UE Fluids Quiz 1 Solution Spring '07

1) (15) a) $Z = h\left[1 - \frac{x^2}{c^2}\right]$, $\frac{dZ}{dx} = -2\frac{h}{c}\frac{x}{c} = \frac{h}{c}(\cos\theta - 1)$

$$A_0 = \alpha - \frac{1}{\pi}\int_0^{\pi}\frac{dZ}{dx}\,d\theta = \alpha - \frac{1}{\pi}\frac{h}{c}\int_0^{\pi}(\underbrace{\cos\theta}_{\to 0}\underbrace{-1}_{\to -\pi})\,d\theta = \alpha + \frac{h}{c}$$

$$A_1 = \frac{2}{\pi}\int_0^{\pi}\frac{dZ}{dx}\cos\theta\,d\theta = \frac{2}{\pi}\frac{h}{c}\int_0^{\pi}(\cos\theta - 1)\cos\theta\,d\theta = \frac{2}{\pi}\frac{h}{c}\left[\frac{\pi}{2} - 0\right] = \frac{h}{c}$$

$$A_2 = \frac{2}{\pi}\int_0^{\pi}\frac{dZ}{dx}\cos 2\theta\,d\theta = \frac{2}{\pi}\frac{h}{c}\int_0^{\pi}(\cos\theta - 1)\cos 2\theta\,d\theta = 0 = A_3 = A_4 \dots$$

(10) b) $$c_\ell = \pi(2A_0 + A_1) = \pi\left[2\left(\alpha + \frac{h}{c}\right) + \frac{h}{c}\right] = 2\pi\left[\alpha + \frac{3}{2}\frac{h}{c}\right]$$

$$C_m = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}\left(0 - \frac{h}{c}\right) = -\frac{\pi}{4}\frac{h}{c}$$

(10) c) $$c_\ell = 2\pi(\cancel{\alpha} - \alpha_{L=0}) = 2\pi\left[\cancel{\alpha} + \frac{3}{2}\frac{h}{c}\right]$$

$$\rightarrow \alpha_{L=0} = -\frac{3}{2}\frac{h}{c}$$

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2) a) $L = \int_{-b/2}^{b/2} \rho V_\infty \Gamma \, dy = \rho V_\infty \Gamma_0 \frac{b}{2} \int_{-1}^{1} \left[1 - \left(\frac{2y}{b}\right)^2\right] d\left(\frac{2y}{b}\right)$

(10)

$$\int_{-1}^{1} (1 - x^2)\,dx = x - \tfrac{1}{3}x^3 \Big|_{-1}^{1} = 2 - \tfrac{2}{3} = \tfrac{4}{3}$$

$$L = \rho V_\infty \Gamma_0 \frac{b}{2} \cdot \frac{4}{3} = \frac{2}{3} \rho V_\infty \Gamma_0 b$$

b) $w(y_0) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma}{dy} \frac{dy}{y - y_0}$; $\frac{d\Gamma}{dy} = \Gamma_0 \left(-2 \frac{4}{b^2} y\right)$

(15)

$$w(y_0) = \frac{\Gamma_0}{4\pi}\left(\frac{-8}{b^2}\right) \int_{-b/2}^{b/2} \frac{y}{y - y_0} dy$$

$t = y - y_0$, $dy = dt$, $\int \frac{y}{y - y_0} dy = \int \frac{y_0 + t}{t} dt$
$= y_0 \ln t + t$
$= y_0 \ln|y - y_0| + y - y_0$

$$w(y_0) = \frac{-2\Gamma_0}{\pi b^2}\left[y_0 \ln|y - y_0| + y\right]_{-b/2}^{b/2} = \frac{-2\Gamma_0}{\pi b^2}\left[y_0 \ln\left|\frac{b/2 - y_0}{b/2 + y_0}\right| + b\right]$$

c) $D_i = \int_{-b/2}^{b/2} L' \alpha_i \, dy = \int_{-b/2}^{b/2} \rho V_\infty \Gamma(y) \left(\frac{-w}{V_\infty}\right) dy = \int_{-b/2}^{b/2} \rho V_\infty \Gamma_0 \left[1 - \left(\frac{2y}{b}\right)^2\right] \frac{2\Gamma_0}{\pi b^2}\left[y \ln\left|\frac{b/2 - y}{b/2 + y}\right| + b\right] dy$

(5)

d) Rewrite D_i a bit... $D_i = \rho V_\infty \Gamma_0^2 \frac{2}{\pi} \underbrace{\int_{-1/2}^{1/2} \left[1 - \left(\frac{2y}{b}\right)^2\right]\left[\frac{y}{b} \ln\left|\frac{1 - \frac{2y}{b}}{1 + \frac{2y}{b}}\right| + 1\right] d\left(\frac{y}{b}\right)}_{\text{doesn't change with } b}$

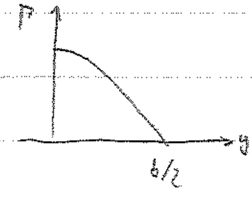
(10)

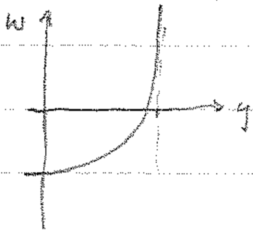
Hence, $L \sim \Gamma_0 b$
$D_i \sim \Gamma_0^2$

i) Γ_0 doubled, b same $\rightarrow$ L doubled, D_i quadrupled

ii) b doubled, Γ_0 same $\rightarrow$ L doubled, D_i same

b) cont'd $w(0) = \frac{-2\Gamma_0}{\pi b}$, $w\left(\frac{b}{4}\right) = \frac{-2\Gamma_0}{\pi b}\left[\frac{1}{4}\ln\left(\frac{1}{3}\right) + 1\right] = \frac{-2\Gamma_0}{\pi b}(0.73)$





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3) a) $C_D = C_d + C_{D_i} = 0.025 + 0.015\,C_L^4 + \dfrac{C_L^2}{\pi AR}$ (5)

b) $\dfrac{C_D}{C_L} = \dfrac{0.025}{C_L} + 0.015\,C_L^3 + \dfrac{C_L}{\pi AR}$

(15) Minimize $\dfrac{C_D}{C_L}$: $\dfrac{d}{dC_L}\left(\dfrac{C_D}{C_L}\right) = -\dfrac{0.025}{C_L^2} + 0.045\,C_L^2 + \dfrac{1}{\pi AR} = 0$

$0.045\,C_L^4 + \dfrac{C_L^2}{\pi AR} - 0.025 = 0$ quadratic for C_L^2

$$C_L^2 = \frac{1}{0.09}\left[-\frac{1}{\pi AR} + \sqrt{\frac{1}{(\pi AR)^2} + 4\cdot 0.045\cdot 0.025}\right] = 0.471$$

$C_L = 0.6865$ $\rightarrow$ plug into $\dfrac{C_D}{C_L}$ expression

$\rightarrow$ min $\dfrac{C_D}{C_L} = 0.0631$ max $\dfrac{C_L}{C_D} = \dfrac{1}{C_D/C_L} = 15.84$

(5) c) $V = \sqrt{\dfrac{2W/S}{\rho C_L}} = \left(\dfrac{2\cdot 10\text{ Pa}}{1.2\text{ kg/m}^3\cdot 0.6865}\right)^{1/2} = 4.927$ m/s