UE Fluids Problem F20 Solution S '07

a) First assume isentropic flow (unchoked, no shock).

$p_o = p_r = p_e\left[1+\frac{\gamma-1}{2}M_e^2\right]^{\frac{\gamma}{\gamma-1}}$, $M_e = \left\{\frac{2}{\gamma-1}\left[\left(\frac{p_r}{p_e}\right)^{\frac{\gamma-1}{\gamma}}-1\right]\right\}^{1/2}$

Using $\frac{p_r}{p_e} = \frac{100\,000}{80\,000} = 1.25$, $M_e = 0.5737$

For this M_e, $A^* = A_e M_e\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{-\frac{\gamma+1}{2(\gamma-1)}} = 0.819\,A_e$

Since this nozzle has $A_t = \frac{A_t}{A_e}\cdot A_e = \frac{0.010}{0.014}A_e = 0.7143\,A_e$
then $A^* > A_t$, so <u>flow is choked</u>.

<u>Try again</u>, using $A^* = A_t = 0.010\ m^2$ instead.

$$\boxed{\dot m = \rho^* a^* A^* = \frac{\gamma\, p_r}{\sqrt{(\gamma-1)h_r}}\left(1+\frac{\gamma-1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\cdot A^* = 2.339\ kg/s}$$

b) Again assume isentropic flow

$\frac{p_r}{p_e} = \frac{100000}{90000} = 1.111$, $M_e = 0.3909$

$A^* = A_e M_e\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{-\frac{\gamma+1}{2(\gamma-1)}} = 0.617\,A_e < A_t$ <u>unchoked</u>

Using this $A^* = 0.617\,A_e = 0.00864$

$$\boxed{\dot m = \frac{\gamma\, p_r}{\sqrt{(\gamma-1)h_r}}\left(1+\frac{\gamma-1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}A^* = 2.021\ kg/s}$$

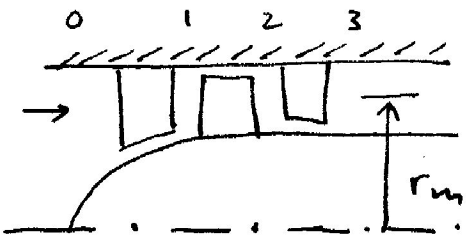
c) For matched supersonic flow, no shocks, $(p_o)_e = p_r$

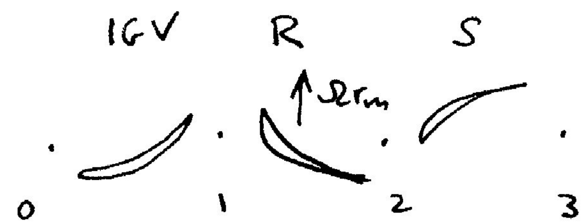
Also, M_e corresponds to $\frac{A_e}{A^*} = \frac{0.014}{0.010} = 1.4 \rightarrow M_e = 1.763$ (Appendix A)

$$\boxed{p_B = p_e = (p_{oe})\left[1+\frac{\gamma-1}{2}M_e^2\right]^{-\frac{\gamma}{\gamma-1}} = 18415\ Pa}$$

