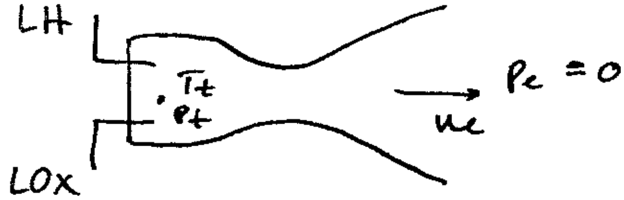
T17



LH, LOX @ T = 0 K (cryogenic) $h^{H_2}_{T=0} = 0$ kJ/kmol

$h^{O_2}_{T=0} = 0$ kJ/kmol

a) $I_{sp} = \frac{u_e}{g}$, 1st law: $u_e^2 = 2 c_p T_t \left(1 - \frac{T_e}{T_t}\right)$ ∧ $\frac{T_e}{T_t} = \left(\frac{p_e}{p_t}\right)^{\frac{\gamma-1}{\gamma}} = 0$

$c_p = \frac{\gamma}{\gamma-1} R$, $R = \frac{\mathcal{R}}{\mathcal{M}}$ and $\mathcal{M} = \frac{\sum n_i \mathcal{M}_i}{\sum n_i}$ combustion products

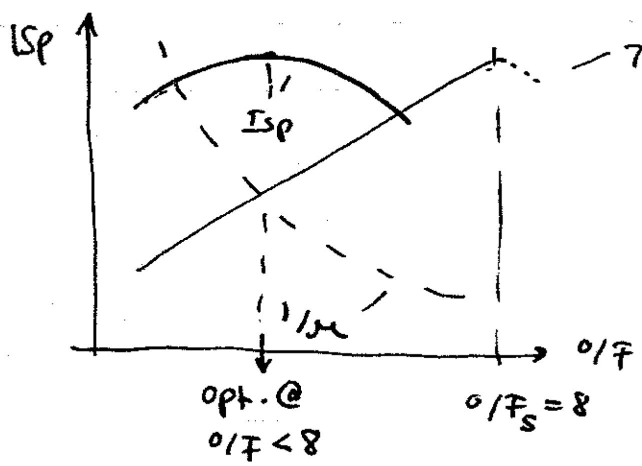
$$I_{sp} = \frac{1}{g}\sqrt{\frac{2\gamma}{\gamma-1}\frac{\mathcal{R}}{\mathcal{M}} T_t}$$

Note: $\mathcal{M}$ and T_t depend on oxidiser to fuel ratio O/F; assume γ constant (to simplify analysis)

‖ T_t highest at stoichiometric conditions

$x H_2 + \frac{1}{2} O_2 \longrightarrow H_2O + (x-1) H_2$ $x_{stoich} = 1$; $\frac{O}{F} = \frac{8}{x}$, $\frac{O}{F_{stoi.}} = 8$

$\mathcal{M}(x) = \frac{18 + 2(x-1)}{x}$ → ‖ $\mathcal{M}(O/F) = 2(O/F + 1)$ $O/F < 8$ fuel rich



No → run at fuel rich conditions! for optimum Isp

b) yes, $O/F = 6$ is fuel rich

→ to reduce $\mathcal{M}$ give low mass H_2 more weight (relative to higher mass O_2)

→ $x = 4/3 > 1$

c) find adiabatic flame temp. T_f: $\sum_{Reactants} n_i \left(h_f^{o(i)} + \Delta h^{(i)}_{298\to T}\right) = \sum_{Products} n_e \left(h_f^{o(e)} + \Delta h^{(e)}_{298\to T_f}\right)$

so $-x h^{H_2}_{298} - \frac{1}{2} h^{O_2}_{298} = h_f^{o\,H_2O} + \bar{c}_{p\,H_2O}(T_f - 298) + \bar{c}_{p\,H_2}(T_f - 298)(x-1)$

