

17.871 - PS3 Solutions

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Due: April 4, 2012

- (a) Black defendants are less likely to be sentenced to death than white defendants. But cases involving white victims (regardless of the race of the defendant) are much more likely to be given the death penalty than cases involving black victims:

```
. tab death bd, col
+-----+
| Key          |
|-----|
| frequency    |
| column percentage |
+-----+
      |          bd
death |          0          1 | Total
-----+-----+-----
      0 |          21          30 |    51
      |          44.68      56.60 |   51.00
-----+-----+-----
      1 |          26          23 |    49
      |          55.32      43.40 |   49.00
-----+-----+-----
      Total |          47          53 |   100
      |          100.00    100.00 |  100.00
```

```
. tab death wv, col
+-----+
| Key          |
|-----|
| frequency    |
| column percentage |
+-----+
      |          wv
death |          0          1 | Total
-----+-----+-----
      0 |          20          31 |    51
      |          76.92      41.89 |   51.00
```

	death	bd	Total
1	6	43	49
	23.08	58.11	49.00
Total	26	74	100
	100.00	100.00	100.00

When we look at just the cases involving white victims,

```

tab death bd if wv==0, col
+-----+
| Key          |
|-----|
| frequency    |
| column percentage |
+-----+
      |          bd
death |          0          1 |          Total
+-----+-----+-----+
      0 |          1          19 |          20
      |          50.00       79.17 |          76.92
+-----+-----+-----+
      1 |          1           5 |           6
      |          50.00       20.83 |          23.08
+-----+-----+-----+
      Total |          2           24 |          26
      |          100.00      100.00 |          100.00
. tab death bd if wv==1, col
+-----+
| Key          |
|-----|
| frequency    |
| column percentage |
+-----+
      |          bd
death |          0          1 |          Total
+-----+-----+-----+
      0 |          20          11 |          31
      |          44.44       37.93 |          41.89
+-----+-----+-----+
      1 |          25          18 |          43
      |          55.56       62.07 |          58.11
+-----+-----+-----+
      Total |          45          29 |          74
      |          100.00      100.00 |          100.00

```

Now we see that if the victim is not white, black defendants are again less likely to receive the death penalty (though keep in mind we only have 2 observations where $wv=0$ and $bd=0$). But if the victim is white, black defendants are much more likely to receive the death penalty.

- (b) Regression output below.

```
. reg death bd
```

Source	SS	df	MS			
Model	.354110799	1	.354110799	Number of obs =	100	
Residual	24.6358892	98	.251386625	F(1, 98) =	1.41	
Total	24.99	99	.252424242	Prob > F =	0.2382	
				R-squared =	0.0142	
				Adj R-squared =	0.0041	
				Root MSE =	.50138	

death	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bd	-.1192292	.1004579	-1.19	0.238	-.3185847	.0801263
_cons	.5531915	.0731345	7.56	0.000	.4080585	.6983245

We'll interpret the constant first: this means when $bd=0$ (a white defendant), there is a 55% probability the defendant in a murder case gets the death penalty. Now the bd coefficient: a one-unit change in bd (that is, moving from being a white defendant to being a black defendant) is associated with a 12 percent decrease in the probability of being assigned the death penalty relative to the baseline of 55%.

- (c) Regression output below.

```
reg death bd wv
```

Source	SS	df	MS			
Model	2.38021397	2	1.19010698	Number of obs =	100	
Residual	22.609786	97	.233090578	F(2, 97) =	5.11	
Total	24.99	99	.252424242	Prob > F =	0.0078	
				R-squared =	0.0952	
				Adj R-squared =	0.0766	
				Root MSE =	.48279	

death	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
bd	.0313217	.1093839	0.29	0.775	-.1857751	.2484185
wv	.3669495	.1244623	2.95	0.004	.1199262	.6139727
_cons	.2018569	.1384194	1.46	0.148	-.0728672	.476581

Start with the constant again: we say that when $bd=0$ and $wv=0$ (thus, white defendant and black victim), the probability of being assigned the death penalty in a murder case is 20%. The “ bd ” coefficient: “holding ‘ wv ’ constant”, black defendants are 3% more likely of being assigned the death penalty relative to the baseline of white defendants and black victims. The “ wv ” coefficient: “holding ‘ bd ’ constant” (thus holding the race of the defendant constant), defendants in murder cases with white victims are 36% more likely to be assigned the death penalty, relative to the baseline of white

defendants and black victims.

