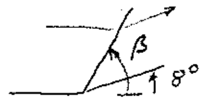
UE Fluids Problem 17 + 18 Solution S '07

a) Usual oblique-shock case, $\theta = 8^\circ$, $M_1 = 1.5 \rightarrow \beta = 52.5^\circ$ from chart



$M_{n_1} = M_1 \sin\beta = 1.19$, plug into normal-shock functions: $\frac{p_2}{p_1} = 1.486$, $M_{n_2} = 0.8486$

$$\boxed{\frac{p_a}{p_\infty} = \frac{p_2}{p_1} = 1.486}, \quad \boxed{M_a = \frac{M_{n2}}{\sin(\beta - \theta)} = 1.211}, \quad \boxed{\frac{p_{oa}}{p_\infty} = \frac{p_a}{p_\infty}\left[1 + \frac{\gamma-1}{2} M_a^2\right]^{\frac{\gamma}{\gamma-1}} = 3.656}$$

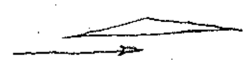
b) Usual expansion fan case, $\theta = 16^\circ$, $M_1 = M_a = 1.211$

$\nu(M_1) = 3.83^\circ$, $\nu(M_2) = \nu(M_1) + \theta = 19.83^\circ \longrightarrow M_2 = M_b = 1.769$ from P-M table

Since fan is isentropic, $p_{ob} = p_{oa} = 3.656\, p_\infty$, and $p_b = p_{ob}\left[1 + \frac{\gamma-1}{2} M_b^2\right]^{-\frac{\gamma}{\gamma-1}}$

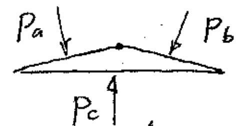
$$\boxed{\frac{p_b}{p_\infty} = \frac{p_{ob}}{p_\infty}\left[1 + \frac{\gamma-1}{2} M_b^2\right]^{-\frac{\gamma}{\gamma-1}} = 0.667}$$

c) There is no shock on bottom ($\theta = 0^\circ$), so $\boxed{\frac{p_c}{p_\infty} = 1}$



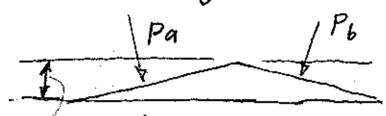
d) $L' = \int_0^c (p_{lower} - p_{upper})\, dx = \int_0^{c/2} (p_c - p_a)\, dx + \int_{c/2}^{c} (p_c - p_b)\, dx = p_\infty \frac{c}{2}\left[\frac{p_c}{p_\infty} - \frac{p_a}{p_\infty} + \frac{p_c}{p_\infty} - \frac{p_b}{p_\infty}\right]$

$$\boxed{L' = p_\infty \frac{c}{2}\left[1 - 1.486 + 1 - 0.667\right] = -p_\infty c \cdot 0.0765}$$



$D' = \int_0^c \left(\left(p \frac{dy}{dx}\right)_{upper} - \left(p \frac{dy}{dx}\right)_{lower}\right) dx$, $\left.\frac{dy}{dx}\right|_{lower} = 0$, $\left.\frac{dy}{dx}\right|_a = \tan 8^\circ = 0.1405$, $\left.\frac{dy}{dx}\right|_b = -0.1405$

$D' = p_a \cdot \tan(8^\circ)\frac{c}{2} - p_b \tan(8^\circ)\frac{c}{2} - p_c \cdot 0$



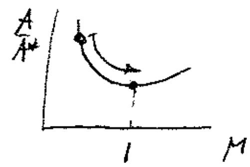
$\frac{c}{2} \cdot \frac{dy}{dx}$ = projected area on which $p_a - p_b$ act

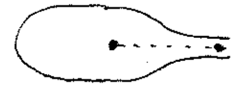
$$\boxed{D' = (p_a - p_b)\tan 8^\circ \cdot \frac{c}{2} = p_\infty c \cdot 0.0576}$$

$$\boxed{C_\ell = \frac{L'}{\frac{1}{2}\rho_\infty V_\infty^2 c}} = \frac{L'}{\frac{\gamma}{2} p_\infty M_\infty^2 c} = \frac{-0.0765\, p_\infty c}{\frac{\gamma}{2} M_\infty^2 p_\infty c} = \boxed{-0.0485}$$

$$\boxed{C_d = \frac{D'}{\frac{1}{2}\rho_\infty V_\infty^2 c}} = \frac{D'}{\frac{\gamma}{2} p_\infty M_\infty^2 c} = \frac{0.0576\, p_\infty c}{\frac{\gamma}{2} M_\infty^2 p_\infty c} = \boxed{0.0365}$$

a) The sonic ($M=1$) condition can only occur at a throat, or area minimum. This can be seen from the A/A^* versus M plot. Since the flow starts at $M \ll 1$ in the bottle, and A monotonically decreases, then M must monotonically increase up to $M=1$ at the nozzle.