(assume avg. c_p over temperature range and constant $\gamma = \frac{\bar{c}_{p\,H_2O}}{\bar{c}_{p\,H_2O} - \mathcal{R}} \approx 1.17$)

$T_t = T_f = 298 - \frac{x h^{H_2}_{298} + \frac{1}{2} h^{O_2}_{298} + h_f^{o\,H_2O}}{x \bar{c}_{p\,H_2} + \bar{c}_{p\,H_2O} - \bar{c}_{p\,H_2}}$ ∧ $x = 8/(O/F)$

for a): $T_f = 4337$ K , $u_e = 5250$ m/s ; $I_{sp} = 535$ s

for b): $T_f = 3555$ K , $u_e = 5389$ m/s ; $\gamma = 1.19$, $I_{sp} = 549$ s

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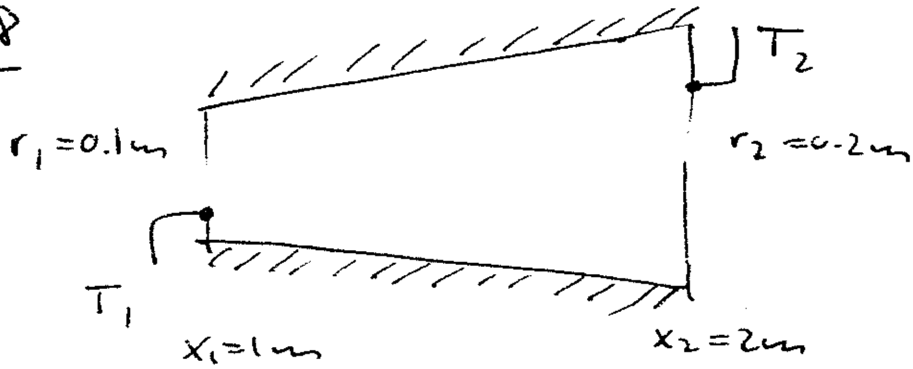
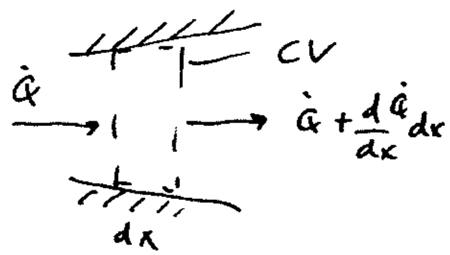
T18



Assume 1-D heat x-fer

a)



1st law: $0 = \dot{Q} - \left(\dot{Q} + \frac{d\dot{Q}}{dx}dx\right)$

$$\frac{d\dot{Q}}{dx} = 0 \quad \text{or} \quad \dot{Q} = \text{const}$$

b)

$\dot{Q} = A(x)\,\dot{q}(x) \qquad A = \pi r^2(x) = 0.01\pi x^2$

$$\frac{d}{dx}(A\dot{q}) = 0; \quad \frac{d\dot{q}}{dx} + \left(\frac{1}{A}\frac{dA}{dx}\right)\dot{q} = 0 \qquad \frac{dA}{dx} = 0.02\pi x$$

c) Fourier: $\dot{q} = -k\frac{dT}{dx}$ solve for $\dot{q}$ first, then for $T(x)$

$$\frac{d\dot{q}}{dx} + \frac{2}{x}\dot{q} = 0 \rightarrow \frac{d\dot{q}}{\dot{q}} = -\frac{2dx}{x}; \quad \ln\dot{q} = -2\ln x + \ln\tilde{C}_1$$

$$\dot{q} = \frac{\tilde{C}_1}{x^2} \quad \text{then} \quad \frac{dT}{dx} = \frac{-C_1}{x^2} \quad \text{and} \quad T(x) = \frac{C_1}{x} + C_2$$

B.C.: $T(1) = T_1 \ : \ T_1 = C_1 + C_2$

$T(2) = T_2 \ : \ T_2 = \frac{C_1}{2} + C_2$

$$T(x) = \frac{2(T_1 - T_2)}{x} + 2T_2 - T_1$$

d)

