1. (35 %) A thin airfoil has an adjustable camberline defined by

\[ Z(x) = h \left[ 1 - \left( \frac{x}{c} \right)^2 \right] \]

where \( h \) is the camberline height at the leading edge.

a) Determine the coefficients \( A_0, A_1, A_2, \ldots \) for this camberline, either by manipulation and inspection of \( dZ/dx \), or by direct Fourier Analysis of \( dZ/dx \). Assume there’s also some arbitrary \( \alpha \) as shown.

b) Determine the airfoil’s \( c_L \) and \( c_{m_{x/L}} \), as functions of \( h/c \) and \( \alpha \).

c) Determine the zero-lift angle \( \alpha_{L=0} \), as a function of \( h/c \).
2. (40 %) A wing operating at velocity $V_\infty$ and air density $\rho$ has the following circulation distribution:

$$\Gamma(y) = \Gamma_0 \left[ 1 - (2y/b)^2 \right]$$

a) Determine the lift $L$.

b) Determine the downwash velocity $w(y)$. Evaluate $w(0)$ at the center of the wing, and also $w(b/4)$ halfway out. Roughly sketch $\Gamma(y)$ and $w(y)$.

c) Write down an expression for the induced drag $D_i$ for this particular wing, but don't bother integrating it (too messy).

d) Determine how both $L$ and $D_i$ will change for each of the following two cases:
   i) $\Gamma_0$ is doubled with the same $b$, and
   ii) $b$ is doubled with the same $\Gamma_0$. 
3. (25 %) An elliptically-loaded wing with aspect ratio $AR = 10$ has an airfoil with the following 2D profile drag polar:

$$c_d(c_l) = 0.025 + 0.015c_l^4$$

We will assume that $c_l = C_L$.

a) Write an expression for the overall drag coefficient $C_D$ of the wing.

b) Determine the maximum lift/drag ratio $C_L/C_D$ (or equivalently, the minimum $C_D/C_L$) that this wing can produce.

c) The airplane using this wing has a wing loading of $W/S = 10$ Pa. Determine the flight speed $V$ it needs to fly so that it operates at its maximum lift/drag ratio. Assume $\rho = 1.2 \text{ kg/m}^3$. 