UE Fluids Quiz 2 Solution S'07

1) $T_{0_i} = T_i\left(1+\frac{\gamma-1}{2}M_i^2\right) = 315\ K$

$T_{0_e} = T_e\left(1+\frac{\gamma-1}{2}M_e^2\right) = 315\ K$

} same T_0 → adiabatic (no heating)

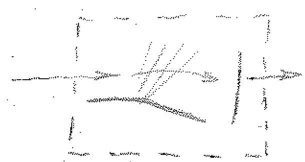
$\rho_{0_i} = \rho_i\left(1+\frac{\gamma-1}{2}M_i^2\right)^{\frac{1}{\gamma-1}} = 1.304\ kg/m^3$

$\rho_{0_e} = \rho_e\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{1}{\gamma-1}} = 1.130\ kg/m^3$

} $\rho_{0_e} < \rho_{0_i}$ → non-isentropic (losses present)

i) Shock - **Yes.** Shock is adiabatic, has losses.

ii) Expansion Fan - **Yes.** Fan is adiabatic, no losses,
But losses could be caused by e.g. shock



iii) Heater - **No.** This would be non-adiabatic

iv) Friction - **Yes.** This is does not add heat, only losses.

2.5 pts each for correct Y/N

5 pts each for correct/sufficient reasoning

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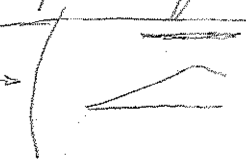
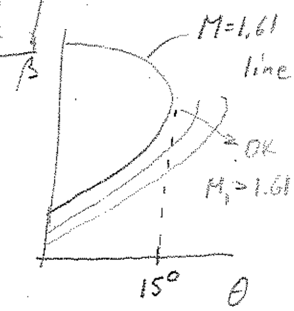
2a) For $\theta = 15^\circ$, **minimum $M_\infty = 1.61$ for oblique shock**

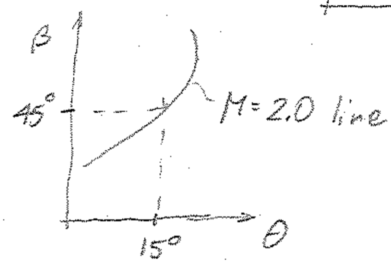
For $M_\infty < 1.61$, will get bow shock →





2b) $\beta = 45^\circ$, $\theta = 15^\circ$, as given.

→ **$M_\infty = 2.0$**



2c) $M_1 = 2.0$, $M_{n_1} = M_1 \sin\beta = 1.414$

From shock table: $\frac{p_2}{p_1} = 2.16$ → **$p_2 = 2.16\, p_\infty$**

2d) From shock table: $M_{n_2} = 0.734$, $M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 1.468$

$p_{o_2} = p_2 \left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\frac{\gamma}{\gamma-1}} = (2.16\, p_\infty)\left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\frac{\gamma}{\gamma-1}} = 7.57\, p_\infty$

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3a) Choking onset is when $A_e^* = A_t = \frac{A_e}{2}$, or $\frac{A_e}{A_e^*} = 2$

From Mach-area relation, for $\frac{A}{A^*} = 2$, $M_e = 0.325$

Since flow is still isentropic, $p_{oe} = p_r$, so $p_e = p_{oe}\left[1 + \frac{\gamma-1}{2}M_e^2\right]^{-\frac{\gamma}{\gamma-1}} = 0.929\, p_r$

Choking onset at $\boxed{p_B = 0.929\, p_r}$

3b) Choked flow: $M_t = 1$, $h_{o_t} = h_r$, $p_{o_t} = p_r$

Use $\dot{m}$ relation, at throat $\boxed{\dot{m} = \frac{\gamma\, p_r}{\sqrt{(\gamma-1) h_r}}\, 1 \left(1 + \frac{\gamma-1}{2} 1^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} A_t = 1.28 \frac{p_r}{\sqrt{h_r}} A_t}$

3c) Flow is isentropic to supersonic exit.

$\frac{A_e}{A_e^*} = 2 \rightarrow M_e = 2.2$

$\boxed{p_e = p_r\left[1 + \frac{\gamma-1}{2}M_e^2\right]^{-\frac{\gamma}{\gamma-1}} = 0.128\, p_r}$

3d) $\theta_2 - \theta_1 = \nu(M_2) - \nu(M_1)$, $M_2 \simeq \infty$ since $p_B \simeq 0$, $M_1 = 2.2$

$\theta_1 = 0°$, $\nu(M_2) = 130.45°$, $\nu(M_1) = 31.7°$

$\rightarrow \boxed{\theta_2 = \theta = 98.8°}$