your results from part c) in the limiting case of an infinite blade spacing. You can hold the circulation around the blade Γ constant.

A subsonic aircraft is flying at speed V_∞, in an atmosphere with p_∞, and ρ_∞.

a) What is the flight Mach number M_∞? Give in terms of the quantities above.

b) Determine the stagnation pressure p_o and the corresponding stagnation-pressure coefficient

$$C_{p_o} \equiv \frac{p_o - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

at the nose of the aircraft in two ways:

i) The exact full compressible equation.
ii) The incompressible Bernoulli equation, pretending $\rho = \rho_\infty$ is constant.

Plot p_o/p_∞ versus M_∞ for the two equations.
Also plot C_{p_o} versus M_∞ for the two equations.

c) What would you judge to be a reasonable upper Mach limit on the validity of the incompressible Bernoulli equation (with the assumption $\rho = \rho_\infty$)? Be sure to state your criterion.

Fluids Problem F15+F16 (Worth 2 Pset Q's)

A rocket motor explodes during a ground test, sending a spherical shock wave traveling away from the explosion into still ambient air which has the following conditions:
$T = 300\,\mathrm{K}$, $p = 100\,\mathrm{KPa}$.

When the shock reaches an observer some distance away from the explosion point, the observer feels a sudden increase in the ambient static pressure of $\Delta p = 50\,\mathrm{KPa}$. (Note: This sounds traumatic, but you feel the same pressure rise when diving down 17 feet underwater).

a) Mach number of the flow into the shock, in the shock's frame.

b) The air temperature behind the shock

c) The velocity of the shock relative to the observer.

d) The air velocity felt by the observer after the shock passes.

e) The observer has a pitot tube which reads $(p_o)_{\mathrm{before}}$ before the shock passes, and then $(p_o)_{\mathrm{after}}$ after the shock passes. Determine these two pressures, applying the usual stagnation-pressure definition in each case,

$$p_o \equiv p\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Compare their ratio given by the stagnation pressure ratio formula for a shock, equation (1) in the F16 notes. Explain any discrepancy.