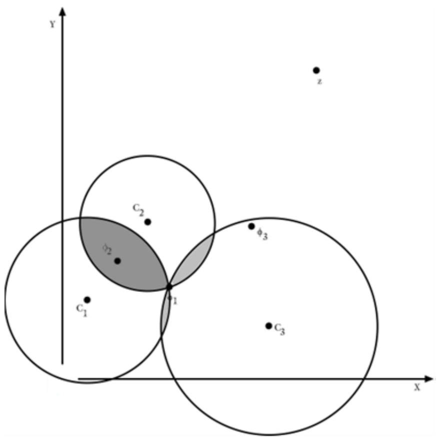
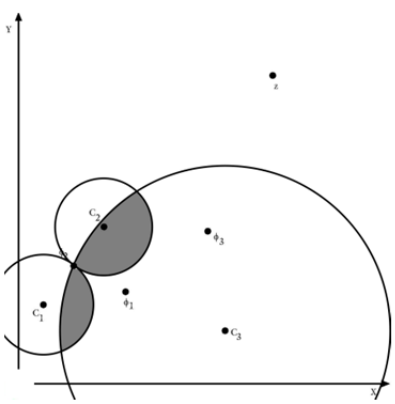
Question 7. In the following figures, the preferred-to-sets against the status quo have been drawn. The shaded areas indicate where at least two preferred-to-sets intersect, forming the win-set.

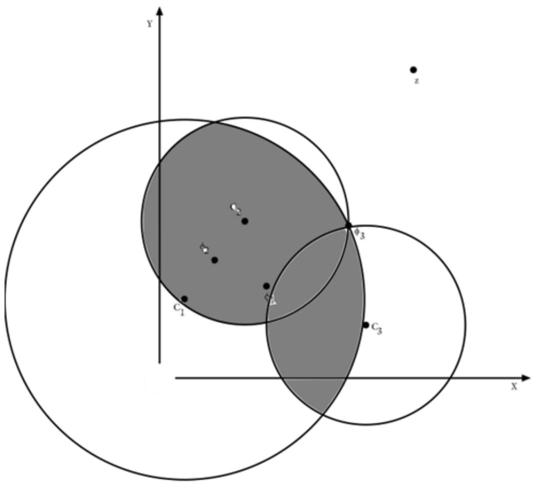
ϕ_1 as status quo:



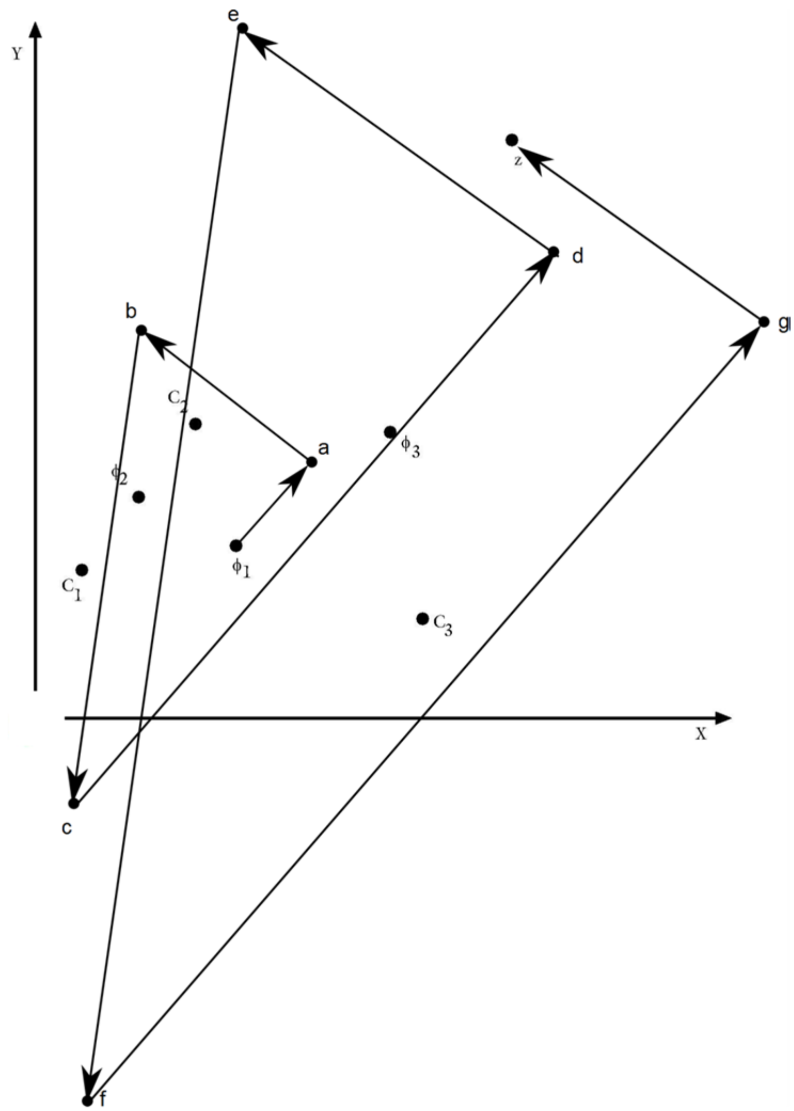
ϕ_2 as status quo:



ϕ_3 as status quo:



Question 8. The following agenda is one of an infinite number of agendas that gets one from the middle of the Pareto set out to point *z*. You will notice that the general pattern of the agenda is to "spiral" the succeeding motions out toward *z*. Such a strategy is perhaps the most direct one that moves far from the Pareto set so quickly.

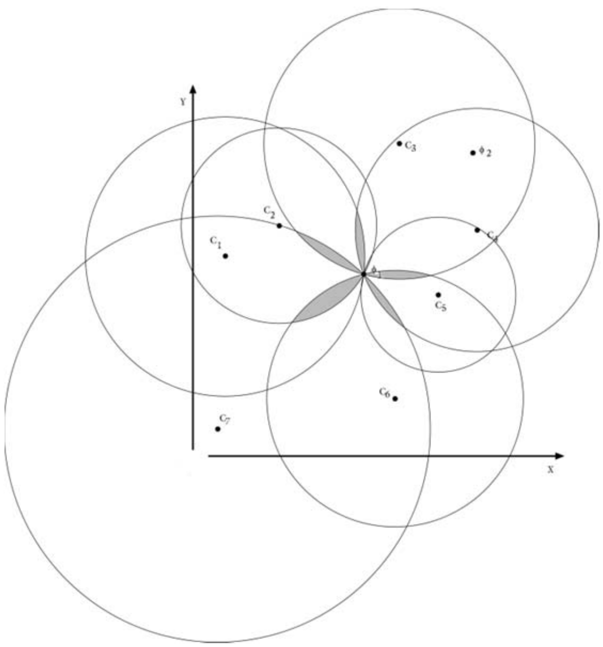


The following chart summarizes the motions and the coalitions voting for the motions:

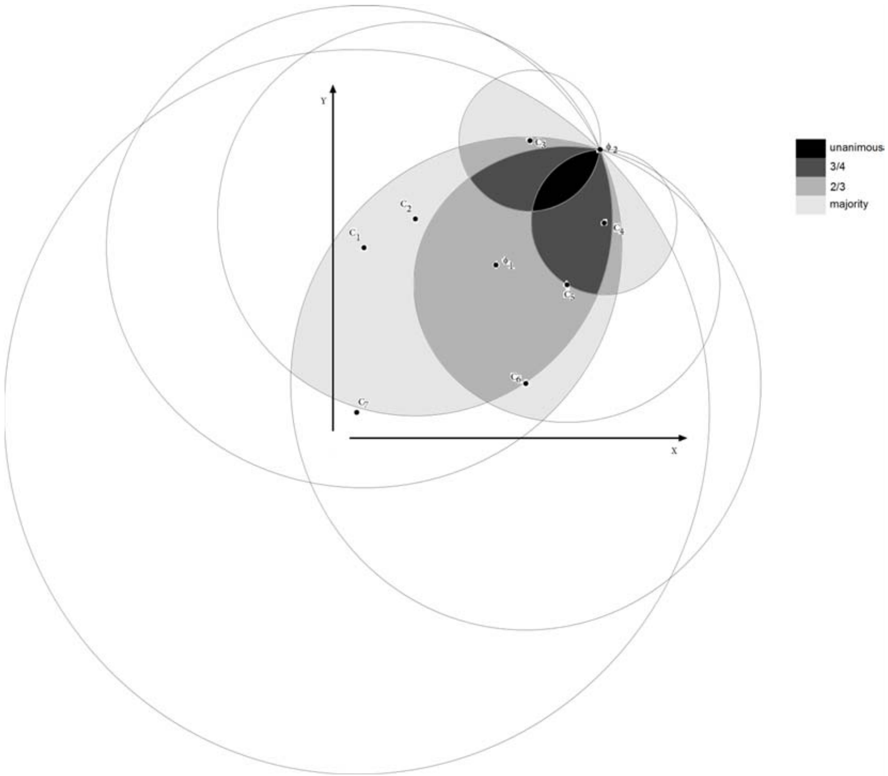
Status quo	Motion	Coalition voting for motion	Coalition voting for status quo
ϕ_1	a	2,3	1
a	b	1,2	3
b	c	1,3	2
c	d	2,3	1
d	e	1,2	3
e	f	1,3	2
f	g	2,3	1
g	z	1,2	3