Spring 2007, 25P

T15





$R = r_H / r_T = 0.8$

$\eta_{ad} = 0.9$

$P_{t0} = 1 \text{bar}$ $T_{t0} = 293\ K$

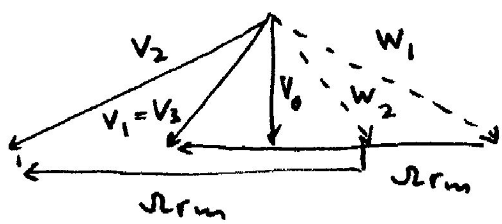
$c_p = 1004\ J/kgK$, $\gamma = 1.4$

abs.: $\alpha_0 = 0$ $\alpha_1 = 40°$ $\alpha_2 = 60°$ $\alpha_3 = 40°$

rel.: $\beta_1 = 60°$ $\beta_2 = 40°$

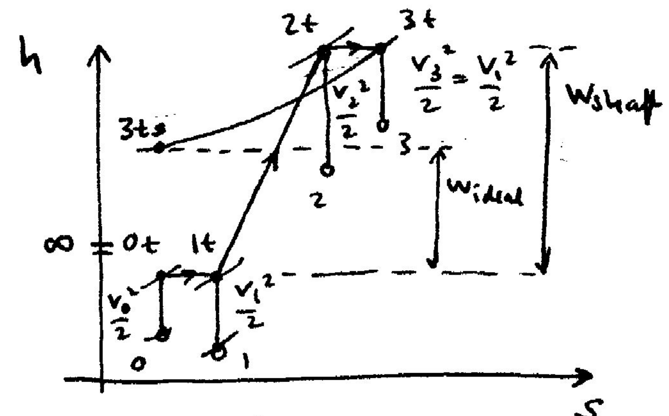
$V_x = 125\ m/s$ constant $r_m = 0.3\,m$

a)



(symmetric) W: relative
V: absolute

b)



adiabatic compressor

c) $V_0 = V_x$; $\Omega r_m = V_0 (\tan\alpha_1 + \tan\beta_1)$

$$\Omega = \frac{V_0}{r_m} (\tan\alpha_1 + \tan\beta_1)$$

get $\underline{\Omega = 10{,}230\ \text{rpm}}$

d) $\dot{m} = \rho_0 A V_0$ $A = \pi (r_T^2 - r_H^2)$ $r_T = \frac{2 r_m}{1+R} = 0.33\,m$, $r_H = r_T \cdot R = 0.27\,m$

$T_0 = T_{t0} - \frac{V_0^2}{2c_p} = 285K$ $M_0 = \frac{V_0}{\sqrt{\gamma R T_0}} = 0.37$ $P_0 = P_{t0} \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{-\gamma/\gamma-1} = 0.91\text{bar}$ $\rho_0 = \frac{P_0}{RT_0} = 1.11 \frac{kg}{m^3}$

$\underline{\dot{m} = 15.7\ kg/s}$

e) $h_{t3s} - h_{t0} = \Omega r_m (V_{\theta 2} - V_{\theta 1}) = \Omega r_m V_0 (\tan\alpha_2 - \tan\alpha_1) = 35.87\ kJ/kg$

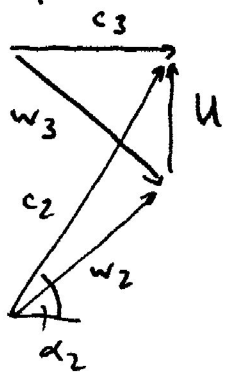
Note: since flow angles assumed to be blade metal angles (have ideal turning) and for this assume inviscid flow (no loss) the above is the ideal work input W_{ideal}

$\underline{\pi = \left(\frac{T_{t3s}}{T_{t0}}\right)^{\gamma/\gamma-1} = 1.5}$

f) $\dot{W}_{shaft} = \frac{1}{\eta_{ad}} \cdot \dot{m}\, W_{ideal} = \underline{626\ KW}$

T16

Impulse Turbine | **Reaction Turbine**

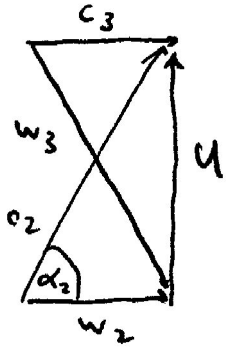


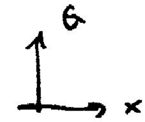
c: abs

w: rel.

