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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 10

Name: ________________________

Due Date: 04/24/2007

	Time Spent (min)
T15	
T16	
F20	
S11	
S12	
Study Time	

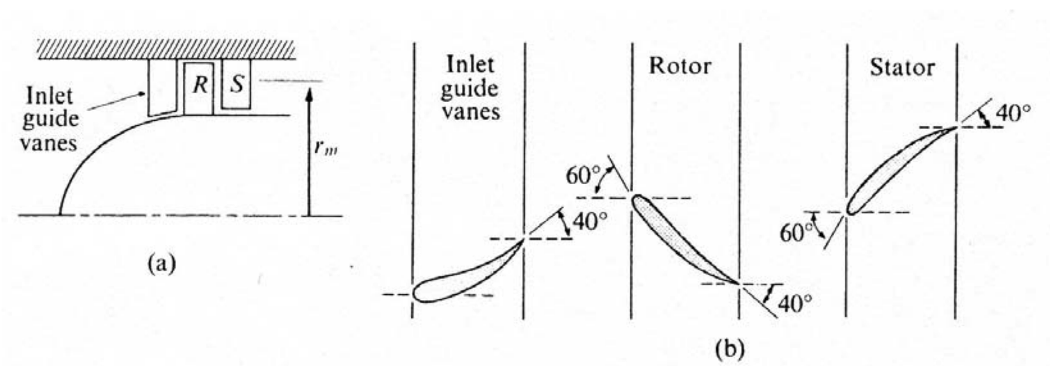
Announcements:

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T15

Consider the single-stage compressor shown schematically in Figure (a). At the mean radius r_m = 0.3 m, the blade configuration is as shown in Figure (b). We wish to estimate the performance of this compressor stage.



The air flow enters the compressor axially and is turned by a row if inlet guide vanes (IGVs) before passing through the rotor and stator blade rows. It can be assumed that, under the operating conditions considered, the flow angles are identical to the blade metal angles. The overall adiabatic efficiency of the stage is 90%. The hub-tip radius ratio is 0.8, high enough so that conditions at the mean radius are a good average of conditions from root to tip. Furthermore the mean radius and hub-tip radius ratio can be assumed constant through the compressor stage. The axial velocity component at design flow rate is uniformly 125 m/s, and the inlet air far upstream is at 1 bar and 293K.

a) Sketch the velocity triangles through the compressor stage.

b) Draw an h-s diagram and sketch the static and the stagnation states through the compressor (label all stations and indicate work and heat transfer if applicable).

c) What should the shaft speed (rpm) be under the operating conditions considered above?

d) Determine the mass flow through the compressor stage.

e) What is the stagnation pressure ratio of the compressor stage?

f) What is the power required to drive the compressor stage?

Clearly state all your assumptions.

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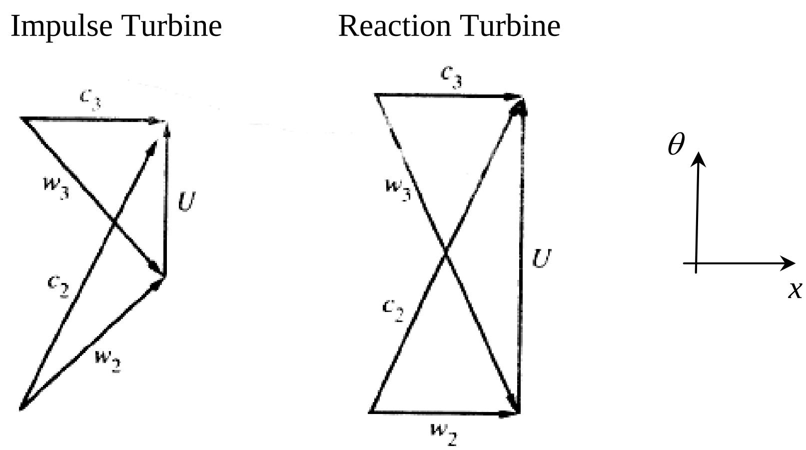
(Add a short summary of the concepts you are using to solve the problem)

Problem T16

Consider the velocity triangles below for an impulse turbine and for a reaction turbine at different rotor speeds. The subscripts 2 and 3 denote conditions before and after the rotor, respectively. In both cases the absolute velocity c_2 is 400 m/s at an angle α_2 = 70°, and the absolute exhaust velocity c_3 is axial. For the impulse turbine the tangential relative velocity components into and out of the rotor $w_{\theta 2}$ and $w_{\theta 3}$ are of the same magnitude. The blades are uncooled and well insulated from the turbine disc such that the flow can be assumed adiabatic. The rotor inlet stagnation temperature is T_{t2} = 1100K.



a) Assuming that the flow into the guide vane is axial, sketch the guide vane and rotor blade profiles for the two turbines.

b) What is the wheel speed U for the impulse turbine? What is it for the reaction turbine?

c) What is the specific work in kJ/kg for the two turbines?

d) What is the change in stagnation temperature across the rotor for the two turbines?

a) What is the change in static temperature across the rotor for the two turbines?

