Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 11

Name: ________________________

Due Date: 05/01/2007

	Time Spent (min)
T17	
T18	
T19	
T20	
S13	
S14	
Study Time	

Announcements:

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T17

Suppose we wish to use hydrogen and oxygen as fuel and oxidizer for rocket propulsion, with the goal of maximizing the velocity (and hence the specific impulse) at which the combustion products leave the exit nozzle of the rocket engine. Assume that the products do not dissociate and that the flow in the nozzle expands isentropically to vacuum.

a) Would a stoichiometric ratio of hydrogen and oxygen be best for this? Why or why not? Bolster your statements with an analysis (interpret your solution, no need to give numerical results here).

b) Would a fuel / oxidizer ratio of 1/6 as used in the space shuttle main engine (SSME) be better? Why or why not? Bolster your statements with an analysis (interpret your solution, no need to give numerical results).

c) For the two cases (a) and (b), what is the nozzle exit velocity? What is the specific impulse?

Use the following properties in your calculations:

- Enthalpy of formation for water vapor: -241,820 kJ/kmol
- Enthalpy of hydrogen at 298K: 8,468 kJ/kmol
- Enthalpy of oxygen at 298K: 8,682 kJ/kmol
- Average specific heat for water vapor: 56.7 kJ/kmol-K
- Average specific heat for hydrogen: 38.2 kJ/kmol-K

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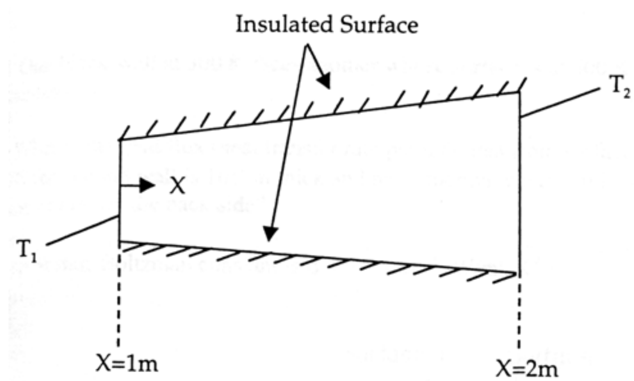
Problem T18

Steady-state heat transfer occurs in a conical section. The cone has a circular cross section. The small end has a radius of 0.1 m and the large end a radius of 0.2 m. The length of the conical section is 1 meter. The thermal conductivity is constant and equal to k.

The temperature of the small end is T_1 and that of the large end is T_2, which is lower than T_1. The sides of the cone are insulated, so that there is no heat transfer out of the sides of the conical section.



If the heat transfer can be considered to be one-dimensional:

a) How does the overall rate of heat transfer (the total rate over the cross section) vary with distance x?

b) Derive an equation for the heat flux (rate of heat transfer per unit area) in the conical section.

c) What is the temperature variation (as a function of x)?

d) What is the overall thermal resistance of the conical section?

e) What is the heat flux variation (as a function of x)?

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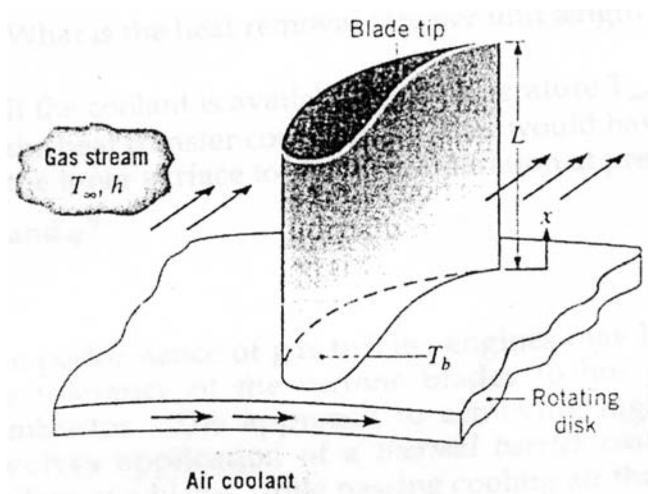
Problem T19

Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200^o C$ and maintains a convection coefficient of h=250 W/m^{2}K over the blade.