U: wheel

$c_p = 1000$ J/kgK



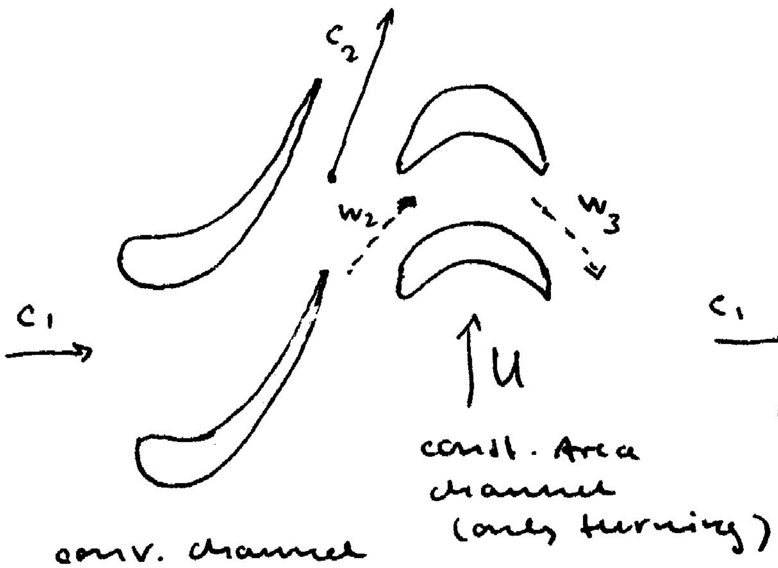


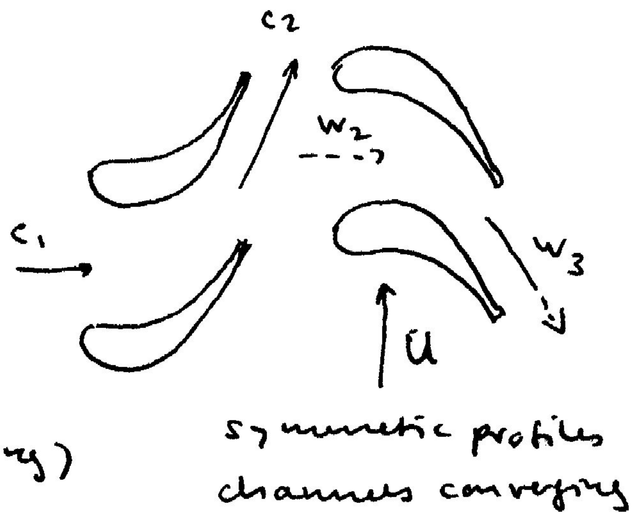
$\alpha_2 = 70°$

$c_2 = 400$ m/s

$T_{t_2} = 1100$ K

a)





b) Impulse: $2U = c_2 \sin\alpha_2$

$\underline{U = 188 \text{ m/s}}$

Reaction: $U = c_2 \sin\alpha_2$

$\underline{U = 376 \text{ m/s}}$

c) Impulse: $W_{shaft} = U(c_{\theta_3} - c_{\theta_2})$, with $c_{\theta_3} = 0$

$\underline{W_{shaft}} = -U c_2 \sin\alpha_2 = -2U^2 = \underline{-70.7 \text{ kJ/kg}}$

Reaction: $W_{shaft} = U(c_{\theta_3} - c_{\theta_2})$

$\underline{W_{shaft}} = -U^2 = \underline{-141.4 \text{ kJ/kg}}$

Note: work output!

d) Impulse: $\Delta T_t = \frac{2U^2}{c_p} = \underline{70.7 \text{ K}}$

Reaction: $\Delta T_t = \frac{U^2}{c_p} = \underline{141.4 \text{ K}}$

e) Impulse: $\Delta T = \Delta T_t - \frac{c_2^2}{2c_p} + \frac{c_3^2}{2c_p}$

$c_3 = c_2 \cos\alpha_2 = 137$ m/s

$\underline{\Delta T = 0 \text{ K}}$ (impulse turbine!)

Reaction: ditto:

$\Delta T = \Delta T_t - \frac{c_2^2}{2c_p} + \frac{c_3^2}{2c_p}$

$\underline{\Delta T = 70.7 \text{ K}}$

Unified Engineering II **Spring 2007**

Problem S11 (Signals and Systems)

1. From the problem statement,

$$\omega_n = \sqrt{2}\,\frac{9.82\text{ m/s}^2}{129\text{ m/s}} = 0.1077\text{ r/s}$$
$$\zeta = \frac{1}{\sqrt{2}(L_0/D_0} = \frac{1}{\sqrt{2}\cdot 15} = 0.0471$$

Therefore,

$$\bar{G}(s) = \frac{1}{s\left(s^2 + 0.01015s + 0.0116\right)}$$

The roots of the denominator are at $s = 0$, and

$$s = \frac{-0.01915 \pm \sqrt{0.01015^2 - 4\cdot 0.0116}}{2}$$
$$= -0.005075 \pm 0.1075j$$

So

$$\bar{G}(s) = \frac{1}{s\left(s - [-0.005075 + 0.1075j]\right)\left(s - [-0.005075 - 0.1075j]\right)}$$

