Problem S21 (Signals and Systems)

Consider the signal

$$g(t) = (1 + |t|)e^{-|t|}$$

1. Plot the signal. Do you expect the signal to have a “good” duration-bandwidth product, meaning that the product is close to the lower bound?
2. Find the duration of the signal, Δt.
3. Find the bandwidth of the signal, $\Delta\omega$. You may want to use the time domain formula for the bandwidth.
4. How close is the answer to the theoretical lower bound? Explain why the answer is or is not close to the bound.

Unified Engineering II **Spring 2004**

Problem S22 (Signals and Systems)

Consider a pulse similar to the Loran-C pulse, given by

$$h(t) = t^3 e^{-t/\tau}\sigma(t)\sin(2\pi f t) = g(t)w(t)$$

where

$$g(t) = te^{-t/\tau}\sigma(t)$$
$$w(t) = \sin(2\pi f t)$$

(a) Find the *centroid* of the pulse envelope, given by

$$\bar{t} = \frac{\int t g^2(t)\,dt}{\int g^2(t)\,dt}$$

(b) Find the duration of the envelope, given by

$$\Delta t = 2\left(\frac{\int (t-\bar{t})^2 g^2(t)\,dt}{\int g^2(t)\,dt}\right)^{\frac{1}{2}}$$

(c)

$$\Delta\omega = 2\left(\frac{\int \dot{g}^2(t)\,dt}{\int g^2(t)\,dt}\right)^{\frac{1}{2}}$$

(d) How does the duration-bandwidth product compare to the theoretical minimum?

Unified Engineering Problem Set **Lectures: M16, M17, M18, M19**
Week 14 Spring, 2007 **Units: M5.1, M5.2, M5.3, M5.4**

(NOTE: _Not_ to be handed in. Solutions to be posted upon handout)

M14.1 A spring is basically a device for storing energy. Therefore, one key consideration in the design of a spring is the maximum energy that can be stored. A second consideration is that the spring should return to its original configuration when unloaded. Thus, the material cannot yield. Consider the simplest model of a spring -- a rod in uniaxial tension.

(a) Express the total energy stored per unit volume in the spring as a function of the applied stress and the modulus of the material.

(b) Identify the combination of material properties that maximize energy storage capacity (without yielding) for:
   (i) a given volume of material;
   (ii) a given mass of material;
   (iii) a given cost of material

(c) Choose amongst the following materials for each of the three criteria listed in part (b). Comment as appropriate.

Material	Modulus E [GPa}	Yield Stress σ_y [MPa]	Density ρ [Mg/m^3]	Price c [$/kg]
Al alloy	70.0	500	2.7	2.0
Spring Steel	210	2400	8.0	3.0
Rubber	0.05	30	0.9	1.3
Titanium	116	1400	4.5	10.1
Nickel	214	2000	8.9	4.3
Graphite/Epoxy	100	650	1.5	200

M14.2 A specially-assembled design team is considering the design of a submarine hull. They have identified some key locations in the hull where the stresses are greatest and expressed these as proportional to some loading characteristic, p. This loading characteristic is related to the external hull pressure, but is not expressed directly in terms of external hull pressure due to *certain concerns*. The four stress conditions are:

Condition A:
$\sigma_{11} = -p$ $\sigma_{12} = 0$
$\sigma_{22} = -p$ $\sigma_{13} = 0$
$\sigma_{33} = -p$ $\sigma_{23} = 0$

Condition B:
$\sigma_{11} = 0.5p$ $\sigma_{12} = 0$
$\sigma_{22} = p$ $\sigma_{13} = 0$
$\sigma_{33} = 2p$ $\sigma_{23} = 0$

Condition C:
$\sigma_{11} = 2p$ $\sigma_{12} = 0$
$\sigma_{22} = -p$ $\sigma_{13} = 0$
$\sigma_{33} = 0.5p$ $\sigma_{23} = 0$

Condition D:
$\sigma_{11} = p$ $\sigma_{12} = 2p$
$\sigma_{22} = 4p$ $\sigma_{13} = 0$
$\sigma_{33} = 0.5p$ $\sigma_{23} = 0$

The designers are currently considering a steel with a yield stress of 1500 MPa.

(a) Using Tresca's yield criterion, calculate the value of the loading characteristic, p, for the onset of yielding and the associated plane on which yield would occur for each.

(b) Repeat this calculation using the von Mises criterion.

(c) Comment on the overall results.

