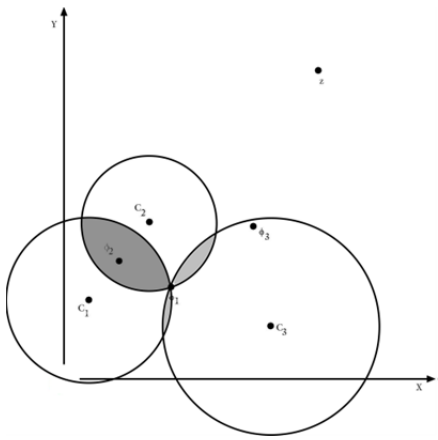
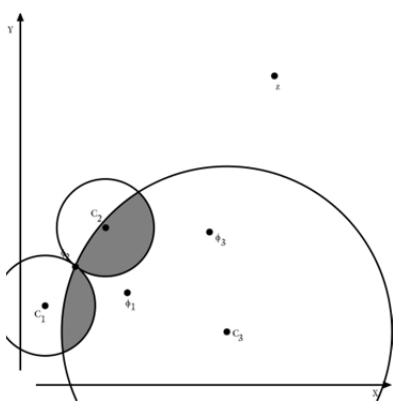
Question 7. In the following figures, the preferred-to-sets against the status quo have been drawn. The shaded areas indicate where at least two preferred-to-sets intersect, forming the win-set.

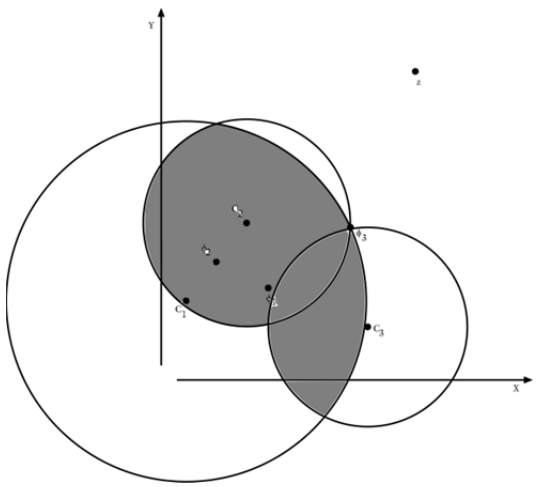
ϕ_1 as status quo:



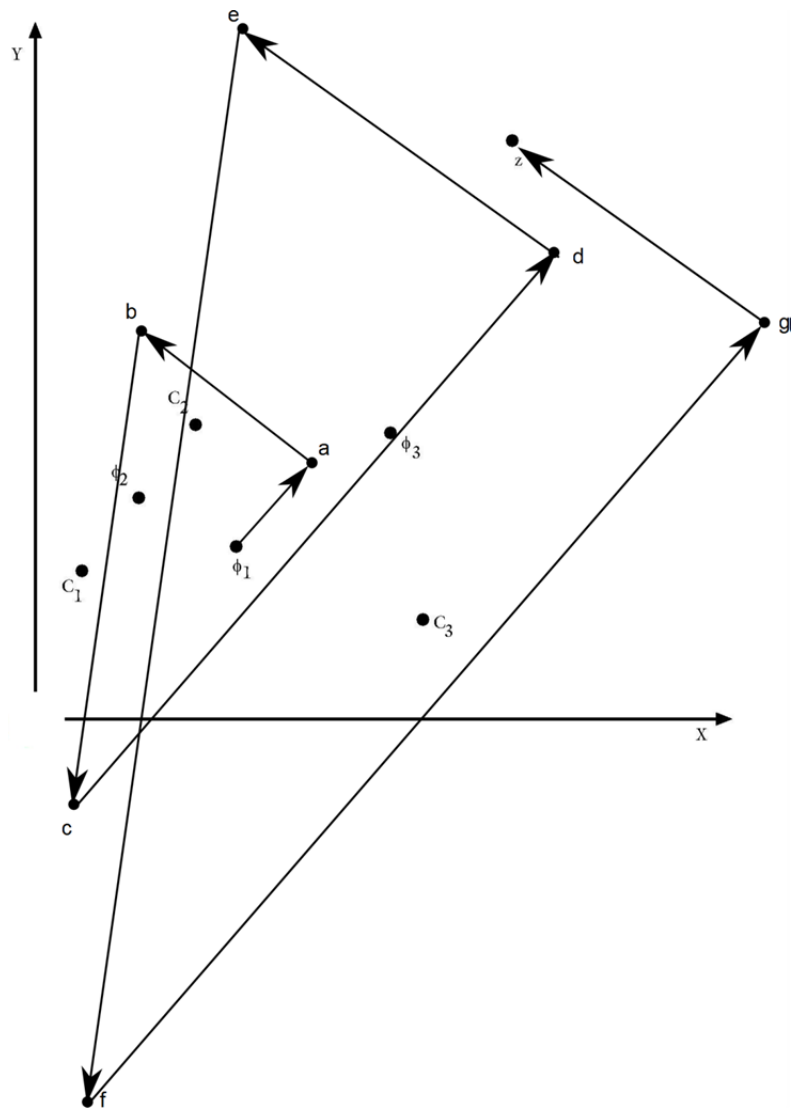
ϕ_2 as status quo:



ϕ_3 as status quo:



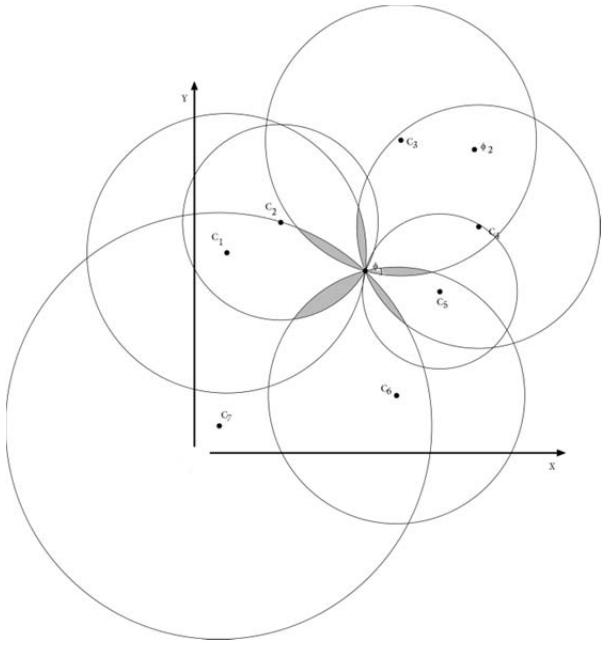
Question 8. The following agenda is one of an infinite number of agendas that gets one from the middle of the Pareto set out to point *z*. You will notice that the general pattern of the agenda is to "spiral" the succeeding motions out toward *z*. Such a strategy is perhaps the most direct one that moves far from the Pareto set so quickly.



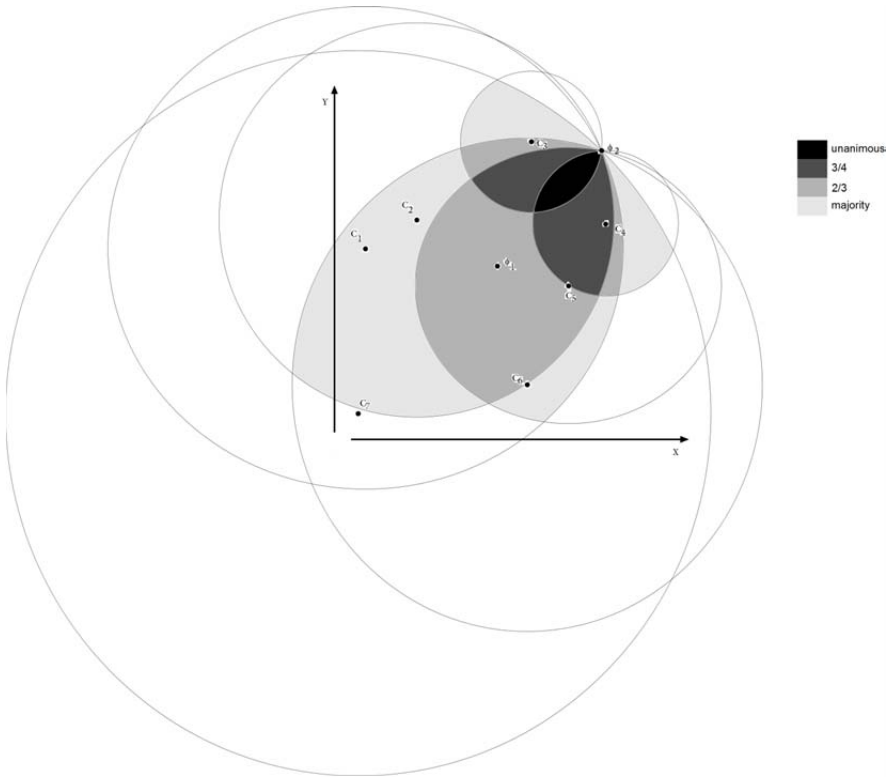
The following chart summarizes the motions and the coalitions voting for the motions:

Status quo	Motion	Coalition voting for motion	Coalition voting for status quo
ϕ_1	a	2,3	1
a	b	1,2	3
b	c	1,3	2
c	d	2,3	1
d	e	1,2	3
e	f	1,3	2
f	g	2,3	1
g	z	1,2	3