Use the coverup method to obtain the partial fraction expansion

$$\bar{G}(s) = \frac{86.283}{s} + \frac{-43.142 + 2.036j}{s - [-0.005075 + 0.1075j]} + \frac{-43.142 - 2.036j}{s - [-0.005075 - 0.1075j]}$$

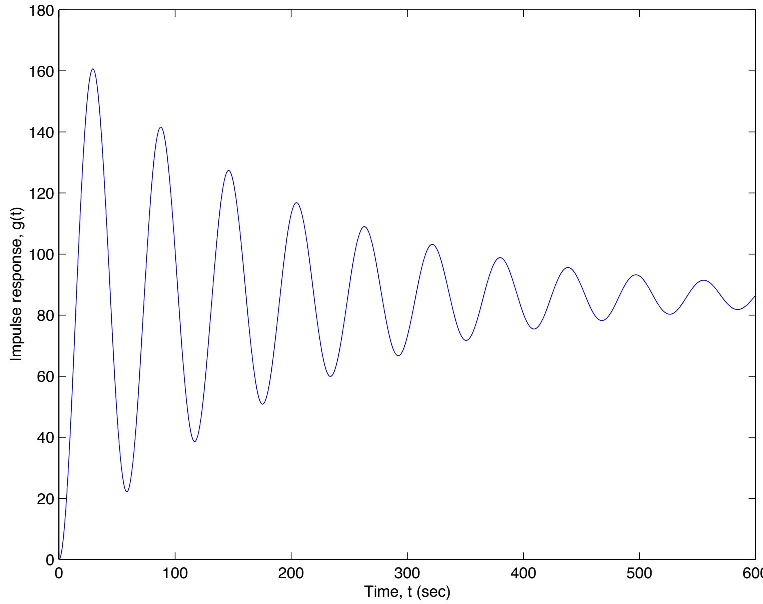
Taking the inverse Laplace transform (assuming that $\bar{g}(t)$ is causal), we have

$$\bar{g}(t) = 86.283\sigma(t) + (-43.142 + 2.036j)e^{(-0.005075+0.1075j)t} + (-43.142 - 2.036j)e^{(-0.005075-0.1075j)t}$$

Therefore,

$$\bar{g}(t) = \sigma(t)\left[86.283 + 2e^{-0.005075t}\left(-43.142\cos\omega_d t - 2.036\sin\omega_d t\right)\right]$$
$$= \sigma(t)\left[86.283 + \left(-86.284\cos\omega_d t - 4.072\sin\omega_d t\right)e^{-0.005075t}\right]$$

where $\omega_d = 0.1075$ r/s. See below for the impulse response.



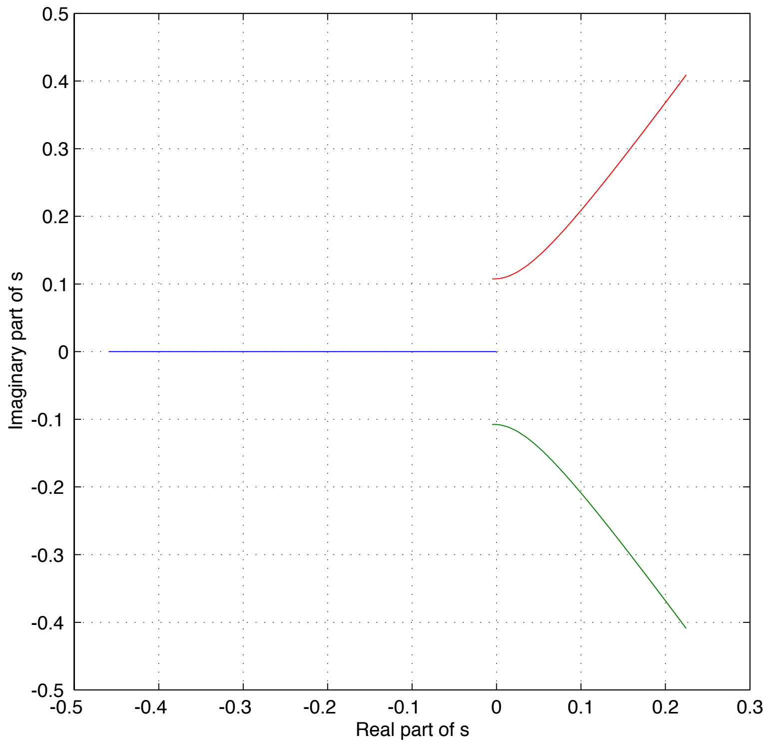
2. From the problem statement,

$$\begin{aligned}\frac{H(s)}{R(s)} &= \frac{k\bar{G}(s)}{1+k\bar{G}(s)} \\ &= \frac{k\,\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2)}}{1+k\,\dfrac{1}{s(s^2+2\zeta\omega_n s+\omega_n^2)}} \\ &= \frac{k}{s^3+2\zeta\omega_n s^2+\omega_n^2 s+k}\end{aligned}$$