b) Stagnation conditions in the bottle, which is in effect a reservoir:

$p_0 = 4 \times 10^5$ Pa (given), $T_0 = 300$ K (given), $\rho_0 = \frac{p_0}{RT_0} = 4.65$ kg/m³

At the throat, where $M=1$, we have

$$p = p^* = p_0 \left[1 + \tfrac{\gamma-1}{2}\right]^{-\frac{\gamma}{\gamma-1}} = 2.11 \times 10^5 \text{ Pa}$$

$$T = T^* = T_0 \left[1 + \tfrac{\gamma-1}{2}\right]^{-1} = 250 \text{ K}$$

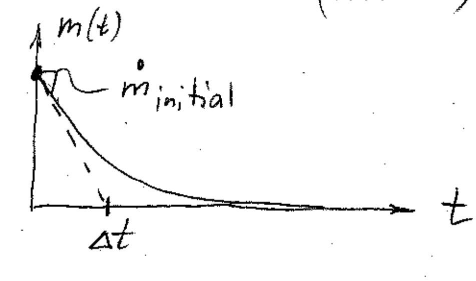
$$\rho = \rho^* = \rho_0 \left(1 + \tfrac{\gamma-1}{2}\right)^{-\frac{1}{\gamma-1}} = 2.95 \text{ kg/m}^3$$

$$u = a^* = \sqrt{\gamma R T^*} = 316.9 \text{ m/s}$$

c) Initial $\dot{m} = \rho u A = 2.95 \text{ kg/m}^3 \cdot 316.9 \text{ m/s} \cdot 0.0004 \text{ m}^2 = 0.374$ kg/s

Initial mass in bottle: $m = \rho_0 \mathcal{V} = 3.318 \text{ kg/m}^3 \cdot 2\ell \, \frac{1}{1000\ell/\text{m}^3} = 0.0093$ kg

If the initial $\dot{m}$ is sustained, the mass will flow out in $\boxed{\Delta t \simeq \frac{m}{\dot{m}} \simeq 0.025 \text{ s}}$ (minimum)
In reality, $\dot{m}$ will decrease as the bottle empties so the actual time will be longer.



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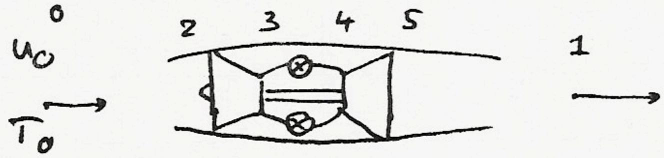
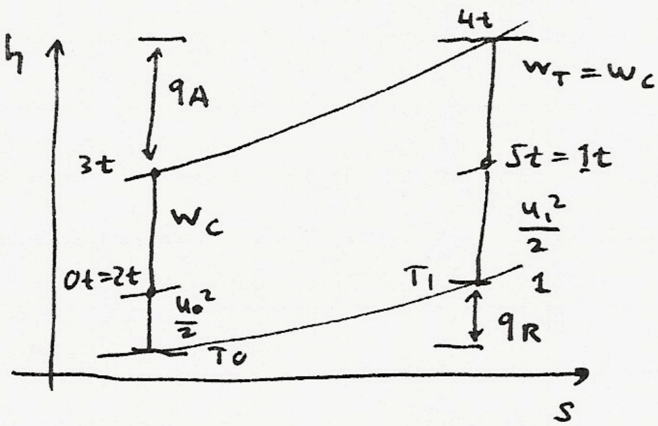
Problem T12



assume ideal cycle

a)



shaft power balance

$w_C = w_T$

$w_{net} = w_{mech} = \frac{1}{2}\left(u_1^2 - u_0^2\right)$

q_A : heat absorbed

q_R : heat rejected

b) ideal cycle, heat rejection at $p = \text{const}$: $T\,ds = dh - \frac{1}{\rho}\cancel{dp}^{\,0} = dq$