Why does the coefficient change on “bd” change? Let’s assume the “right” model for the death penalty outcome can be written,

$$death_i = \alpha + \beta_1 bd_i + \beta_2 wv_i + \epsilon_i$$

where i indexes death penalty cases. In part (b) we ran a bivariate regression. Recalling the lecture notes on multivariate regression, in this context the coefficient estimate can be written as,

$$\begin{aligned} \hat{\beta}^B &= Cov(bd, death)/Var(bd) \\ &= Cov(bd, \alpha + \beta_1 bd + \beta_2 wv + \epsilon_i)/Var(bd) \\ &= \frac{Cov(bd, \alpha) + \beta_1 Cov(bd, bd) + \beta_2 Cov(bd, wv) + Cov(bd, \epsilon_i)}{Var(bd)} \\ &= \frac{0 + \beta_1 Var(bd) + \beta_2 Cov(bd, wv) + 0}{Var(bd)} \\ &= \beta_1 + \beta_2 \frac{Cov(bd, wv)}{Var(bd)} \\ &= \text{”true } \beta_1 \text{”} + BIAS \end{aligned}$$

where between the second and third line, the first and fourth terms in the numerator are zero by assumption, and for the second term we use the fact that $Cov(X, X) = Var(X)$ for any random variable X . Thus we see that the estimate $\hat{\beta}^B$ is biased. Now take the multivariate estimate we obtained in (c). This estimate can be written,

$$\begin{aligned} \hat{\beta}^M &= \frac{Cov(bd, death)}{Var(bd)} - \beta_2 \frac{Cov(bd, wv)}{Var(bd)} \\ &= \hat{\beta}^B - BIAS \\ &= (\beta_1 + BIAS) - BIAS \\ &= \beta_1 \end{aligned}$$

Thus the multiple regression estimate “accounts for” the correlation between the variables and bd and wv when it calculates the estimate for the correlation between bd and death. How much of this correlation is there to account for? Let’s look at the correlation matrix between the three variables.

```
. cor death bd wv
(obs=100)
-----+-----
      death |    1.0000
      bd   |   -0.1190    1.0000
      wv   |    0.3074   -0.4668    1.0000
```

Thus we see that `bd` and `wv` are indeed highly (negatively) correlated, which explains (together with the math above) why the coefficients changed so much between the regressions. If we want the whole correlation matrix, it is:

```
. corr
(obs=100)
-----+-----
      |      id      death      bd      wv      ac      fv      vs      v2      ms      yv
-----+-----
id    |      1.0000
death |      0.5035      1.0000
bd    |     -0.2398     -0.1190      1.0000
wv    |      0.6002      0.3074     -0.4668      1.0000
ac    |      0.3630      0.1725      0.1062      0.0866      1.0000
fv    |     -0.0094     -0.0082     -0.0620     -0.0463     -0.3245      1.0000
vs    |      0.5638      0.2805      0.2393     -0.0337      0.1096     -0.1215      1.0000
v2    |     -0.0938     -0.0496     -0.1397     -0.0237     -0.1001     -0.0212     -0.1810      1.0000
ms    |      0.1556      0.0689     -0.0222      0.0084     -0.1928      0.1517     -0.2536     -0.0603      1.0000
yv    |     -0.1188     -0.0678      0.0561     -0.0773     -0.0263      0.0414     -0.1534      0.1997     -0.1089      1.0000
```