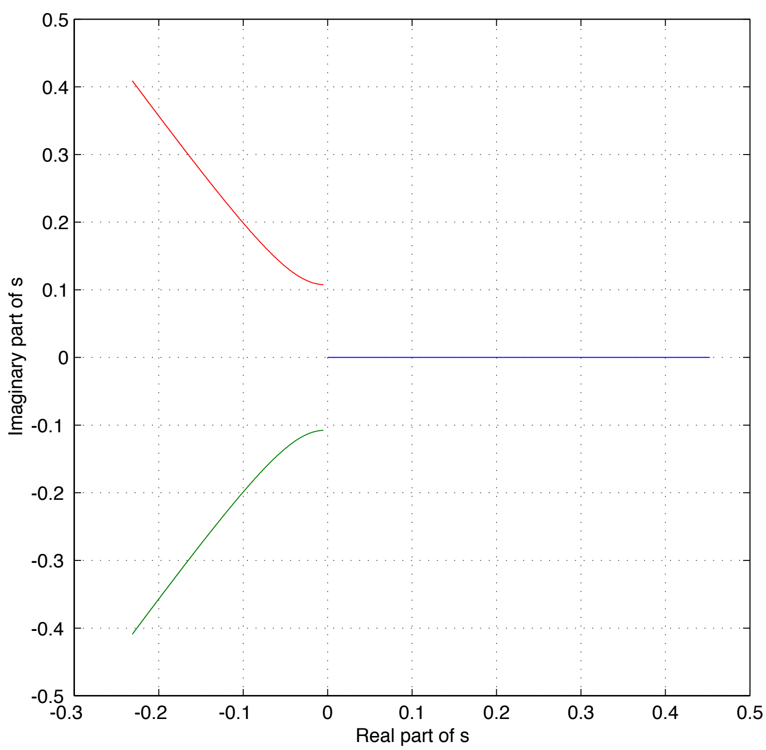
So the poles of the system are the roots of the denominator polynomial,

$$\phi(s) = s^3 + 2\zeta\omega_n s^2 + \omega_n^2 s + k = 0$$

The roots can be found using Matlab, a programmable calculator, etc. The plot of the roots (the "root locus") is shown below. Note that the oscillatory poles go unstable at a gain of only $k = 0.000118$.



3. The roots locus for negative gains can be plotted in a similar way, as below. Note that the real pole is unstable for all negative k.



Unified Engineering II **Spring 2005**

Problem S12 (Signals and Systems)

1. $g(t) = \sin(at)\sigma(-t)$. To do this problem, expand the sinusoid as complex exponentials, so that

$$g(t) = \left[\frac{e^{ajt} - e^{-ajt}}{2j}\right]\sigma(-t)$$

Therefore, the LT is given by

$$G(s) = \int_{-\infty}^{0} \left[\frac{e^{ajt} - e^{-ajt}}{2j}\right] e^{-st}\,dt$$

For the LT to converge, the integrand must go to zero as t goes to $-\infty$. Therefore, the integral converges only for $\text{Re}[s] < 0$. The integral is then

$$\begin{aligned}
G(s) &= \int_{-\infty}^{0} \left[\frac{e^{ajt} - e^{-ajt}}{2j}\right] e^{-st}\,dt \\
&= \frac{1}{2j}\left[\frac{1}{-s+aj} e^{(aj-s)t}\Big|_{-\infty}^{0} - \frac{1}{-s-aj} e^{(-aj-s)t}\Big|_{-\infty}^{0}\right] \\
&= \frac{1}{2j}\left[\frac{1}{-s+aj} - \frac{1}{-s-aj}\right] \\
&= \frac{-a}{s^2+a^2}, \qquad \text{Re}[s] < 0
\end{aligned}$$