$\underline{q_R = c_p(T_1 - T_0)}$

net heat input: $q_A - q_R = \frac{1}{2}(u_1^2 - u_0^2)$

c) 1st law: $w_{net} = q_A - q_R$, $q_A = q_R + w_{mech} \rightarrow \underline{q_A = c_p(T_1 - T_0) + \frac{1}{2}(u_1^2 - u_0^2)}$

d)
$$\eta_{th} = \frac{w_{net}}{q_A} = \frac{\frac{1}{2}(u_1^2 - u_0^2)}{\frac{1}{2}(u_1^2 - u_0^2) + c_p(T_1 - T_0)} = \underline{\frac{1}{1 + \frac{T_1 - T_0}{\frac{u_1^2}{2c_p} - \frac{u_0^2}{2c_p}}}}$$

e)
$$\eta_{overall} = \eta_{th} \cdot \eta_{prop}$$

$$\eta_{overall} = \frac{F u_0}{\dot{m} f \Delta h} = \frac{F u_0}{\dot{m} q_A} = \frac{(u_1 - u_0)u_0}{q_A}$$

$$\eta_{overall} = \frac{u_0}{\frac{1}{2}(u_1 + u_0) + c_p\frac{T_1 - T_0}{u_1 - u_0}} = \underline{\frac{2}{1 + \frac{u_1}{u_0} + \frac{2c_p}{u_0}\frac{T_1 - T_0}{u_1 - u_0}}}$$

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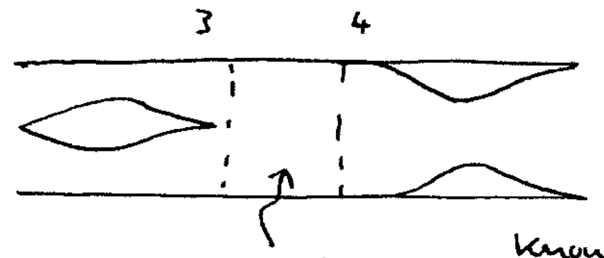
Problem T13

p_0
T_0 $\quad u_0 \longrightarrow$



$\longrightarrow u_5$

Know: T_{t4}, Δh_f
T_0, p_0, u_0
$p_5 = p_0$
$\mathcal{P} = F u_0$

Assume: no losses, no shocks

a) $T_{t0} = T_{t3}$; $T_{t0} = T_0 + \frac{u_0^2}{2c_p}$ 1st law

isentropic compression $\quad P_{t0} = P\left(1 + \frac{\gamma-1}{2} M_0^2\right)^{\gamma/\gamma-1} = P_{t3}$

$M_0 = \frac{u_0}{\sqrt{\gamma R T_0}}$, $M_0 = 3.06$, $\underline{T_{t3} = 638\text{ K}}$, $\underline{P_t = 8.04\text{ bar}}$

b) $M_0 = M_5$ (same PR → same TR, hence same M)

$T_5 = T_{t4}\left(1 + \frac{\gamma-1}{2} M_5^2\right)^{-1} \qquad u_5 = M_5\sqrt{\gamma R T_5} \qquad \underline{u_5 = 1736.7\text{ m/s}}$

c) $F = \dot{m}(u_5 - u_0)$, $\mathcal{P} = F u_0$ so $\underline{F = 8.2\text{ kN}}$

$\dot{m} = \frac{\mathcal{P}}{u_0(u_5 - u_0)} \qquad \underline{\dot{m} = 9.97\text{ kg/s}}$

d) $W_{net} = W_{mech} = \frac{1}{2}\left(u_5^2 - u_0^2\right) \qquad \underline{W_{net} = 1.09\text{ MJ/kg}}$

e) $\dot{Q}_A = \dot{m} c_p (T_{t4} - T_{t3}) = \dot{m}_f \Delta h_f \qquad f = \frac{c_p(T_{t4} - T_{t3})}{\Delta h_f}$

$\underline{f = 0.0388}$ $\quad \dot{m}_f = f \cdot \dot{m} \longrightarrow \underline{\dot{m}_f = 0.387\text{ kg/s}}$

f) $\eta_{prop} = \frac{2}{1 + \frac{u_5}{u_0}} \longrightarrow \underline{\eta_{prop} = 0.69}$

g) $\eta_{overall} = \eta_{th} \times \eta_{prop} \qquad \eta_{th} = \frac{W_{net}}{f \Delta h_f} = 0.65$

