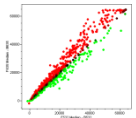
Bivariate Relationships

17.871

2013

Testing associations (not causation!)

- Continuous data
  - Scatter plot (always use first!)



  - (Pearson) correlation coefficient (somewhat common)
  - (Spearman) rank-order correlation coefficient (rare)
  - Regression coefficient (very common)
- Discrete data
  - Cross tabulations
  - χ^2
  - Gamma, Beta, etc.

Continuous DV, continuous EV

- Dependent Variable: DV
- Explanatory (or independent) Variable: EV (not IV)

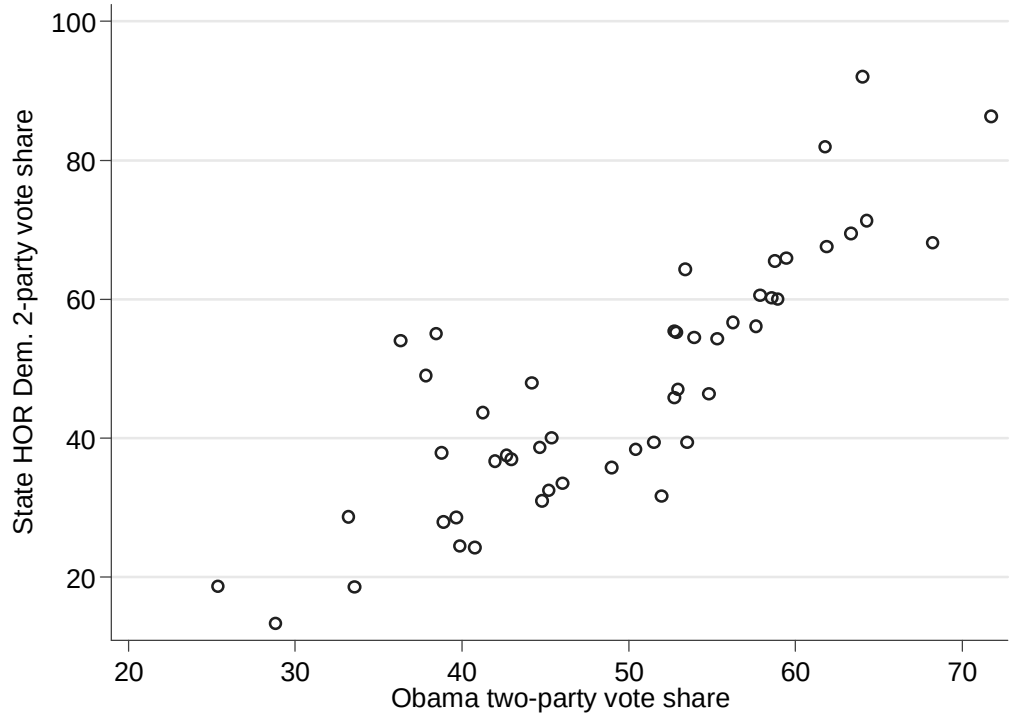
- Example: What is the relationship between Democratic percent in state legislatures and Democratic vote for president?

Regression interpretation

Three key things to learn (today)

1. Where does regression come from
2. To interpret the regression coefficient
3. To interpret the confidence interval
   - We will learn how to calculate confidence intervals in a couple of weeks

Linear Relationship between Support for Obama and Democrats in State Legislatures

