Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 12

Name: ______________________

Due Date: 05/08/2007

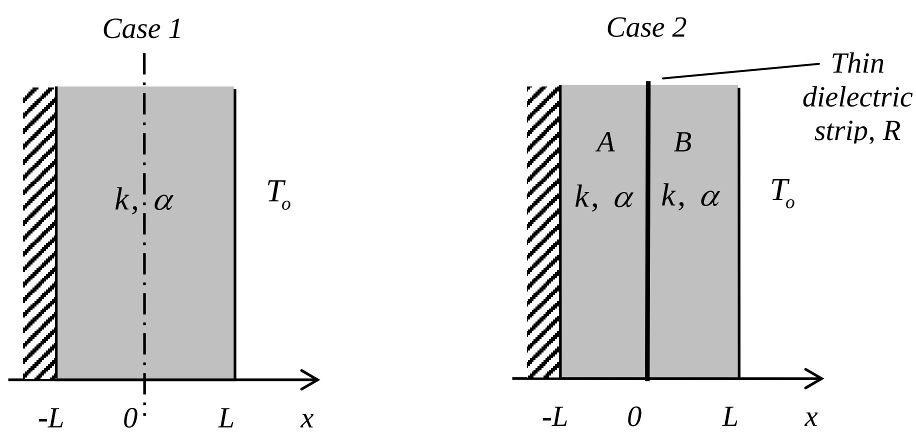
	Time Spent (min)
T21	
T22	
S15	
S16	
M12.1	
M12.2	
Study Time	

Announcements:

(Add a short summary of the concepts you are using to solve the problem)

Problem T21

A plane wall of thickness *2L* and thermal conductivity *k* experiences a uniform volumetric heat source generating heat per unit volume at a rate α. As shown in the sketch for case 1, the surface at *x* = -*L* is perfectly insulated, while the other surface is maintained at a uniform, constant temperature T_o. For case 2, a very thin dielectric strip is inserted at the midpoint of the wall (*x* = *0*) in order to electrically isolate the two sections, A and B. The thermal resistance of the strip is *R* = 0.0005 m²K/W. The parameters associated with the wall are *k* = 50 W/mK, *L* = 20 mm, $\alpha = 5\times10^6$ W/m³, and $T_o = 50^o$C.



a) Derive an equation for the temperature distribution in case 1.

b) Solve for the temperature distribution in case 1 and sketch it on a *T*-*x* plot. Describe the key features of this distribution.

c) Where in the wall for case 1 is the location of maximum temperature? Find the maximum temperature.

d) Repeat a) through b) for case 2.

e) What is the temperature difference between the two walls at *x* = *0* for case 2?

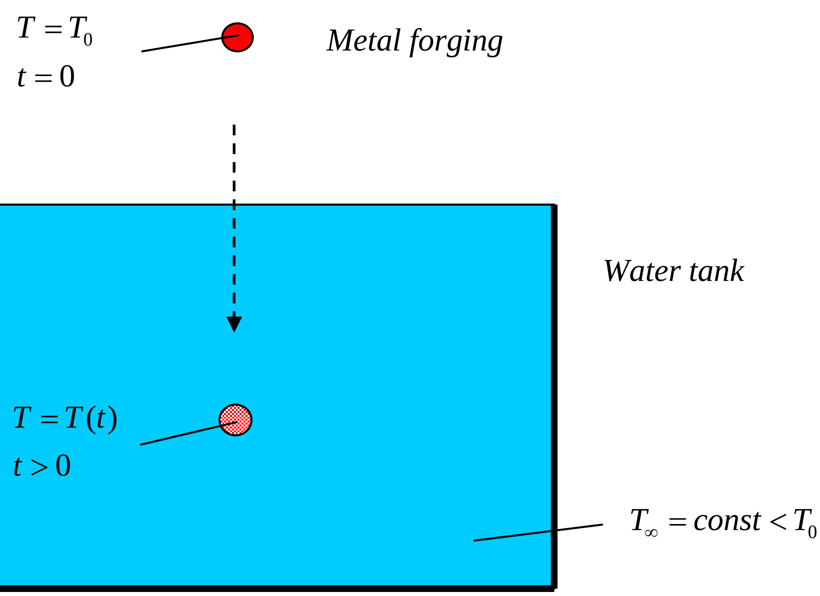
f) Where in the wall for case 2 is the location of maximum temperature? Find the maximum temperature.

