UE Fluids Problem 14 Solution S'07

a) $a_\infty = \sqrt{\gamma p_\infty / \rho_\infty}$, $\boxed{M_\infty \equiv \frac{V_\infty}{a_\infty} = \frac{V_\infty}{\sqrt{\gamma p_\infty/\rho_\infty}}}$

b) i) $p_{o_\infty} = p_\infty \left[1 + \frac{\gamma-1}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}$, also $\rho_\infty V_\infty^2 = \gamma p_\infty M_\infty^2$

Since there are no bow shock losses, at nose, $p_o = p_{o_\infty}$, hence

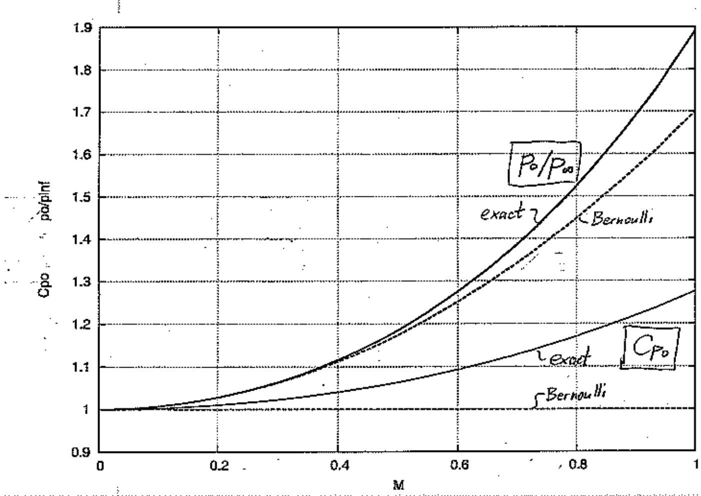
$$\boxed{C_{p_o} \equiv \frac{p_o - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{p_\infty\left[1+\frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} - p_\infty}{\frac{1}{2}\gamma p_\infty M_\infty^2} = \frac{2}{\gamma M_\infty^2}\left[\left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]}$$

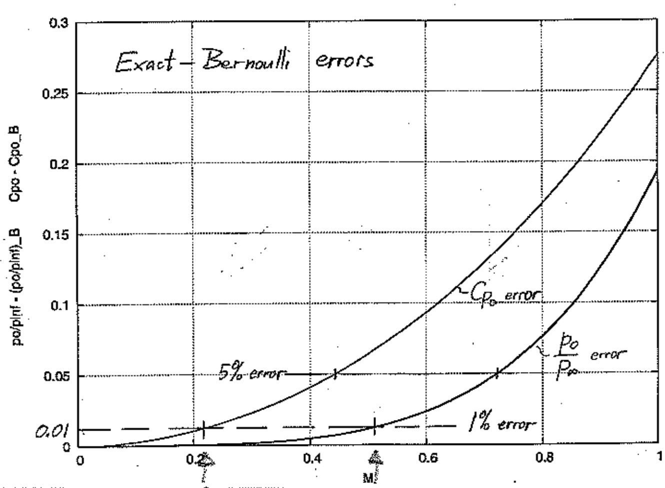
ii) $p_o = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2$ (assumed)

$$\boxed{C_{p_o} \equiv \frac{p_o - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\frac{1}{2}\rho_\infty V_\infty^2}{\frac{1}{2}\rho_\infty V_\infty^2} = 1}$$

Also for plotting: i) $\frac{p_o}{p_\infty} = \frac{p_\infty\left[1+\frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}{p_\infty} = \left[1+\frac{\gamma-1}{2}M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}$

ii) $\frac{p_o}{p_\infty} = \frac{p_\infty + \frac{1}{2}\rho_\infty V_\infty^2}{p_\infty} = 1 + \frac{\gamma}{2}M_\infty^2$





c) If we arbitrarily set 1% max error, then the maximum M_∞ for Bernoulli validity are $M_\infty \leq 0.22$ for C_{p_o}, and $M_\infty \leq 0.51$ for p_o/p_∞. With 5% error, the limits are $M_\infty \leq 0.45, 0.72$ respectively

UE Fluids   Problem 15 + 16   Solution   S '07

a) Shock static pressures: $p_1 = 100$ KPa, $P_2 = 100$ KPa $+ 50$ KPa $= 150$ KPa.