Question 10. All of the circular indifference curves in the figure below are drawn through $\phi_{1..}$ The shaded area is the region that beats $\phi_{1.}$ by a simple majority. There is no region that a two-thirds, three-quarters, or unanimous majority prefers compared to $\phi_{1.}$

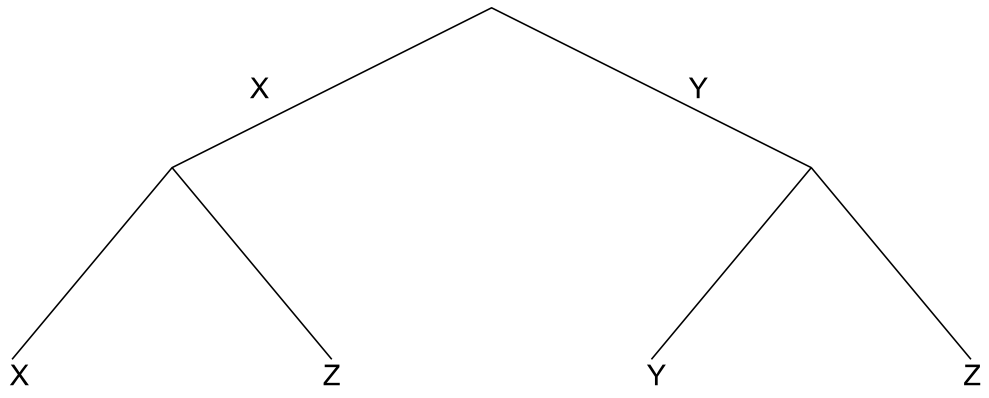


All of the circular indifference curves in the figure below are drawn through $\phi_{2..}$ The shaded regions denote the areas that beat ϕ_2 by different majorities. With an electorate of seven members, a simple majority requires four votes, a 2/3 majority requires five, a 3/4 majority requires six, and unanimity requires seven.

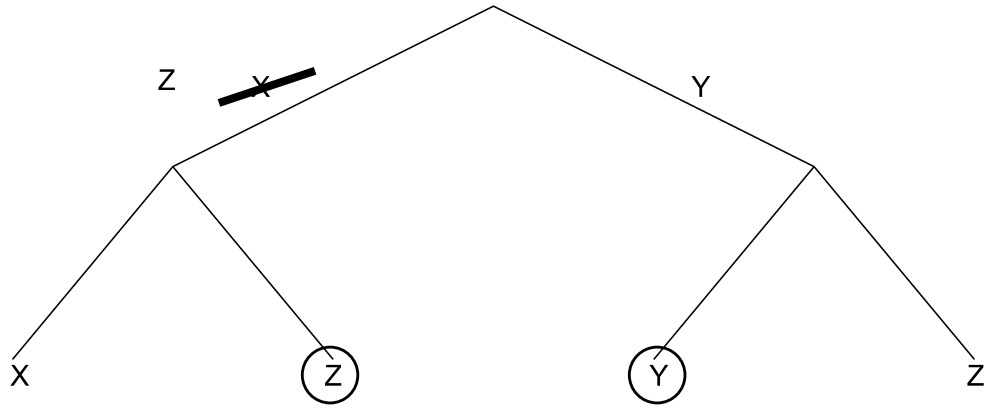


Question 11. With sincere voting, each legislator simply consults his/her preference ordering, voting for the options s/he prefers more. On the first vote, *X* is paired against *Y*. Legislator 1 and 3 prefer *X* to *Y*, while Legislator 2 prefers *Y* to *X*. Therefore, alternative *X* wins the first round. The second round pits *X* against *Z*. Legislator 1 prefers *X* to *Z*, while Legislators 2 and 3 prefer *Z* to *X*. Therefore alternative *Z* prevails under sincere voting.

With sophisticated voting, it is best to draw out the game tree and then implement backward induction. Here is the game tree:



Under backward induction, we start at the bottom of the game tree, calculate which alternative would prevail on a majority vote at that level, and then adjust the prior voting level according to the winner at the last level. (With a longer game tree, we would iterate up through the tree, until we get to the top.) In this case, *Z* beats *X* in a majority vote, while *Y* beats *Z*. We can indicate this on the game tree by replacing the sincere outcomes with the "sophisticated equivalent" as follows:



On the left-hand branch, we know that if *X* and *Z* are paired against each other *Z* prevails. We therefore circle it, cross-off the *X* on the branch above, and replace it with the *Z*. On the right-hand branch, we know that if *Y* and *Z* are paired, *Y* prevails. This is the alternative in the branch immediately above, therefore, we leave it unchanged. The graph reveals to us that the sophisticated equivalent of voting for *X* on the first round is eventual victory for *Z*. Therefore, a

sophisticated voter would treat the first round of voting as a contest between (and eventual victory for) *Z* against *Y*. Because a majority prefer *Y* to *Z*, it prevails on the first round, and then on the second round. *Y* wins under sophisticated voting.