Unified Engineering Spring 2007
Thermodynamics & Propulsion Z. S. Spakovszky

(Add a short summary of the concepts you are using to solve the problem)

Problem T22

A small spherical metal forging of radius R is dropped into a large tank of cold water. You can assume a spatially uniform temperature in the forging (temperature gradients within the solid are negligible). The specific heat and the density of the metal are c and ρ respectively, and the very large tank of water is at constant temperature T_∞ .



a) What is the dominant heat transfer mode at play? Define the appropriate heat transfer coefficient (introduce it as a known parameter).

b) Write down the 1st law for this problem. Is the situation steady or unsteady?

c) Find an equation to determine the temperature of the metal forging.

d) Solve for the metal temperature and plot it as a function of time.

e) What is the time constant of the cooling process? Interpret your result and indicate the time constant in your plot.

f) In analogy to electrical circuits, what would be the appropriate thermal network model?

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Problem S15 (Signals and Systems)

Find the Fourier transforms of the following signals:

1. $$g(t) = \delta(t - T)$$

   Note: The system with impulse response $g(t)$ produces an output that is the input delayed by T. Since delays occur frequently in signal processing, $G(j\omega)$ is an important transfer function.

2. $$g(t) = \begin{cases} 1, & |t| \leq T \\ 0, & |t| > T \end{cases}$$

   Note: Because $g(t)$ is symmetric, $G(j\omega)$ should be real. Please express your answer so that it is apparent that the answer is real.

3. $$g(t) = \frac{1}{t^2 + T^2}$$

   Hint: If you find the integral hard to do, you might be able to find the answer using duality.

4. $$g(t) = \frac{\sin \pi t/T}{\pi t/T}$$

   Hint: You almost certainly won't be able to do the FT integral directly. Use duality and the results of (2) above to find the answer. The $g(t)$ in this problem has important connections to, among other things, CD players!

5. Find the inverse transform of

   $$G(j\omega) = \left(\frac{\sin \omega T}{\omega T}\right)^2$$

   using the results of part (2), and FT properties. Hint: You will have to use the result that $\mathcal{F}[g * h] = \mathcal{F}[g]\mathcal{F}[h]$.

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Problem S16 (Signals and Systems)

1. In class, we found that the Fourier transform of a complex exponential is given by

$$\mathcal{F}\left[e^{j\omega_0 t}\right] = 2\pi\delta(\omega - \omega_0)$$

We showed that this result is correct by working backwards, that is, by showing that

$$\mathcal{F}^{-1}\left[2\pi\delta(\omega - \omega_0)\right] = e^{j\omega_0 t}$$

Prove the result directly, by finding the FT of the signal

$$g_a(t) = e^{j\omega_0 t}e^{-a|t|}$$

and taking the limit as $a \to 0$.