Shock pressure ratio $p_2/p_1 = 150/100 = 1.5$

From $\frac{p_2}{p_1}(M_1)$ shock function (shock table in App. B), this corresponds to $\boxed{M_1 = 1.195}$

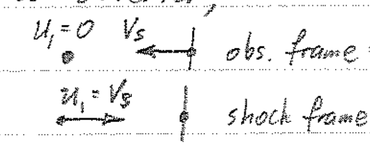
or approximately $\boxed{M_1 \approx 1.2}$

b) For this M_1, the table gives: $T_2/T_1 = 1.125$

hence, $\boxed{T_2 = T_1 \cdot \frac{T_2}{T_1} = 300K \cdot 1.125 = \underline{337.5\ K}} = 64.5\ C = 148\ F$ toasty

c) Since the air upstream of shock is still relative to observer,

the shock velocity $= u_1$ as seen by shock



$u_1 = M_1 a_1$, $a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \cdot 287 \cdot 300} = 347.2$ m/s

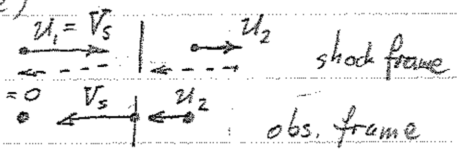
$\boxed{u_1 = 1.195 \cdot 347.2 \text{ m/s} = 414.9 \text{ m/s} = V_s}$

d) In shock frame: $u_2 = M_2 a_2$

From shock table: $M_2 = 0.8454$, also $a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \cdot 287 \cdot 337.5} = 368.1$ m/s

$\rightarrow u_2 = 0.8454 \cdot 368.1 \text{ m/s} = 311.2$ m/s (in shock frame)

In the observer's frame:



$u_2\big|_{obs} = u_2\big|_{shock} - V_s = 311.2 - 414.9 = -103.6$ m/s

(to the left)

e) In the observer's frame: $M_1\big|_{obs} = 0$, $p_{o1}\big|_{obs} = p_1 = 100$ KPa

$M_2\big|_{obs} = u_2\big|_{obs} / a_2 = 103.6 \text{ m/s} / 368.1 \text{ m/s} = 0.282$

$$p_{o2}\big|_{obs} = p_2\left[1 + \frac{\gamma-1}{2} M_2\big|_{obs}^2\right]^{\frac{\gamma}{\gamma-1}} = 150 \text{ KPa}\left[1 + 0.2 \cdot 0.282^2\right]^{3.5} = 158.5 \text{ KPa.}$$

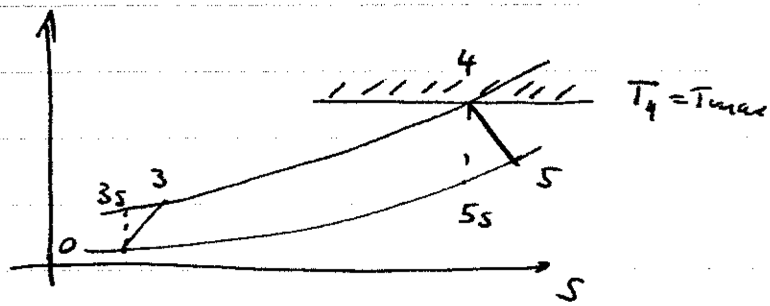
$\therefore \boxed{\frac{p_{o2}}{p_{o1}}\bigg|_{obs} = \frac{158.5}{100} = 1.585}$, shock table: $\boxed{\frac{p_{o2}}{p_{o1}} = 0.993}$ or equation (1) in F16

They are different because the observer is <u>not</u> in the steady shock frame.

Problem T10

MIT μ-engine cycle

$\pi = 3$, $\tau_{max} = \frac{T_4}{T_0} = 5$, $\eta_T = 0.7$



Concepts: 1st and 2nd law, definition of thermal eff, adiab. eff.

a) $\eta_{th} = \frac{w_{net}}{q_A} = 1 - \frac{q_R}{q_A} = 1 - \frac{T_5 - T_0}{T_4 - T_3}$ define $\tau_s = \pi^{\frac{\gamma-1}{\gamma}}$

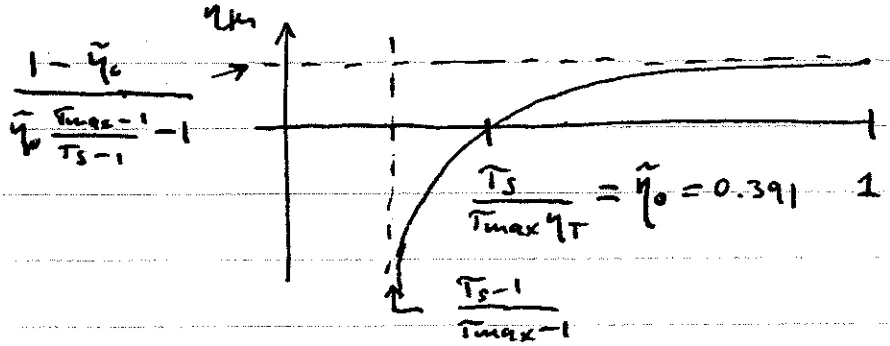
Using def. of η_T and isentropic relations: $T_5 = T_4\left(1 - \eta_T\left(1 - \frac{1}{\tau_s}\right)\right)$

similarly for compressor: $T_3 = T_0\left(1 + \frac{1}{\eta_c}(\tau_s - 1)\right)$