$\longrightarrow \eta_{overall} = 0.45$

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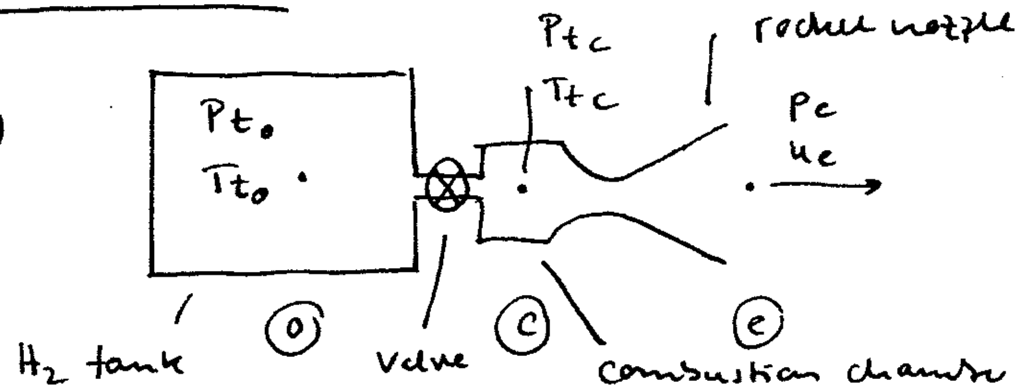
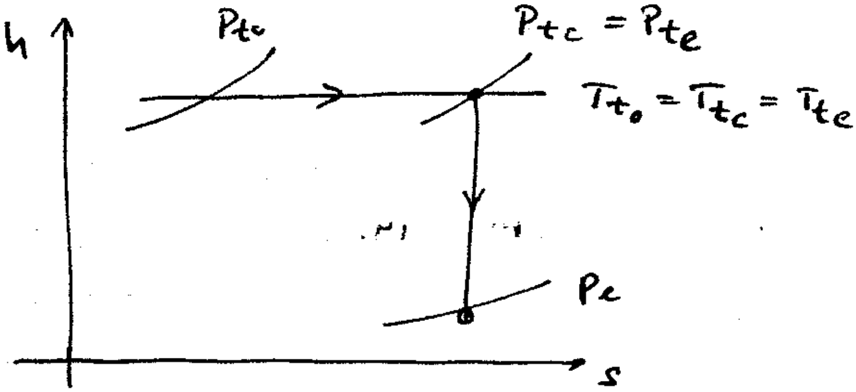
Problem T14

a)



b)



H_2: $c_p = \frac{7}{2}\frac{\mathcal{R}}{\mathcal{M}} = 14.5\ \text{kJ/kgK}$

$R = \frac{\mathcal{R}}{\mathcal{M}} = 4155\ \text{J/kgK}$

c) $$I_{sp} = \frac{F}{\dot{m}_f g} = \frac{\dot{m} u_e}{\dot{m} g} = \frac{u_e}{g}$$

1st Law: $h_t = \text{const} \rightarrow T_{t0} = T_{te} = T_e + \frac{u_e^2}{2c_p}$

$$u_e = \sqrt{2c_p(T_{t0} - T_e)} \qquad T_e = T_{t0}\left(\frac{P_{t0}}{p_e}\right)^{-\frac{\gamma-1}{\gamma}}$$

$$I_{sp} = \frac{u_e}{g} \;\wedge\; u_e = \sqrt{2c_pT_{t0}\left(1 - \left(\frac{p_e}{P_{t0}}\right)^{\frac{\gamma-1}{\gamma}}\right)} = 2237\ \text{m/s} \qquad \underline{I_{sp} = 228\ \text{s}}$$

d) $$\dot{m}_f = \frac{F}{I_{sp} g} \qquad \dot{m}_f = \dot{m} \rightarrow \underline{\dot{m} = 0.089\ \text{kg/s}}$$

e) thrusting requirement: $F\Delta t = \dot{m} u_e \Delta t = \rho_0 V_0 u_e$

$$\underline{V_0} = \frac{F\Delta t}{\rho_0 u_e} = \frac{F\Delta t\, R T_{t0}}{P_{t0} u_e} = \underline{0.139\ \text{m}^3}$$

(equivalent to sphere of 0.32 m radius)