M14.3 Much of an airplane fuselage can be modeled to first order as a pressurized cylinder with superposed longitudinal and torsional loads due to the empennage. Consider a mid-sized airplane with a fuselage radius of 6 feet and a thickness of 0.035 inches. Given the "skeleton" construction of the fuselage that includes longerons and frames, assume that the skin only carries 50% of the applied loads due to pressure and from the empennage. For such a "thin shell" construction, the stresses due to pressure can be shown to be proportional to the pressure differential, p, and radius, R, and inversely proportional to the thickness, t. The "hoop stress" is in the circumferential direction and is:

$$\sigma_{hoop} = pR/t$$

while the stress in the longitudinal direction of the cylinder is:

$$\sigma_{long} = pR/2t$$

The limit condition for flight is 10 psi differential between the cabin pressure and the exterior pressure.

(a) Consider a piece of material on the fuselage that is stressed as noted (including the 50% factor). The fuselage is made of 2024 aluminum that has a modulus of 10.1 Msi and a yield stress of 50 ksi. Using limit condition and the Tresca failure criterion, determine an "operating stress envelope" for this piece of material with axes of applied longitudinal stress and applied torsional stress (due to the applied longitudinal and torsional loads *separate* from the pressure-induced stresses).

(b) A *damage tolerant* approach is now taken such that the material must tolerate a through-crack of 0.25 inches in length that can be detected nondestructively in scheduled inspection intervals. The fracture toughness of the 2024 aluminum is 31 ksi in$^{1/2}$. In applying the fracture mechanics criterion, ignore all stresses except that perpendicular to the crack. Determine the "operating stress envelope" for this piece of the same piece of material using limit condition and the damage tolerant approach.

(c) Compare the two results and make relevant comments.

M14.4 This problem is provided as a STUDY HELPER in relation to Units M5.1 and M5.2 (mainly the latter). Eight questions and eight answers are provided. You are to match the answers to the appropriate questions.
Reading the assigned sections and chapters for these units will be of help.

Questions:

1. In ancient times, swordsmiths would use bronze, an alloy of copper and tin, to make their swords. Why did they use this alloy rather than pure copper?
2. These swordsmiths would manufacture these swords by repeated hammering rather than by melting the metal and then casting the molten metal to the appropriate shape. What did this repeated hammering achieve?
3. Why can the yield strength of very fine (approximately 1 μm in diameter) needlelike crystals approach the theoretical strength (modulus/15), whereas bulk specimens of the metal do not come close?
4. Aluminum alloys are generally not considered viable for use on critically exposed surfaces of supersonic alloys. Why not?
5. Thin metals wires (such as electrical connectors and cables) are made by *drawing*, i.e. pulling, a thicker cross-section through a die. Why is there an upper limit to the reduction in the cross-sectional area (known as the draw ratio) that can be achieved in this process?
6. Why do wires become hot during the drawing process?
7. A metal developer is trying to strengthen an aluminum alloy by adding aluminum oxide powder. Why do the company researchers find that a smaller particle size produces a higher yield strength for a given volume fraction of powder?
8. Why do pure metals generally have lower yield stresses than ceramics?

Answers:

A. Several factors contribute to this. One, there are less likely to be imperfections, therefore less dislocations are available to cause yield. Two, overall directions may be oriented favorably relative to the loading direction such that the shear stresses acting on the slip planes will be small. Three, the surface provides a barrier to dislocation motion.

B. The cross-section is reduced by plastic deformation and this deformation is achieved by applying a uniaxial tensile stress in the wires. A plastic instability can result, thereby causing the material to neck down and rupture.

C. Yield is determined by dislocation motion. Metals generally have more close-packed crystal structure than ceramics. Therefore, there are more slip planes on which dislocations can glide in metals. Furthermore, ceramics are often covalently bonded giving a large intrinsic lattice resistance to dislocations since covalent bonds are more directional than metallic bonds and thus cannot switch as easily between neighboring atoms.

D. The atoms of the second material locally distort the FCC crystal lattice of the primary material. This has the effect of "roughening" the slip planes which increases the resistance to dislocation glide. This results in an increase in the yield stress thereby producing a harder material.

E. Particles "pin" dislocations, thereby increasing the resistance to glide along the slip planes. The increase in the shear yield stress is inversely proportional to the spacing of the particles. A smaller size for a given volume fraction results in more particles with closer spacing.

F. Nearly all of the mechanical work done to plastically deform the material is converted to heat as plastic deformation is an irreversible process.

G. This is a form of work hardening. Extra hardness can be obtained by work hardening by introducing plastic deformation. More dislocations are introduced. These interfere with each other and further increase the yield stress.

H. The material is exposed to sustained temperature in the vicinity of 300°F and higher. Creep can occur at these temperature under sustained load.