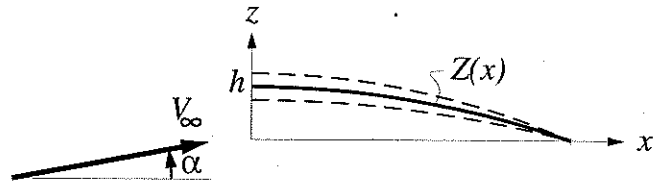


1. (35 %) A thin airfoil has an adjustable camberline defined by

$$Z(x) = h \left[ 1 - \left( \frac{x}{c} \right)^2 \right]$$



where  $h$  is the camberline height at the leading edge.

- Determine the coefficients  $A_0, A_1, A_2, \dots$  for this camberline, either by manipulation and inspection of  $dZ/dx$ , or by direct Fourier Analysis of  $dZ/dx$ . Assume there's also some arbitrary  $\alpha$  as shown.
- Determine the airfoil's  $c_l$  and  $c_{m_{c/4}}$ , as functions of  $h/c$  and  $\alpha$ .
- Determine the zero-lift angle  $\alpha_{L=0}$ , as a function of  $h/c$ .

2. (40 %) A wing operating at velocity  $V_\infty$  and air density  $\rho$  has the following circulation distribution:

$$\Gamma(y) = \Gamma_0 [1 - (2y/b)^2]$$

- a) Determine the lift  $L$ .
- b) Determine the downwash velocity  $w(y)$ . Evaluate  $w(0)$  at the center of the wing, and also  $w(b/4)$  halfway out. Roughly sketch  $\Gamma(y)$  and  $w(y)$ .
- c) Write down an expression for the induced drag  $D_i$  for this particular wing, but don't bother integrating it (too messy).
- d) Determine how both  $L$  and  $D_i$  will change for each of the following two cases:
  - i)  $\Gamma_0$  is doubled with the same  $b$ , and
  - ii)  $b$  is doubled with the same  $\Gamma_0$ .

3. (25 %) An elliptically-loaded wing with aspect ratio  $AR = 10$  has an airfoil with the following 2D profile drag polar:

$$c_d(c_\ell) = 0.025 + 0.015c_\ell^4$$

We will assume that  $c_\ell = C_L$ .

- a) Write an expression for the overall drag coefficient  $C_D$  of the wing.
- b) Determine the maximum lift/drag ratio  $C_L/C_D$  (or equivalently, the minimum  $C_D/C_L$ ) that this wing can produce.
- c) The airplane using this wing has a wing loading of  $W/S = 10$  Pa. Determine the flight speed  $V$  it needs to fly so that it operates at its maximum lift/drag ratio. Assume  $\rho = 1.2$  kg/m<sup>3</sup>.