Massachusetts Institute of Technology
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16.003/16.004 Unified Engineering III, IV
Spring 2007

Problem Set 13

Name: ________________________

Due Date: 05/15/2007

	Time Spent (min)
S17	
S18	
S19	
S20	
M13.1	
M13.2	
M13.3	
Study Time	

Announcements:

Unified Engineering II **Spring 2007**

Problem S17 (Signals and Systems)

Consider the *quadrature modulation/demodulation* system shown below. The purpose of the system is to transmit two signals, $x_1(t)$ and $x_2(t)$, over the same frequency band simultaneously. $x_1(t)$ and $x_2(t)$ are bandlimited signals, with bandwidth W. That is, their Fourier transforms $X_1(f)$ and $X_2(f)$ satisfy

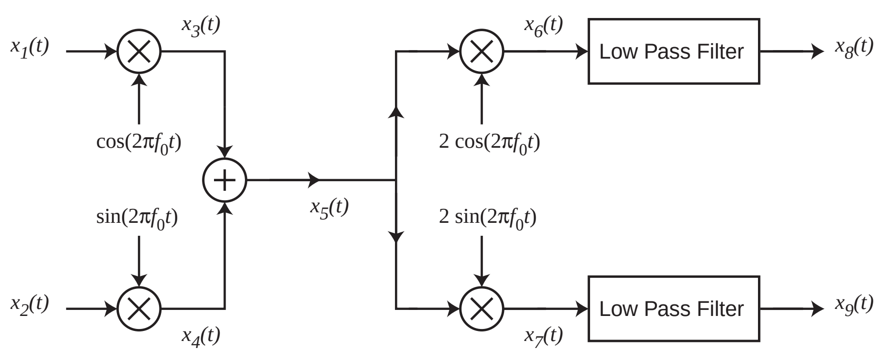
$$X_1(f) = 0, \quad |f| \geq W$$

$$X_2(f) = 0, \quad |f| \geq W$$

The bandwidth is much less than the modulation frequency, f_0. The lowpass filters shown in the diagram are ideal, with transfer function

$$L(f) = \begin{cases} 1, & |f| < W \\ 0, & |f| > W \end{cases}$$

Find the Fourier transforms of the signals $x_3(t)$, $x_4(t)$, $x_5(t)$, $x_6(t)$, $x_7(t)$, $x_8(t)$, and $x_9(t)$ in terms of $X_1(f)$ and $X_2(f)$.



The modulation / demodulation scheme here is very similar to that in AM-DSB/SC — if you understand this problem, you should understand AM-DSB/SC.

Problem S18 (Signals and Systems)

Do problem 8.26 from Openheim and Willksy, *Signals and Systems*, reprinted below:

8.26. In Section 8.2.2, we discussed the use of an envelope detector for asynchronous demodulation of an AM signal of the form $y(t) = [x(t) + A]\cos(\omega_c t + \theta_c)$. An alternative demodulation system, which also does not require phase synchronization, but does require frequency synchronization, is shown in block diagram form in Figure P8.26. The lowpass filters both have a cutoff frequency of ω_c. The signal $y(t) = [x(t) + A]\cos(\omega_c t + \theta_c)$, with θ_c constant but unknown. The signal $x(t)$ is band limited with $X(j\omega) = 0,\ |\omega| > \omega_M$, and with $\omega_M < \omega_c$. As we required for the use of the envelope detector, $x(t) + A > 0$ for all t.

Show that the system in Figure P8.26 can be used to recover $x(t)$ from $y(t)$ without knowledge of the modulator phase θ_c.

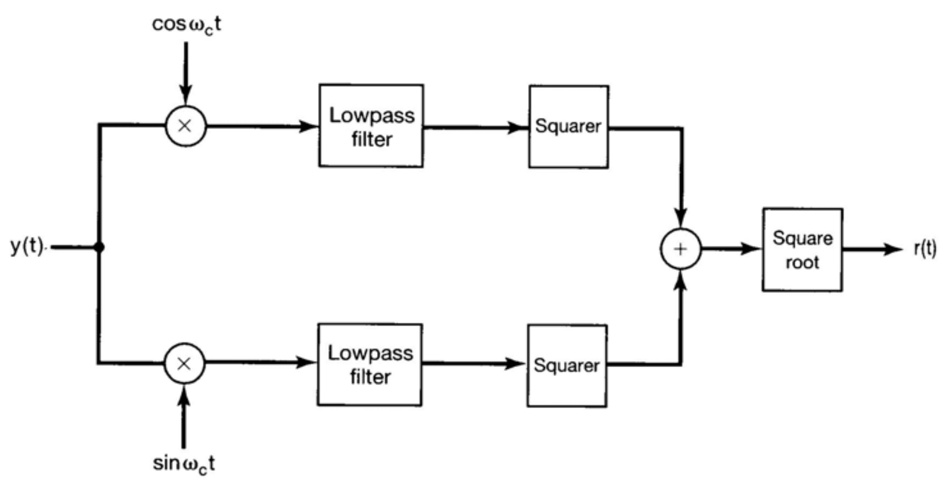


Figure P8.26

Problem S19 (Signals and Systems)