The blades, which are fabricated from Inconel, k=20 W/mK, have a length of L=50 mm. The blade profile has a uniform cross-sectional area of A_c=6x10^{-4}m^2 and a perimeter of P=110 mm. A proposed blade-cooling scheme, which involves routing air through the supporting disk, is able to maintain the base of each blade at a temperature of T_b=300^{o}C. The blade tip may be assumed to be adiabatic.

a) Considering an elementary slice (blade profile) of height dx, derive an equation for the temperature distribution along the blade span.

b) What are the boundary conditions?

c) Solve for the temperature distribution. If the maximum allowable blade temperature is 1050^{o}C, is the proposed cooling scheme satisfactory?

d) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

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Problem T20

The MIT-CU *Silent Aircraft Initiative* (SAI) has developed a conceptual aircraft sufficiently quiet that outside the airport perimeter aircraft noise is less than the background noise of a well populated area. A flying wing type airframe was considered (see illustration below) due to acoustic advantages. This "silent aircraft" will need some way to prevent ice from forming on its exterior. One solution would be to install electric resistance heating elements under the skin near the leading edge of the flying wing. You are asked to evaluate this proposal. You can assume that the silent aircraft is in steady level flight and that ice will not form near the leading edge when the surface temperature is kept at $T_S = 5^\circ C$.

***S**ilent **A**ircraft e**X**perimental design SAX-40*

The following data is available:

Body surface area (wetted):	$A = 800$ m²
Total cruise thrust (3 engine clusters):	$F = 70{,}000$ N
Cruise Mach:	$M = 0.8$
Cruise conditions:	$p_\infty = 18{,}754$ Pa
	$T_\infty = 217$ K

a) If the wing was adiabatic, what is the adiabatic wall temperature at cruise? Is heating of the wing surface required to avoid the formation of ice?

b) Estimate the electrical power needed to maintain a surface temperature of $T_S = 5^\circ C$ near the leading edge of area $A_{LE} = 20$ m². You can assume that the total drag is only due to friction on the flying wing of wetted area A.

Problem S13 (Signals and Systems)

The point of this problem is to give you practice doing inverse Laplace Transforms. Please don't use bibles for this problem — it defeats the whole point, which is for you to practice! For each of the following Laplace transforms, find the inverse Laplace transform. Note that you can use the results of Problem S10 to help with the partial fraction expansions, but you will need to use the region of convergence to determine the inverse transform.

1. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad -2 < \text{Re}[s] < 2$

2. $G(s) = \dfrac{3s^2 + 3s - 10}{s^2 - 4}, \quad \text{Re}[s] < -2$

3. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad -2 < \text{Re}[s] < -1$

4. $G(s) = \dfrac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}, \quad -3 < \text{Re}[s] < -2$

5. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad -2 < \text{Re}[s] < -1$

6. $G(s) = \dfrac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad \text{Re}[s] < -2$

7. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad -1 < \text{Re}[s] < 0$

8. $G(s) = \dfrac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad \text{Re}[s] < -1$

9. $G(s) = \dfrac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)}, \quad \text{Re}[s] < 0$

Problem S14 (Signals and Systems)

This is one of those classic problems that helps demonstrate an important point. Please don't use bibles on this problem! In class, you learned about a *smoother*, with transfer function

$$G_1(s) = \frac{-a^2}{(s-a)(s+a)}$$

The smoother is an example of a *low-pass filter*, which means that it tends to attenuate high-frequency sine waves, but "pass" low-frequency sine waves. Unfortunately, the smoother is non-causal, which means that it can't be implemented in real time. A similar causal low-pass filter is

$$G_2(s) = \frac{a^2}{(s+a)^2}$$

In this problem, you will compare these two low-pass filters, to see how they affect sinusoidal inputs. Consider an input signal

$$u(t) = \cos \omega t$$

1. Find the transfer function, $G_1(j\omega)$, as a function of frequency, ω.

2. Since the transfer function is complex, it can be represented as

   $$G_1(j\omega) = A_1(\omega)e^{j\phi_1(\omega)}$$

   where the amplitude of the transfer function is $A_1(\omega)$, and the phase of the transfer function is $\phi_1(\omega)$. Find $A_1(\omega)$ and $\phi_1(\omega)$.

3. Find the transfer function, $G_2(j\omega)$, as a function of frequency, ω, as well as $A_2(\omega)$ and $\phi_2(\omega)$.

4. For the input $u(t)$ above, show that the output of the system G_1 is

   $$y_1(t) = A_1(\omega)\cos(\omega t + \phi_1(\omega))$$

   and do likewise for system G_2.

5. A_1 and A_2 determine how much the magnitude of the input cosine wave is affected by each filter. Ideally, A_1 and A_2 would be 1, meaning that the filters don't change the magnitude of the input sine at all. Which filter (if either) changes the magnitude the least?

6. ϕ_1 and ϕ_2 determine how much the phase of the input cosine wave is affected by each filter. Non-zero values of ϕ correspond to a shifting left or right (i.e., advancing or delaying) the sine wave. Ideally, ϕ_1 and ϕ_2 would be zero, meaning that the filters don't change the phase of the input sine at all. Which filter (if either) produces the least phase shift?

7. Explain why the non-causal filter is preferred in signal processing applications where it can be applied.