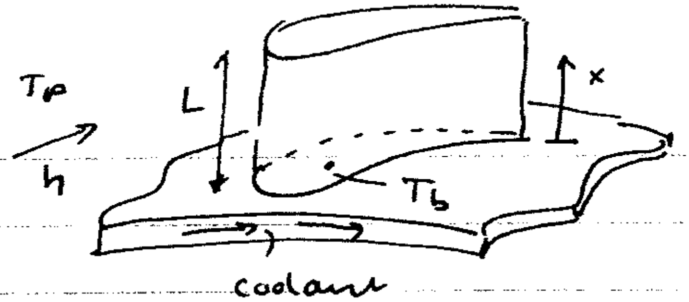
$$\dot{Q} = -k\,0.01\pi \left.\frac{dT}{dx}\right|_{x=1} = 0.02\pi k(T_1 - T_2)$$

$$R = \frac{T_1 - T_2}{\dot{Q}} \quad \text{so} \quad R = \frac{1}{0.02\pi k}$$

e)

$$\dot{q} = -k\frac{dT}{dx} = 2k\frac{T_1 - T_2}{x^2} \qquad \dot{q}(x) = 2k\frac{T_1 - T_2}{x^2}$$

T19



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$T_\infty = 1200°C \quad h = 250\ W/m^2K$

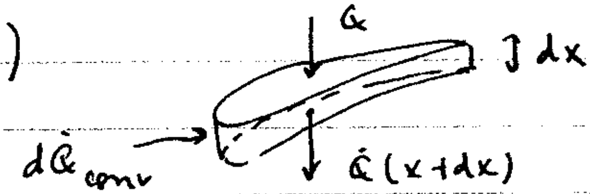
$k = 20\ W/mK \quad L = 50\ mm$

$A_c = 6 \cdot 10^{-4}\ m^2 \quad P = 110\ mm$

$T_b = 300°C$

Assume: blade tip adiabatic

a)



1st law:

$$0 = \dot{Q}(x) - \dot{Q}(x+dx) + d\dot{Q}_{conv}$$

$$d\dot{Q}_{conv} = h(T_\infty - T(x))P\,dx$$

$$\dot{Q}(x) = -kA_c \frac{dT}{dx} \quad \text{and} \quad \dot{Q}(x+dx) = \dot{Q}(x) + \frac{d\dot{Q}}{dx}dx$$

get thus
$$\frac{d^2T}{dx^2} + \frac{hP}{kA_c}(T_\infty - T) = 0$$

b) B.C.: $x=0$: $T(0) = T_b$, $x = L$: $\dot{Q}(x=L) = 0 \quad \left.\frac{dT}{dx}\right|_L = 0$

c) use $\Theta = T - T_\infty$, $c^2 = \frac{hP}{kA_c}$ $\Bigg\|$ $\frac{d^2\Theta}{dx^2} - c^2\Theta = 0 \ \wedge\ \Theta(0) = T_b - T_\infty$, $\frac{d\Theta}{dx}(L) = 0$

$$\Theta = \tilde{C}_1 e^{-cx} + \tilde{C}_2 e^{+cx}; \quad \tilde{C}_1 = \tilde{C}_2 e^{2cL}, \quad \tilde{C}_2 = \frac{T_b - T_\infty}{1 + e^{2cL}}$$

so
$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{e^{\sqrt{\frac{hP}{kA_c}}(2L - x)} + e^{\sqrt{\frac{hP}{kA_c}}x}}{1 + e^{2L\sqrt{\frac{hP}{kA_c}}}} = \frac{\cosh\left(L\sqrt{\frac{hP}{kA_c}}\left(1 - \frac{x}{L}\right)\right)}{\cosh\left(L\sqrt{\frac{hP}{kA_c}}\right)}$$