Substitute in above: $$\eta_{th} = 1 - \frac{T_4/T_0\left(1 - \eta_T\left(1 - \frac{1}{\tau_s}\right)\right) - 1}{T_4/T_0 - \left(1 + 1/\eta_c(\tau_s - 1)\right)}$$

Using $\tau_{max} = T_4/T_0$ find:

$$\eta_{th} = \frac{\eta_c\left[\tau_{max}\eta_T(1 - \tau_s^{-1})\right] - (\tau_s - 1)}{\eta_c\left[\tau_{max} - 1\right] - (\tau_s - 1)}$$



b) If $\eta_{th} > 0$ will produce net work so

$w_{net} > 0$ for $\eta_c > \frac{\tau_s}{\tau_{max}\eta_T}$

$\eta_c > 0.391$

c) $P = \dot{m} w_{net}$, $w_{net} = c_p T_4 \eta_T (1 - \tau_s^{-1}) - \frac{c_p T_0}{\eta_c}(\tau_s - 1)$

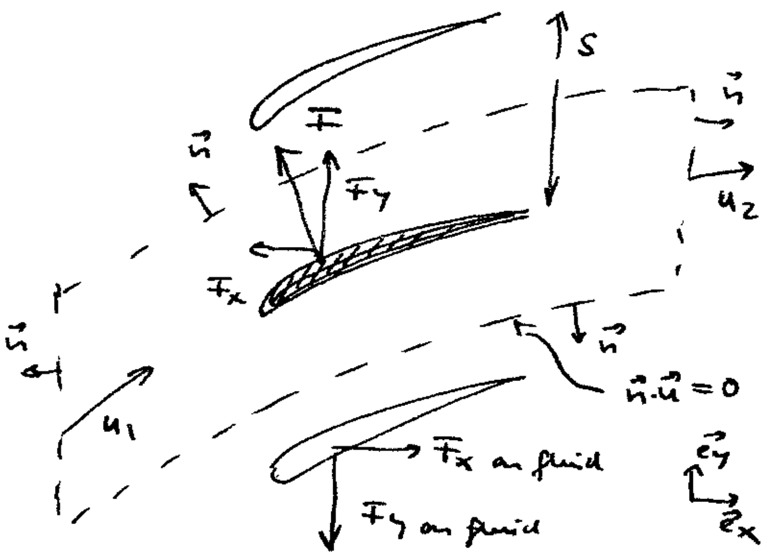
for $\eta_c = 0.55 \to w_{net} = 82$ kJ/kg

$\dot{m} = \rho A_{in} u_{in} = \frac{p_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}} A_{in} M_{in} \to \dot{m} = 0.38$ g/s

$P = 31.2$ W

Spring 2007, ZSP

Problem T11



Concepts: integral mom. theorem
Bernoulli equation (cons. of energy)

Assume: inviscid, steady incompressible flow
depth of control surfaces = 1

a) account for mom. flux
pressure force, blade force

$$-\rho s u_{x_1}^2 + \rho s u_{x_2}^2 = +F_x + s(p_1 - p_2)$$, mass conserved so $u_{x_1} = u_{x_2}$

$F_x = s(p_2 - p_1)$, p_t = const along streamline (Bernoulli)

so $p_2 + \frac{1}{2}\rho(u_{x_2}^2 + u_{y_2}^2) = p_1 + \frac{1}{2}\rho(u_{x_1}^2 + u_{y_1}^2)$ $\rightarrow$ $\underline{F_x = \frac{s}{2}\rho(u_{y_1}^2 - u_{y_2}^2)}$

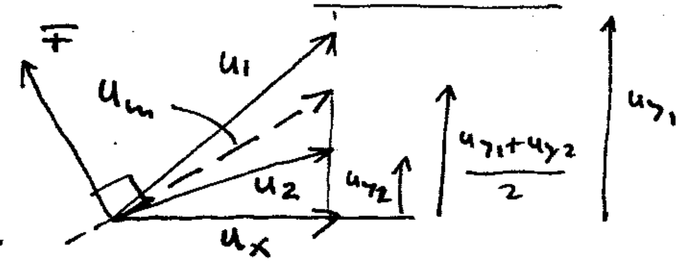
b) $-\rho u_{x_1} u_{y_1} s + \rho u_{x_2} u_{y_2} s = -F_y$ pressure force cancels

$$\underline{F_y = s\rho u_x (u_{y_1} - u_{y_2})}$$

c) $\Gamma = -\oint \vec{u}\, d\vec{s}$; $\Gamma = s(u_{y_1} - u_{y_2})$ so $\underline{F_y = \rho u_x \Gamma}$

and $\underline{F_x = \rho \frac{u_{y_1} + u_{y_2}}{2} \Gamma}$

so total blade force $\underline{F = \rho \Gamma u_m}$



d) Γ = const so if $s \to \infty$ then $u_{y_2} \to u_{y_1}$ and $u_m \to u_1 = u_\infty$

such that $\underline{F = \rho \Gamma u_\infty}$ (Kutta-Joukowski formula)