Do problem 8.8 from Openheim and Willksy, *Signals and Systems*, reprinted below:

8.8. Consider the modulation system shown in Figure P8.8. The input signal $x(t)$ has a Fourier transform $X(j\omega)$ that is zero for $|\omega| > \omega_M$. Assuming that $\omega_c > \omega_M$, answer the following questions:

(a) Is $y(t)$ guaranteed to be real if $x(t)$ is real?

(b) Can $x(t)$ be recovered from $y(t)$?

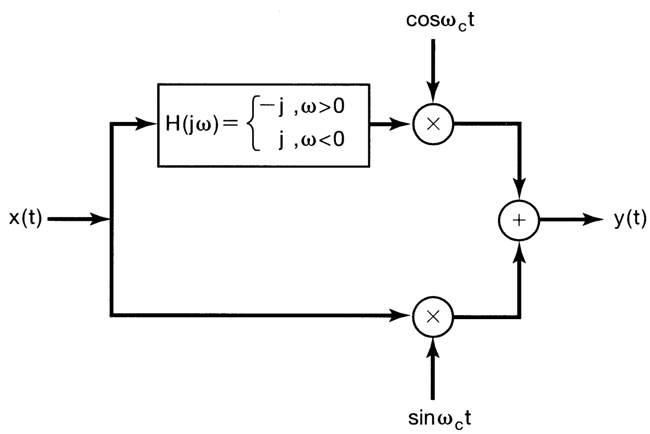


Figure P8.8

Note that this system implements a type of single sideband amplitude modulation.

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Problem S20 (Signals and Systems)

Do problem 7.23 from Openheim and Willksy, *Signals and Systems*, reprinted below:

7.23. Shown in Figure P7.23 is a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

(a) For $\Delta < \pi/(2\omega_M)$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.

(b) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $x_p(t)$.

(c) For $\Delta < \pi/(2\omega_M)$, determine a system that will recover $x(t)$ from $y(t)$.

(d) What is the *maximum* value of Δ in relation to ω_M for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$?

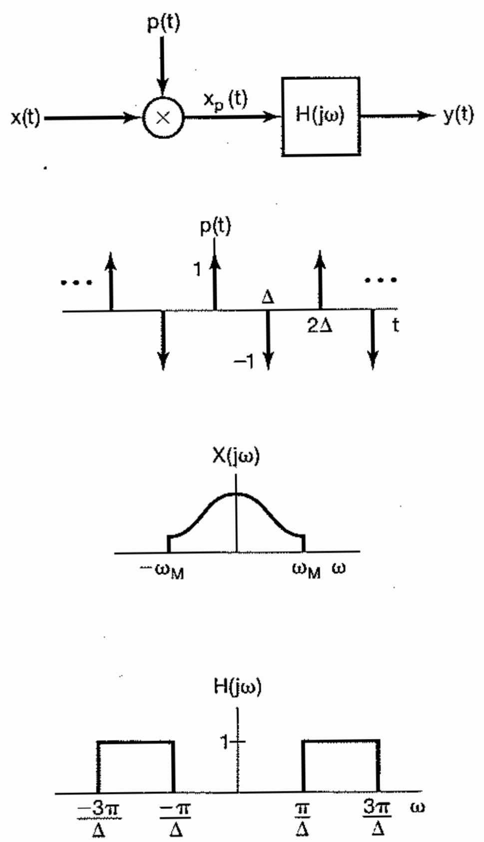
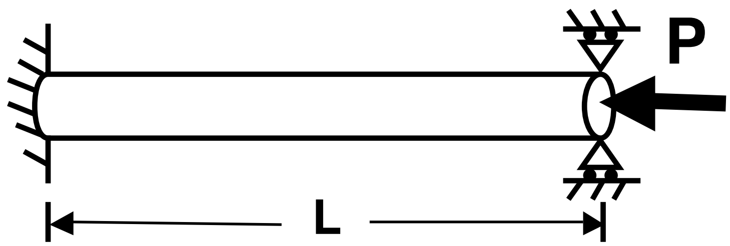