Question 10. All of the circular indifference curves in the figure below are drawn through ϕ_{1}..

The shaded area is the region that beats ϕ_{1}. by a simple majority. There is no region that a two-thirds, three-quarters, or unanimous majority prefers compared to ϕ_{1}.

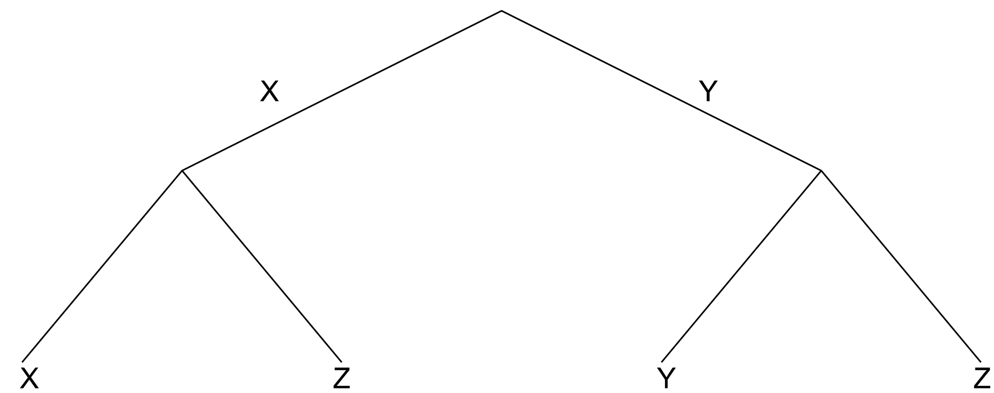


All of the circular indifference curves in the figure below are drawn through ϕ_{2}.. The shaded regions denote the areas that beat ϕ_2 by different majorities. With an electorate of seven members, a simple majority requires four votes, a 2/3 majority requires five, a 3/4 majority requires six, and unanimity requires seven.

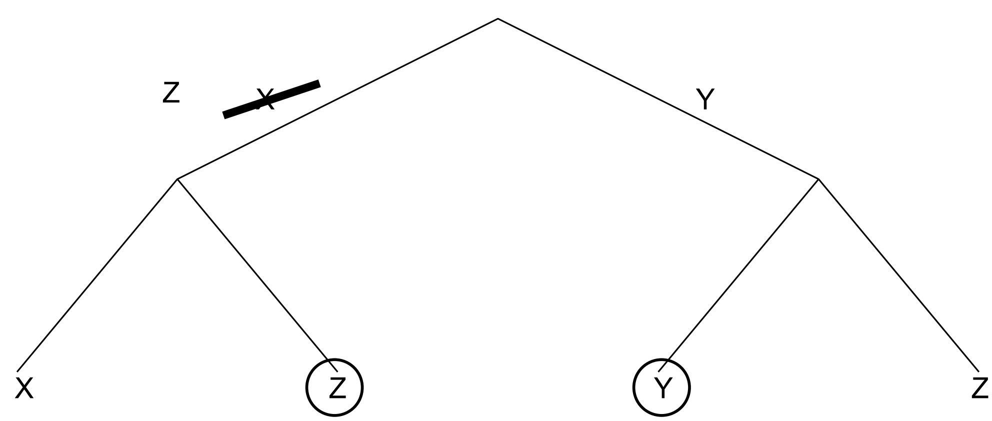


Question 11. With sincere voting, each legislator simply consults his/her preference ordering, voting for the options s/he prefers more. On the first vote, *X* is paired against *Y*. Legislator 1 and 3 prefer *X* to *Y*, while Legislator 2 prefers *Y* to *X*. Therefore, alternative *X* wins the first round. The second round pits *X* against *Z*. Legislator 1 prefers *X* to *Z*, while Legislators 2 and 3 prefer *Z* to *X*. Therefore alternative *Z* prevails under sincere voting.