Plug in numbers get $T(L) = 1037°C$ → yes, scheme is satisfactory

d) $\dot{Q} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$ $\quad \frac{dT}{dx} = \frac{d\Theta}{dx} = c(T_b - T_\infty)\frac{1 - e^{2cL}}{1 + e^{2cL}} = c(T_b - T_\infty)\tanh(cL)$

$$\dot{Q} = \sqrt{hPkA_c}\,(T_\infty - T_b)\tanh\left(\sqrt{\frac{hP}{kA_c}L^2}\right) \qquad \dot{Q} = 508\ W$$

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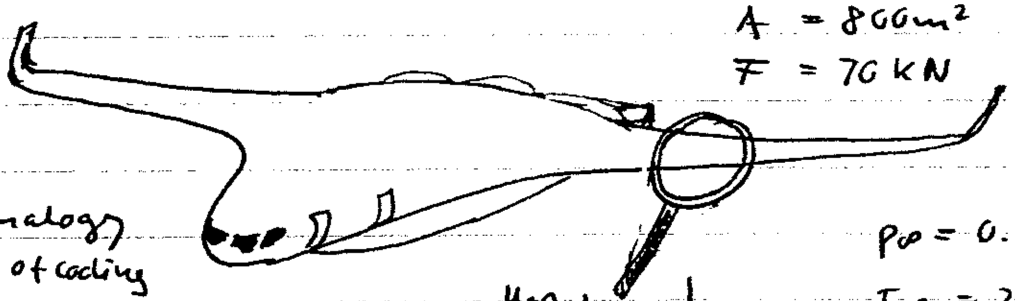
$M_\infty = 0.8$

$A = 800\,m^2 \quad A_{LE} = 20\,m^2$

$F = 70\,kN$



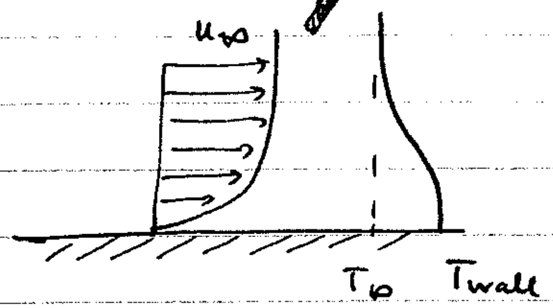
Concepts:

- Reynolds Analogy
- Newton's law of cooling
- Adiab. wall temp.

$p_\infty = 0.187\,bar$

$T_\infty = 217\,K$

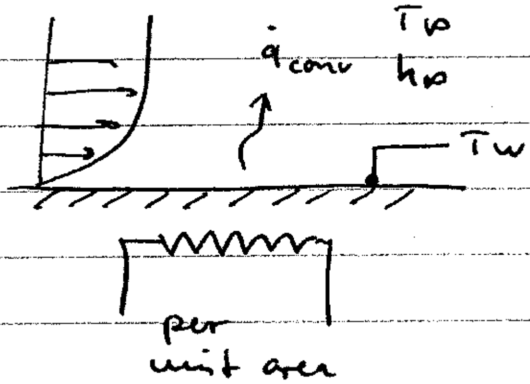
a) adiabatic wing



$$T_{wall} = T_{t\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right) T_\infty$$

$\underline{T_{wall} = 245\,K}$ → $-28°C$ need surface heating

b)



level flight:

$$F = D = \frac{1}{2} \rho_\infty u_\infty^2 A\, c_D$$

$$c_D = \frac{F\, R T_\infty}{\frac{1}{2} p_\infty u_\infty^2 A}$$

$$u_\infty^2 = M_\infty^2 \gamma R T_\infty$$

$$c_D = \frac{2F}{\gamma p_\infty M_\infty^2 A} = 0.01$$

Reynolds analogy $\quad St = \frac{c_D}{2} = \frac{h_\infty}{\rho_\infty u_\infty c_p} \rightarrow h_\infty = 372\,W/m^2$