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Fluids Problem F20

An air reservoir with pressure and enthalpy of $p_r = 100\,000\,\mathrm{Pa}$, $h_r = 300\,000\,\mathrm{m^2/s^2}$ exhausts through a Laval nozzle with throat and exit areas $A_t = 0.010\,\mathrm{m}^2$, $A_e = 0.014\,\mathrm{m}^2$.

a) Determine the mass flow when the back pressure is $p_{\mathrm{B}} = 80\,000\,\mathrm{Pa}$

b) Determine the mass flow when the back pressure is $p_{\mathrm{B}} = 90\,000\,\mathrm{Pa}$

c) Determine the back pressure which will give matched supersonic flow at the exit.

Problem S11 (Signals and Systems)

Note: This problem has been given before. Please do not use bibles to solve or check your answer. There have been a number of aviation accidents in which the aerodynamic controls (elevator, etc.) were lost. Usually this is a result of a complete hydraulic failure, due to some other cause, such as a catastrophic engine failure. In these cases, pilots have attempted to control altitude with throttle variations, with mixed success. In the crash of a DC10 in Sioux City, Iowa, where the aircraft lost elevator control, the captian was able to land the airplane with 184 survivors of 296 on board using throttle only to control altitude. It can be done, but it is very hard. This problem examines why that is so.

Consider an aircraft flying in cruise at 250 knots, so that

$$v_0 = 129 \text{ m/s}$$

Assume that the aircraft has lift-to-drag ratio

$$\frac{L_0}{D_0} = 15$$

Then the transfer function from changes in thrust to changes in altitude is

$$G(s) = \frac{2g}{mv_0} \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

where the *natural frequency* of the phugoid mode is

$$\omega_n = \sqrt{2}\frac{g}{v_0}$$

the *damping ratio* is

$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0)}$$

and $g = 9.82$ m/s is the acceleration due to gravity. The transfer function can be normalized by the constant factor $\frac{2g}{mv_0}$, so that

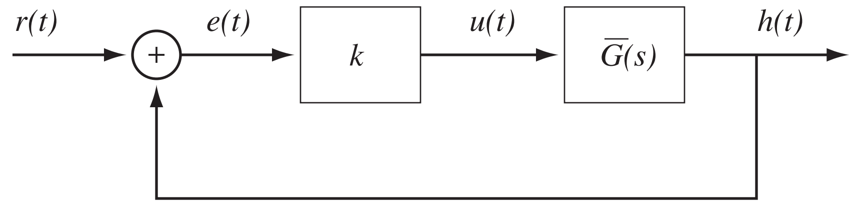
$$\bar{G}(s) = \frac{1}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

is the normalized transfer function, corresponding to normalized input

$$u(t) = \frac{2g}{mv_0}\delta T$$

1. Find and plot the impulse response corresponding to the transfer function $\bar{G}(s)$, using partial fraction expansion and inverse Laplace techniques. Hint: The poles of the system are complex, so you will have to do complex arithmetic. You may *check* your response using the Matlab command `impulse`.

2. Suppose we try to control the altitude through a feedback loop, as shown below



That is, the control input $u(t)$ (normalized throttle) is a gain k times the error, $e(t)$, which is the difference between the altitude $h(t)$ and the altitude reference $r(t)$. The transfer function from $r(t)$ to $h(t)$ can be shown to be

$$\frac{H(s)}{R(s)} = \frac{k\bar{G}(s)}{1 + k\bar{G}(s)}$$

For the gain k in the range $[0, 0.1]$, plot the poles of the closed-loop system in the complex plane. You should find that for any positive k, the complex poles are made less stable. What gain k makes the complex poles unstable, i.e., for what gain is the damping ratio zero?

3. For the gain k in the range $[-0.1, 0]$, plot the poles of the closed-loop system in the complex plane. You should find that for any negative k, the real pole is unstable.

Note that neither positive gain or negative gain makes the system more stable than without feedback control. It is possible to do better with a dynamic gain, but this problem should give you an idea of why the phugoid dynamics are so hard to control with throttle only.

Problem S12 (Signals and Systems)

For each signal below, find the bilateral Laplace transform (includingthe region of convergence) by directly evaluating the Laplace transform integral. If the signal does not have a transform, say so.

1. $g(t) = \sin(at)\sigma(-t)$
2. $g(t) = e^{at}\sigma(-t)$
3. $g(t) = te^{at}\sigma(-t)$
4. $g(t) = \cos(\omega_0 t)\, e^{-a|t|}$, for all t
5. $g(t) = e^{2t}\sigma(t) - e^{-t}\sigma(-t)$