- (d) Remember that we found in part (a) that black defendants with white victims are much more likely to receive the death penalty than white defendants with white victims. What are some possible confounds? We want to think of things that may be correlated with both X, being a black defendant, and Y, getting the death penalty. One that comes to mind is that if we have a black defendant and a white victim, it's more likely that the victim was a stranger, `vs=1`. We see above in (c) that there is a positive correlation between `vs` and `death` and between `vs` and `bd`, about .28 and .24, respectively. Also if there were more aggravating circumstances, then this would probably increase the probability of the death penalty. I wouldn't expect this to be correlated with race of defendant necessarily, but there are two stories I might tell. One is that police discrimination results in greater fabrication of `ac`. Another is that we only have 100 observations, so even if race of defendant were randomly assigned (which it very much is not!) we would expect some imbalances on these variables.
- (e) Regression output below.

```
. reg death bd wv ac fv vs v2 ms yv
Source |      SS      df      MS              Number of obs =      100
-----+-----              F( 8, 91) =      3.45
Model  |  5.81563266      8      .726954083      Prob > F      =      0.0016
Residual |  19.1743673      91      .210707333      R-squared      =      0.2327
-----+-----              Adj R-squared =      0.1653
Total  |      24.99      99      .252424242      Root MSE      =      .45903
-----+-----
death |      Coef.      Std. Err.      t      P>|t|      [95% Conf. Interval]
-----+-----
bd    |     -.0869168      .1102374     -0.79      0.432      -.3058898      .1320562
wv    |      .3052246      .1207463      2.53      0.013      .065377      .5450722
ac    |      .4071931      .2228501      1.83      0.071      -.0354711      .8498574
fv    |      .0790273      .1061283      0.74      0.458      -.1317836      .2898382
vs    |      .3563889      .101464      3.51      0.001      .1548431      .5579348
v2    |      .0499414      .1394044      0.36      0.721      -.2269684      .3268511
ms    |      .2836468      .1517671      1.87      0.065      -.0178198      .5851134
yv    |      .050356      .1773002      0.28      0.777      -.3018291      .4025411
_cons |     -.1189227      .1782999     -0.67      0.506      -.4730936      .2352482
```

Do the results change? Well the first result in part (a) was that there was that the bivariate relationship between bd and death was negative. That stays the same here. The second result in (a) was that wv has a positive association with death, especially when $bd=1$. We aren't conditioning on $bd=1$ here, but we do see that the wv variable is 0.31. This means that, "holding all the other variables constant", moving from a black victim to a white victim is associated with a 31% increase in the probability of the death penalty over the baseline. (Note that the constant no longer has such a nice interpretation as before. Technically, it still tells us the probability of death penalty when all the variables are zero—which never happens. It's also less than 0, which is impossible for a probability.)

- (f) Since all the variables are rescaled from 0-1, then we can just compare the coefficients to one another in terms of their magnitude. "ac" has the largest "effect", at 0.40. This tells us that moving from the minimum on "ac" to the max on "ac" (note this is the same as a one-unit increase) is associated with a 40% increase in the probability of receiving the death penalty, holding all else fixed.
- (g) In the included "coefs.xls" spreadsheet, I calculate that the Simpson case had an 85% chance of receiving the death penalty. I decided to set $ac = .4$, which is between 3 and 4 on the recoded ac scale, and to set vs to .5, given Simpson knew two of the victims. Obviously, if we had inferred from this prediction that Simpson would receive the death penalty, then we would have been wrong. It doesn't seem fair to me to say that the prediction was wrong—it said there was an 85% chance, which there may have been. But let's say for the sake of argument that the prediction was "wrong". What happened? It turns out I think there a lot of reasons you can think of for why the prediction was off. For one, these data were from Georgia, which no doubt has a very different justice system and very different cultural attitudes towards race compared to LA. For another, Simpson was a celebrity, and this was a celebrity trial. So, if we really wanted to ask whether "someone in a case similar to Simpson's could expect the death penalty," I would say that this is the wrong model to answer that question. But if we were faced with a more obscure case from a southern state, my expectation is we would be closer to a "true" prediction. (One last thing to note: when we change the "bd" value to 0 in our prediction spreadsheet, the probability of death is 8 points higher!)