1st law:

$$\dot{W}_{el} = A_{LE}\, h_\infty (T_w - T_\infty) \quad \wedge \quad T_w = 5°C$$

$$\underline{\dot{W}_{el} = 453\,W}$$

Problem S13 (Signals and Systems) SOLUTION

1. The LT is

$$G(s) = \frac{3s^2+3s-10}{s^2-4}, \quad -2 < \text{Re}[s] < 2$$

Because the numerator is the same order as the denominator, the partial fraction expansion will have a constant term:

$$\begin{aligned} G(s) &= \frac{3s^2+3s-10}{s^2-4} \\ &= \frac{3s^2+3s-10}{(s-2)(s+2)} \\ &= a + \frac{b}{s-2} + \frac{c}{s+2} \end{aligned}$$

To find a, b, and c, use coverup method:

$$\begin{aligned} a &= G(s)\big|_{s=\infty} = 3 \\ b &= \left.\frac{3s^2+3s-10}{s+2}\right|_{s=2} = 2 \\ c &= \left.\frac{3s^2+3s-10}{s-2}\right|_{s=-2} = 1 \end{aligned}$$

So

$$G(s) = 3 + \frac{2}{s-2} + \frac{1}{s+2}, \qquad -2 < \text{Re}[s] < 2$$

We can take the inverse LT by simple pattern matching, taking note of the fact that the pole at $s = 2$ must be anticausal, since it is to the right of the ROC. The result is that

$$g(t) = 3\delta(t) - 2e^{2t}\sigma(-t) + e^{-2t}\sigma(t)$$

2. The LT is

$$G(s) = \frac{3s^2+3s-10}{s^2-4}, \quad \text{Re}[s] < -2$$

that is, the same as in part (1) except for the ROC. Therefore,

$$G(s) = 3 + \frac{2}{s-2} + \frac{1}{s+2}, \qquad \text{Re}[s] < -2$$

The inverse LT is then

$$g(t) = 3\delta(t) - 2e^{2t}\sigma(-t) - e^{-2t}\sigma(-t)$$

3. The LT is given by

$$G(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)}, \quad -2 < \text{Re}[s] < -1$$

The partial fraction expansion is

$$\begin{aligned} G(s) &= \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)} \\ &= \frac{a}{s+1}+\frac{b}{s+2}+\frac{c}{s+3} \end{aligned}$$

where

$$\begin{aligned} a &= \left.\frac{6s^2+26s+26}{(s+2)(s+3)}\right|_{s=-1} = 3 \\ b &= \left.\frac{6s^2+26s+26}{(s+1)(s+3)}\right|_{s=-2} = 2 \\ c &= \left.\frac{6s^2+26s+26}{(s+1)(s+2)}\right|_{s=-3} = 1 \end{aligned}$$

So

$$G(s) = \frac{3}{s+1}+\frac{2}{s+2}+\frac{1}{s+3}, \qquad -2 < \mathrm{Re}[s] < -1$$

The inverse LT is given by

$$g(t) = \left(e^{-3t}+2e^{-2t}\right)\sigma(t) - 3e^{-t}\sigma(-t)$$

4. The LT is

$$G(s) = \frac{6s^2+26s+26}{(s+1)(s+2)(s+3)}, \quad -3 < \mathrm{Re}[s] < -2$$

This is the same as in part (3), except for the ROC. Therefore, the partial fraction expansion is

$$G(s) = \frac{3}{s+1}+\frac{2}{s+2}+\frac{1}{s+3}, \qquad -3 < \mathrm{Re}[s] < -2$$

and the inverse LT is

$$g(t) = e^{-3t}\sigma(t) - \left(2e^{-2t}+3e^{-t}\right)\sigma(-t)$$

5. The LT is

$$G(s) = \frac{4s^2+11s+9}{(s+1)^2(s+2)}, \quad -2 < \mathrm{Re}[s] < -1$$

This one is a little tricky — there is a second order pole at $s = -1$. So the partial fraction expansion is

$$G(s) = \frac{4s^2+11s+9}{(s+1)^2(s+2)} = \frac{a}{s+1}+\frac{b}{(s+1)^2}+\frac{c}{s+2}$$