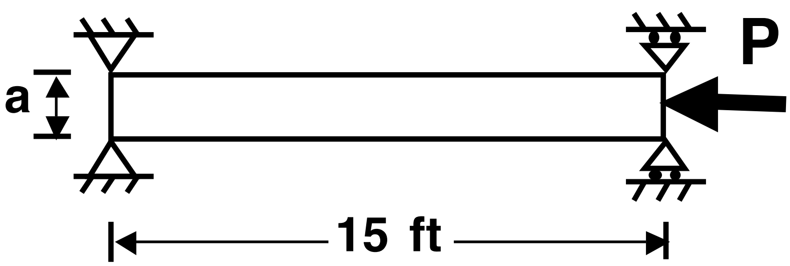
Figure P7.23

Unified Engineering Problem Set **Lectures: M13, M14, M15**
Week 13 Spring, 2007 **Units: M4.7, M4.8**

M13.1 ***(10 points)*** A column of length L is clamped at one end with a compressive load P applied at the other end which is supported by roller supports (think of it as a ring of roller balls). The column has a constant cross-section with area A and moment of inertia I, and is made of a material with modulus of E. Determine the expressions to find the buckling load and the buckling mode.



M13.2 ***(10 points)*** A series of 15-foot long columns is to be produced using aluminum (modulus = 10.3 Msi, σ_{cu} = 63 ksi, σ_{cy} = 55 ksi). These columns will be simply-supported and have square cross-sections with various side lengths. A compressive load will be applied at the column end with the roller supports. The supports at each end are attached totally around the cross-section edges. You are asked to determine a design chart showing the maximum load-carrying capability of these products as a function of the side length, a. Be sure to clearly indicate your reasoning and "important points" on the design chart.

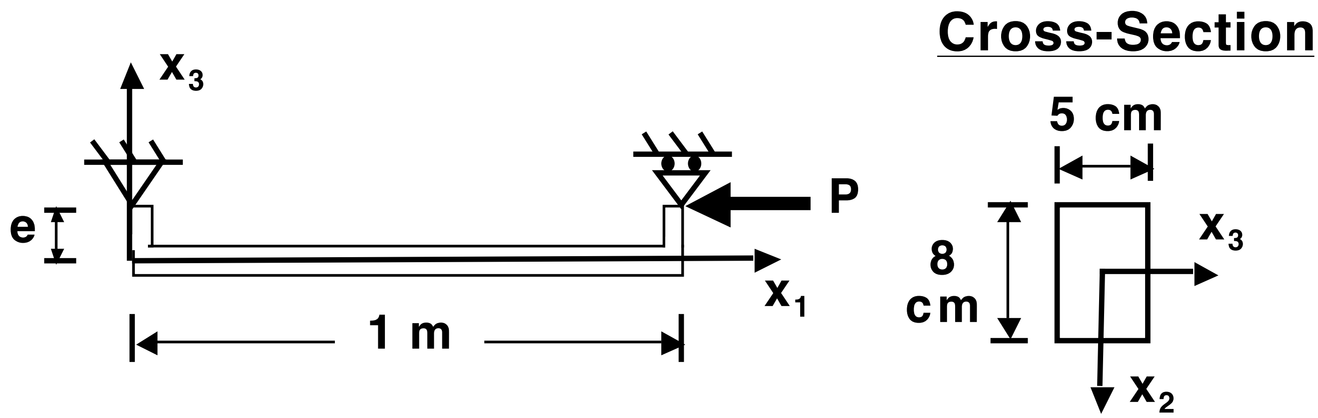


(a) Determine the buckling load for this configuration as a function of the side length of the aluminum columns.

(b) Determine the squashing load for this configuration as a function of the side length of the aluminum columns.

(c) Determine and sketch the design chart.

M13.3 ***(10 points)*** A steel (E = 200 GPa, □ = 0.3, $\square_{cu}$ = 1375 MPa, $\square_{cy}$ = 1305 MPa) column is 1 meter long and has a rectangular cross-section of 8 centimeters by 5 centimeters. The column is assumed simply-supported and is used to support a compressive load of magnitude P. Assume that the load is applied off the centerline of the column by an eccentricity of value e. Consider five cases of eccentricity normalized by the total length of the column: 0, 0.01, 0.02, 0.05, 0.1.



(a) For each case, determine the maximum load the column can carry.

(b) For each case, determine a normalized relationship between the applied load and the lateral deflection of the center of the column. Normalize the center deflection of the column by the specimen length and normalize the column load, P, by the critical buckling load, P_{cr}.

(c) Plot the normalized center deflection of the column versus the normalized load for the five cases of eccentricity.