With sophisticated voting, it is best to draw out the game tree and then implement backward induction. Here is the game tree:



Under backward induction, we start at the bottom of the game tree, calculate which alternative would prevail on a majority vote at that level, and then adjust the prior voting level according to the winner at the last level. (With a longer game tree, we would iterate up through the tree, until we get to the top.) In this case, *Z* beats *X* in a majority vote, while *Y* beats *Z*. We can indicate this on the game tree by replacing the sincere outcomes with the "sophisticated equivalent" as follows:

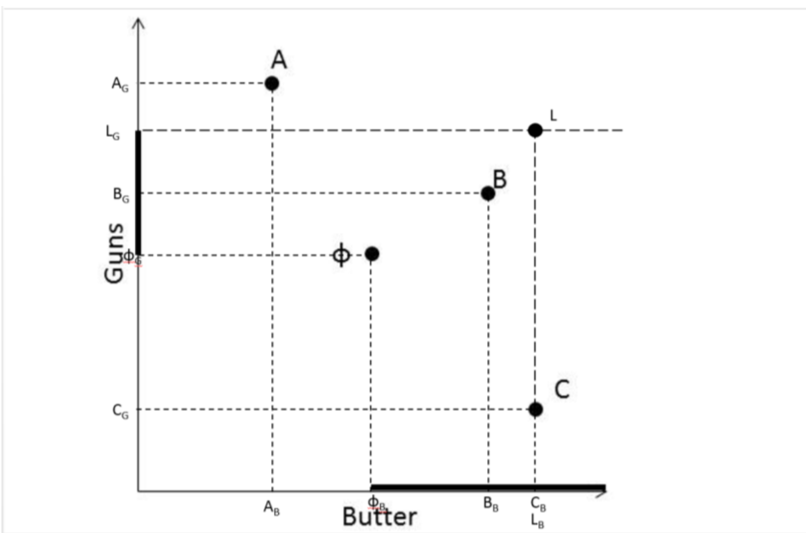


On the left-hand branch, we know that if *X* and *Z* are paired against each other *Z* prevails. We therefore circle it, cross-off the *X* on the branch above, and replace it with the *Z*. On the right-hand branch, we know that if *Y* and *Z* are paired, *Y* prevails. This is the alternative in the branch immediately above, therefore, we leave it unchanged. The graph reveals to us that the sophisticated equivalent of voting for *X* on the first round is eventual victory for *Z*. Therefore, a sophisticated voter would treat the first round of voting as a contest between (and eventual victory for) *Z* against *Y*. Because a majority prefer *Y* to *Z*, it prevails on the first round, and then on the second round. *Y* wins under sophisticated voting.

B1. Members A and C are the agenda-setters for the Guns and Butter dimensions, respectively. They can each "open the gates," that is, decide whether to allow a proposal to change policy onto the agenda in their respective dimensions. If a bill is allowed onto the agenda, the respective member's proposal is decided on a take-it-or-leave-it basis, that is, a generic "closed rule." Thus, in each dimension, we first want to know whether there is a bill that the agenda-setter could offer on that dimension that will make both the median on that dimension *and* the agenda-setter better off, compared to the status quo.

To answer this question, we need to turn the graphical representation into two one-dimensional problems. This is accomplished by projecting the ideal points onto the respective dimensions, as is shown below. The tightly dashed lines show where the ideal points are projected. These are the "induced ideal points" along each dimension.

Note that along each dimension, the agenda-setter has extreme ("outlying") preferences compared to the rest of the chamber. Thus each agenda-setter would like to make policy extreme. The question is whether the median in each dimension would go along with an extreme policy proposal. To answer this question, we draw the preferred-to set for the median in each dimension. That's what the thick line segments in the graph show.



For the Guns dimension, notice that the median's preferred-to set does not encompass A's ideal point. However, it is possible for A to propose a bill right on the edge of the preferred-to set, labeled L_G, which is preferred to over the status quo by the median voter. Thus, A proposes a bill at this location, and it passes.

For the Butter dimension, C's induced ideal point is encompassed by B's preferred-to set. Thus, C proposes a bill at C's ideal point, and it passes. The graph finally shows the location of the resulting policy in the two dimensional space, by the point labeled L.

B2. The following takes the previous graph, cleans it up, and draws the preferred-to set against L in two dimensions for each member. The shaded area is the win set. The roughly triangular region of the win set in the middle is the space where a bill could beat L via a unanimous vote. The rest of the win set shows regions where a 2-1 majority would beat the status quo.