We can find b and c by the coverup method:

$$\begin{aligned} b &= \left.\frac{4s^2+11s+9}{s+2}\right|_{s=-1} = 2 \\ c &= \left.\frac{4s^2+11s+9}{(s+1)^2}\right|_{s=-2} = 3 \end{aligned}$$

So

$$G(s) = \frac{a}{s+1} + \frac{2}{(s+1)^2} + \frac{3}{s+2}$$

To find a, pick a value of s, and plug into the equation above. The easiest value to pick is $s = 0$. Then

$$G(0) = \frac{a}{1} + \frac{2}{(1)^2} + \frac{3}{2} = \frac{9}{2}$$

Solving, we have

$$a = 1$$

Therefore,

$$G(s) = \frac{1}{s+1} + \frac{2}{(s+1)^2} + \frac{3}{s+2}, \qquad -2 < \text{Re}[s] < -1$$

The inverse LT is then

$$g(t) = 3e^{-2t}\sigma(t) - \left(e^{-t} + 2te^{-t}\right)\sigma(-t)$$

6. The LT is

$$G(s) = \frac{4s^2 + 11s + 9}{(s+1)^2(s+2)}, \quad \text{Re}[s] < -2$$

This problem is similar to above. The partial fraction expansion is

$$G(s) = \frac{1}{s+1} + \frac{2}{(s+1)^2} + \frac{3}{s+2}, \qquad \text{Re}[s] < -2$$

The inverse LT is then

$$g(t) = -\left(e^{-t} + 2te^{-t} + 3e^{-2t}\right)\sigma(-t)$$

7. The LT is

$$G(s) = \frac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad -1 < \text{Re}[s] < 0$$

We can find b and d by the coverup method

$$\begin{aligned} b &= \left.\frac{4s^3 + 11s^2 + 5s + 2}{(s+1)^2}\right|_{s=0} = 2 \\ d &= \left.\frac{4s^3 + 11s^2 + 5s + 2}{s^2}\right|_{s=-1} = 4 \end{aligned}$$

So

$$G(s) = \frac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2} = \frac{a}{s} + \frac{2}{s^2} + \frac{c}{s+1} + \frac{4}{(s+1)^2}$$

To find a and c, pick two values of s, say, $s = 1$ and $s = 2$. Then

$$\begin{aligned} G(1) &= \frac{4 + 11 + 5 + 2}{1^2(1+1)^2} = \frac{a}{1} + \frac{2}{1^2} + \frac{c}{1+1} + \frac{4}{(1+1)^2} \\ G(2) &= \frac{4 \cdot 2^3 + 11 \cdot 2^2 + 5 \cdot 2 + 2}{2^2(2+1)^2} = \frac{a}{2} + \frac{2}{2^2} + \frac{c}{2+1} + \frac{4}{(2+1)^2} \end{aligned}$$

Simplifying, we have that

$$\begin{aligned} a + \frac{c}{2} &= \frac{5}{2} \\ \frac{a}{2} + \frac{c}{3} &= \frac{3}{2} \end{aligned}$$

Solving for a and c, we have that

$$\begin{aligned} a &= 1 \\ c &= 3 \end{aligned}$$

So

$$G(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{3}{s+1} + \frac{4}{(s+1)^2}, \qquad -1 < \mathrm{Re}[s] < 0$$

The inverse LT is then

$$g(t) = \left(3e^{-t} + 4te^{-t}\right)\sigma(t) - (1+2t)\,\sigma(-t)$$

8. The LT is

$$G(s) = \frac{4s^3 + 11s^2 + 5s + 2}{s^2(s+1)^2}, \quad \mathrm{Re}[s] < -1$$

From above, the PFE is

$$G(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{3}{s+1} + \frac{4}{(s+1)^2}, \qquad \mathrm{Re}[s] < -1$$