2. Use the result above to find

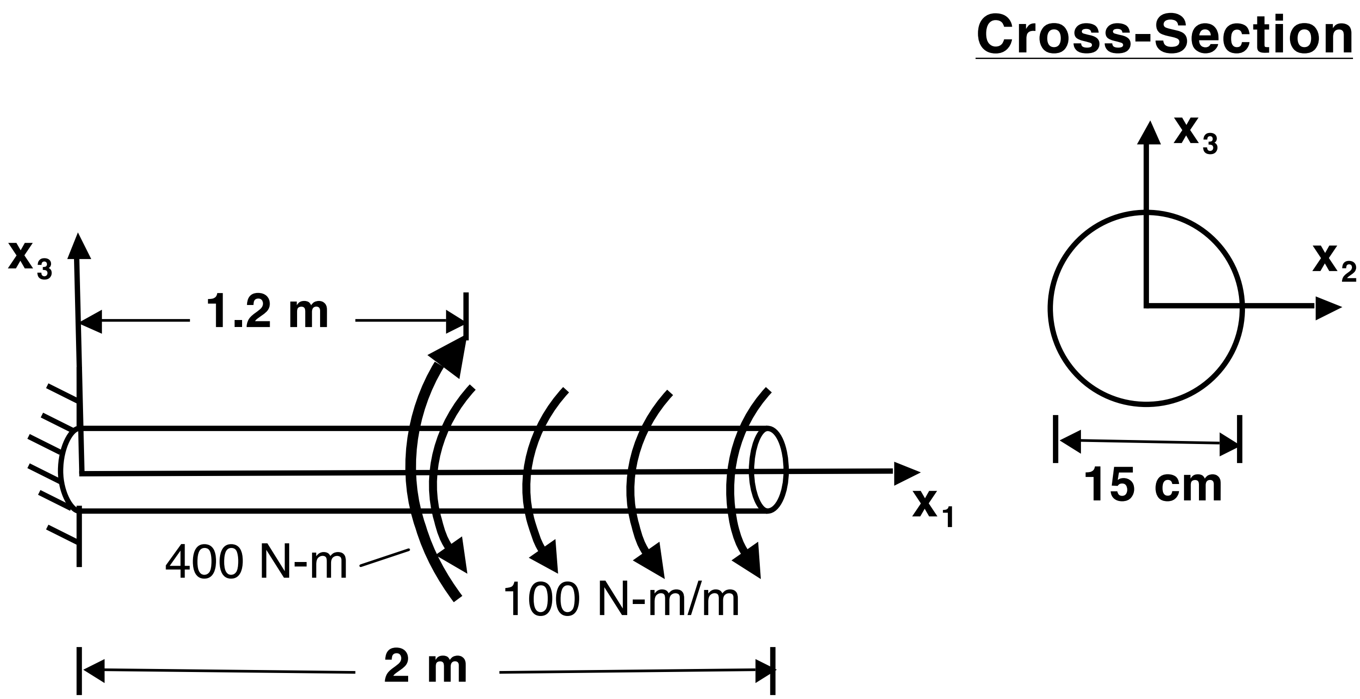
$$\mathcal{F}[\cos at]$$

and

$$\mathcal{F}[\sin at]$$

Unified Engineering Problem Set **Lectures: M11, M12**
Week 12 Spring, 2007 **Units: M4.6**

M12.1 ***(10 points)*** A steel rod has a solid circular cross-section with a 15-cm diameter and is 2 meters in length. The rod is clamped to a solid wall at one end, and is subjected to a concentrated negative torque of 400 N-m at a point 1.2 m from the clamped end. In addition, there is a distributed positive torque of 100 N-m/m along the latter end of the rod from the point of application of the concentrated torque to the free edge. The configuration is shown below. The modulus of steel is 200 GPa and the Poisson's ratio is 0.3.



(a) Determine the torque distribution in the rod structural configuration and sketch this as a function of x_1.

(b) Determine the twist at the tip of the rod.

(c) Determine the maximum shear stress in the rod and its location.

(d) If the rod were a hollow tube with the same outer radius and a wall with a thickness of 3 mm, how would these answers change?

M12.2 ***(10 points)*** You are asked to serve as a consultant for a company designing a device to measure torque. Their current preliminary design entails a circular rod clamped at one end with a pointer at the other end which will be calibrated to a linear angular scale. There will be a gear at the center of the shaft which will be connected by a chain to allow the proper torque measurement.

The designers want to maximize the sensitivity of the rod. The variables they have are the material of the rod and the rod cross-section and its size (maintaining the circular constraint). The length is relatively constrained.

Please comment on the issues which must be considered. Use equations as needed/helpful. A hand sketch of a model of the configuration would be helpful. The answer should be on the order of one or two paragraphs (hand-written) to the company giving your recommendations and reasoning. An appendix showing the derivation of any pertinent equations can be included as deemed necessary.