2. $g(t) = e^{at}\sigma(-t)$. The LT is given by

$$G(s) = \int_{-\infty}^{0} e^{at}e^{-st}\,dt = \int_{-\infty}^{0} e^{(a-s)t}\,dt$$

For the LT to converge, the integrand must go to zero as t goes to $-\infty$. Therefore, the integral converges only for $\text{Re}[s] < a$. Then

$$\begin{aligned}
G(s) &= \int_{-\infty}^{0} e^{(a-s)t}\,dt \\
&= \frac{1}{a-s} e^{(a-s)t}\Big|_{-\infty}^{0} \\
&= 0 - \frac{1}{a-s} \\
&= -\frac{1}{s-a}, \qquad \text{Re}[s] < a
\end{aligned}$$

3. $g(t) = te^{at}\sigma(-t)$. The LT is given by

$$G(s) = \int_{-\infty}^{0} te^{at}e^{-st}\,dt = \int_{-\infty}^{0} te^{(a-s)t}\,dt$$

For the LT to converge, the integrand must go to zero as t goes to $-\infty$. Therefore, the integral converges only for $\text{Re}[s] < a$. To find the integral, integrate by parts:

$$
\begin{aligned}
G(s) &= \int_{-\infty}^{0} t e^{(a-s)t}\, dt \\
&= t \frac{1}{a-s} e^{(a-s)t} \Big|_{-\infty}^{0} - \frac{1}{a-s} \int_{-\infty}^{0} e^{(a-s)t}\, dt \\
&= 0 - \frac{1}{a-s} \int_{-\infty}^{0} e^{(a-s)t}\, dt \\
&= -\frac{1}{(a-s)^2} e^{(a-s)t} \Big|_{-\infty}^{0} \\
&= -\frac{1}{(s-a)^2}, \qquad \text{Re}[s] < a
\end{aligned}
$$

4. $g(t) = \cos(\omega_0 t)\, e^{-a|t|}$, for all t. The LT is given by

$$G(s) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-a|t|} e^{-st}\, dt$$

For the LT to converge, the integrand must go to zero as t goes to $-\infty$ and ∞. Therefore, the integral converges only for $-a < \text{Re}[s] < a$. The integral is given by

$$
\begin{aligned}
G(s) &= \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-a|t|} e^{-st}\, dt \\
&= \int_{-\infty}^{0} \cos(\omega_0 t)\, e^{at} e^{-st}\, dt \quad + \quad \int_{0}^{\infty} \cos(\omega_0 t)\, e^{-at} e^{-st}\, dt
\end{aligned}
$$

Expanding the cosine term as

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

yields

$$
\begin{aligned}
G(s) &= \int_{-\infty}^{0} \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\, e^{at} e^{-st}\, dt \quad + \quad \int_{0}^{\infty} \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\, e^{-at} e^{-st}\, dt \\
&= \int_{-\infty}^{0} \frac{e^{(j\omega_0 + a - s)t} + e^{(-j\omega_0 + a - s)t}}{2}\, dt \quad + \quad \int_{0}^{\infty} \frac{e^{(j\omega_0 - a - s)t} + e^{(-j\omega_0 - a - s)t}}{2}\, dt \\
&= \frac{1}{2} \left[\frac{1}{j\omega_0 + a - s} + \frac{1}{-j\omega_0 + a - s} - \frac{1}{j\omega_0 - a - s} - \frac{1}{-j\omega_0 - a - s} \right] \\
&= \frac{-s}{s^2 - 2as + a^2 + \omega_0^2} + \frac{s}{s^2 + 2as + a^2 + \omega_0^2}, \qquad -a < \text{Re}[s] < a
\end{aligned}
$$

5. $g(t) = e^{2t}\sigma(t) - e^{-t}\sigma(-t)$. The LT of $e^{2t}\sigma(t)$ is given by

$$\frac{1}{s-2}, \quad \text{Re}[s] > 2$$

The LT of $-e^{-t}\sigma(-t)$ is given by

$$\frac{-1}{s+1}, \quad \text{Re}[s] < -1$$

For the LT of $g(t)$ to converge, both parts of the integral must converge, so the r.o.c. is the *intersection* of the two r.o.c.'s above. Since the intersection is empty, there is no LT for any value of s.