The inverse LT is then

$$g(t) = -\left(3e^{-t} + 4te^{-t} + 1 + 2t\right)\sigma(-t)$$

9. The LT is

$$G(s) = \frac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)}, \quad \mathrm{Re}[s] < 0$$

$G(s)$ can be expanded as

$$\begin{aligned} G(s) &= \frac{s^3 + 3s^2 + 9s + 12}{(s^2+4)(s^2+9)} \\ &= \frac{s^3 + 3s^2 + 9s + 12}{(s+2j)(s-2j)(s+3j)(s-3j)} \\ &= \frac{a}{s+2j} + \frac{b}{s-2j} + \frac{c}{s+3j} + \frac{d}{s-3j} \end{aligned}$$

The coefficients can be found by the coverup method:

$$\begin{aligned} a &= \left.\frac{s^3 + 3s^2 + 9s + 12}{(s-2j)(s+3j)(s-3j)}\right|_{s=-2j} = 0.5 \\ b &= \left.\frac{s^3 + 3s^2 + 9s + 12}{(s+2j)(s+3j)(s-3j)}\right|_{s=+2j} = 0.5 \\ c &= \left.\frac{s^3 + 3s^2 + 9s + 12}{(s+2j)(s-2j)(s-3j)}\right|_{s=-3j} = 0.5j \\ d &= \left.\frac{s^3 + 3s^2 + 9s + 12}{(s+2j)(s-2j)(s+3j)}\right|_{s=+3j} = -0.5j \end{aligned}$$

Therefore

$$G(s) = \frac{0.5}{s+2j} + \frac{0.5}{s-2j} + \frac{0.5j}{s+3j} + \frac{-0.5j}{s-3j}, \qquad \text{Re}[s] < 0$$

and the inverse LT is

$$g(t) = -0.5\left(e^{-2jt} + e^{2jt} + je^{-3jt} - je^{3jt}\right)\sigma(-t)$$

This can be expanded using Euler's formula, which states that

$$e^{ajt} = \cos at + j\sin at$$

Applying Euler's formula yields

$$g(t) = -\left(\cos 2t + \sin 3t\right)\sigma(-t)$$

Problem S14 (Signals and Systems) SOLUTION

The transfer functions needed in this problem are the (noncausal) smoother,

$$G_1(s) = \frac{-a^2}{(s-a)(s+a)}$$

and a similar (causal) low-pass filter

$$G_2(s) = \frac{a^2}{(s+a)^2}$$

The input is assumed to be

$$u(t) = \cos \omega t$$

1. Find the transfer function, $G_1(j\omega)$, as a function of frequency, ω. **Solution:** Simply replacing s by $j\omega$, we have

$$G_1(j\omega) = \frac{-a^2}{(j\omega - a)(j\omega + a)} = \frac{a^2}{\omega^2 + a^2}$$

Note that even though $j\omega$ is complex, $G_1(j\omega)$ is real for all ω.

2. Since the transfer function is complex, it can be represented as

$$G_1(j\omega) = A_1(\omega)e^{j\phi_1(\omega)}$$

where the amplitude of the transfer function is $A_1(\omega)$, and the phase of the transfer function is $\phi_1(\omega)$. Find $A_1(\omega)$ and $\phi_1(\omega)$. **Solution:** The expression above can be expanded as

$$G_1(j\omega) = A_1(\omega)\left(\cos\left(\phi_1(\omega)\right) + j\sin\left(\phi_1(\omega)\right)\right)$$

and hence

$$\phi_1 = \tan^{-1}\left(\frac{\mathrm{Im}(G_1)}{\mathrm{Re}(G_1)}\right)$$
$$A_1 = \mathrm{abs}\left(G_1\right)$$

Since in the present case, $G_1(j\omega)$ is real, we must have that the phase $\phi_1 = 0$, and therefore

$$A_1(\omega) = \frac{a^2}{\omega^2 + a^2}$$
$$\phi_1\omega = 0$$

3. Find the transfer function, $G_2(j\omega)$, as a function of frequency, ω, as well as $A_2(\omega)$ and $\phi_2(\omega)$. **Solution:** Express G_2 as

$$G_2(j\omega) = \frac{a^2}{(j\omega + a)^2}$$

Since the magnitude of a product is the product of the magnitudes,

$$\begin{aligned} A_2(\omega) &= \frac{a^2}{\text{abs}(j\omega + a)^2} \\ &= \frac{a^2}{\left(\sqrt{\omega^2 + a^2}\right)^2} \\ &= \frac{a^2}{\omega^2 + a^2} \end{aligned}$$

That is, $A_2(\omega)$ and $A_1(\omega)$ are exactly the same!

Next, find ϕ_2:

$$\phi_2(\omega) = -2\tan^{-1}(\omega/a)$$

The minus sign is because the term $j\omega + a$ is in the denominator of $G_2(j\omega)$; the factor of 2 is because there are two such terms. So the phases of the transfer functions are different, even though the amplitudes are the same.

4. For the input $u(t)$ above, show that the output of the system G_1 is

$$y_1(t) = A_1(\omega)\cos(\omega t + \phi_1(\omega))$$

and do likewise for system G_2. **Solution:** It's enough to show the results for $y_1(t)$, since the result for the case $y_2(t)$ results just by changing subscripts. Since the input is

$$u(t) = \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

the output is given by

$$\begin{aligned} y_1(t) &= \frac{G_1(j\omega)e^{j\omega t} + G_1(-j\omega)e^{-j\omega t}}{2} \\ &= \frac{A_1(\omega)e^{j\phi_1(\omega)}e^{j\omega t} + A_1(-\omega)e^{j\phi_1(-\omega)}e^{-j\omega t}}{2} \end{aligned}$$

But $A_1(-\omega) = A_1(\omega)$, and $\phi_1(-\omega) = -\phi_1(\omega)$. This result is in fact valid for any transfer function $G(j\omega)$ that results from a real impulse response $g(t)$. Therefore,

$$\begin{aligned} y_1(t) &= \frac{A_1(\omega)e^{j\phi_1(\omega)}e^{j\omega t} + A_1(\omega)e^{-j\phi_1(\omega)}e^{-j\omega t}}{2} \\ &= A_1(\omega)\frac{e^{j\phi_1(\omega)}e^{j\omega t} + e^{-j\phi_1(\omega)}e^{-j\omega t}}{2} \\ &= A_1(\omega)\cos(\omega t + \phi_1(\omega)) \end{aligned}$$

as required.

5. A_1 and A_2 determine how much the magnitude of the input cosine wave is affected by each filter. Ideally, A_1 and A_2 would be 1, meaning that the filters don't change the magnitude of the input sine at all. Which filter (if either) changes the magnitude the least? **Solution:** $A_1(\omega) = A_2(\omega)$, so the two filters have exactly the same magnitude response — neither is better in that regard.

6. ϕ_1 and ϕ_2 determine how much the phase of the input cosine wave is affected by each filter. Non-zero values of ϕ correspond to a shifting left or right (i.e., advancing or delaying) the sine wave. Ideally, ϕ_1 and ϕ_2 would be zero, meaning that the filters don't change the phase of the input sine at all. Which filter (if either) produces the least phase shift? **Solution:** $\phi_1(\omega) = 0$ for all ω, so it has the least phase shift.

7. Explain why the non-causal filter is preferred in signal processing applications where it can be applied. **Solution:** For many signal processing applications, the noncausal filter would be preferred, since it produces no change in the phase of the signal from input to output. Conversely, real-time filters almost always unavoidably change the phase from input to output.