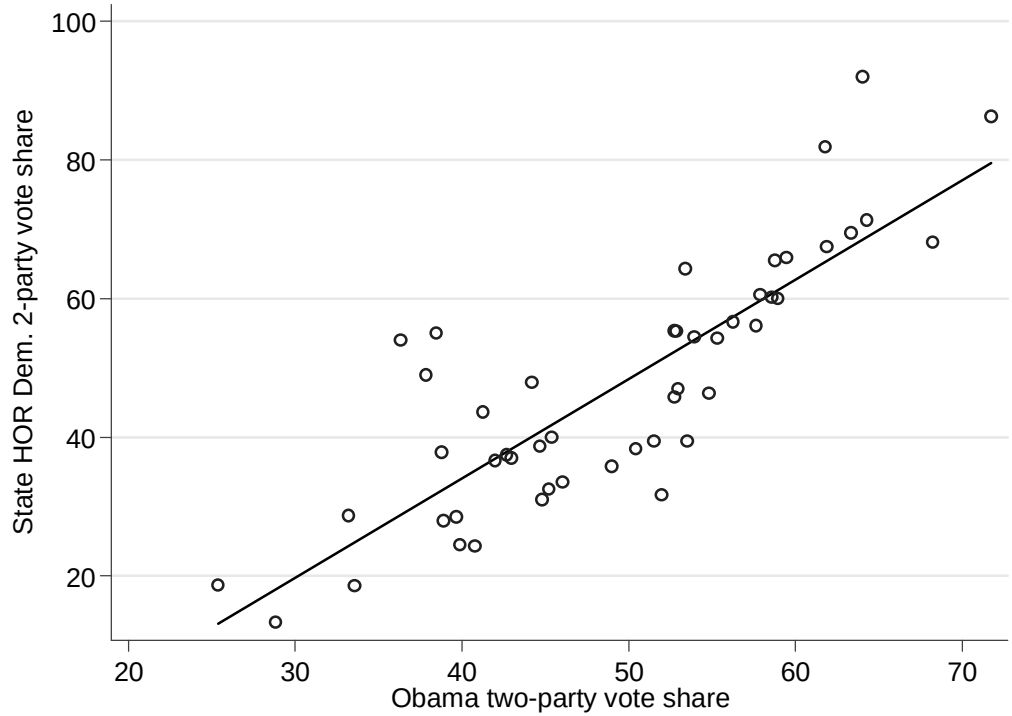


The linear relationship between two variables

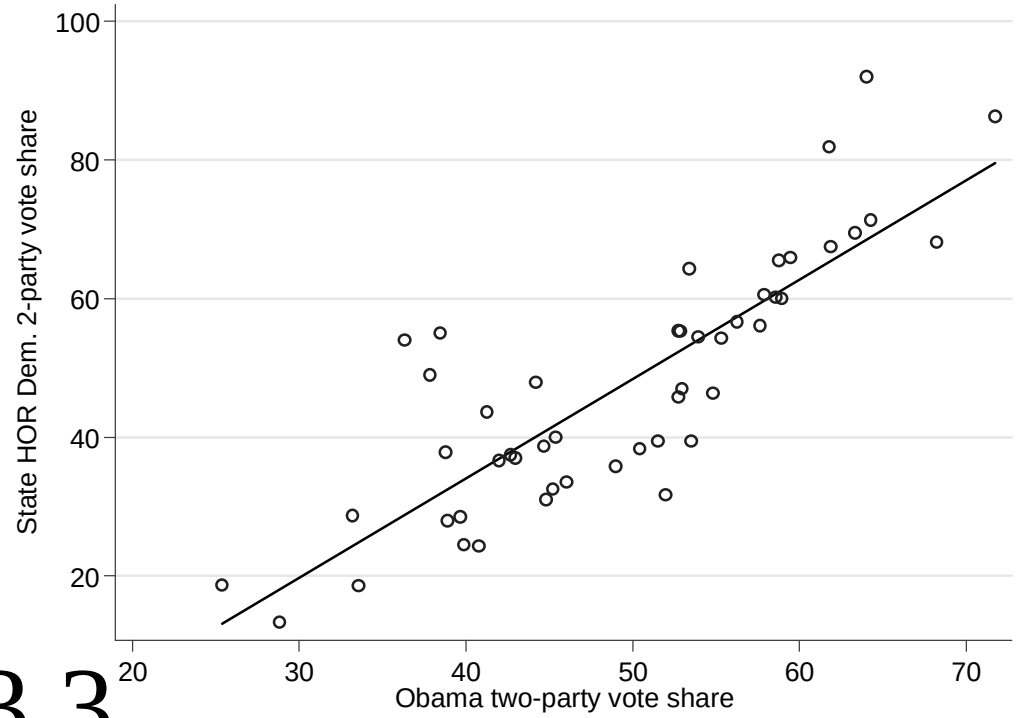
$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Regression quantifies how one variable can be described in terms of another

Linear Relationship between Support for Obama and Democrats in State Legislatures



Linear Relationship between Support for Obama and Democrats in State Legislatures



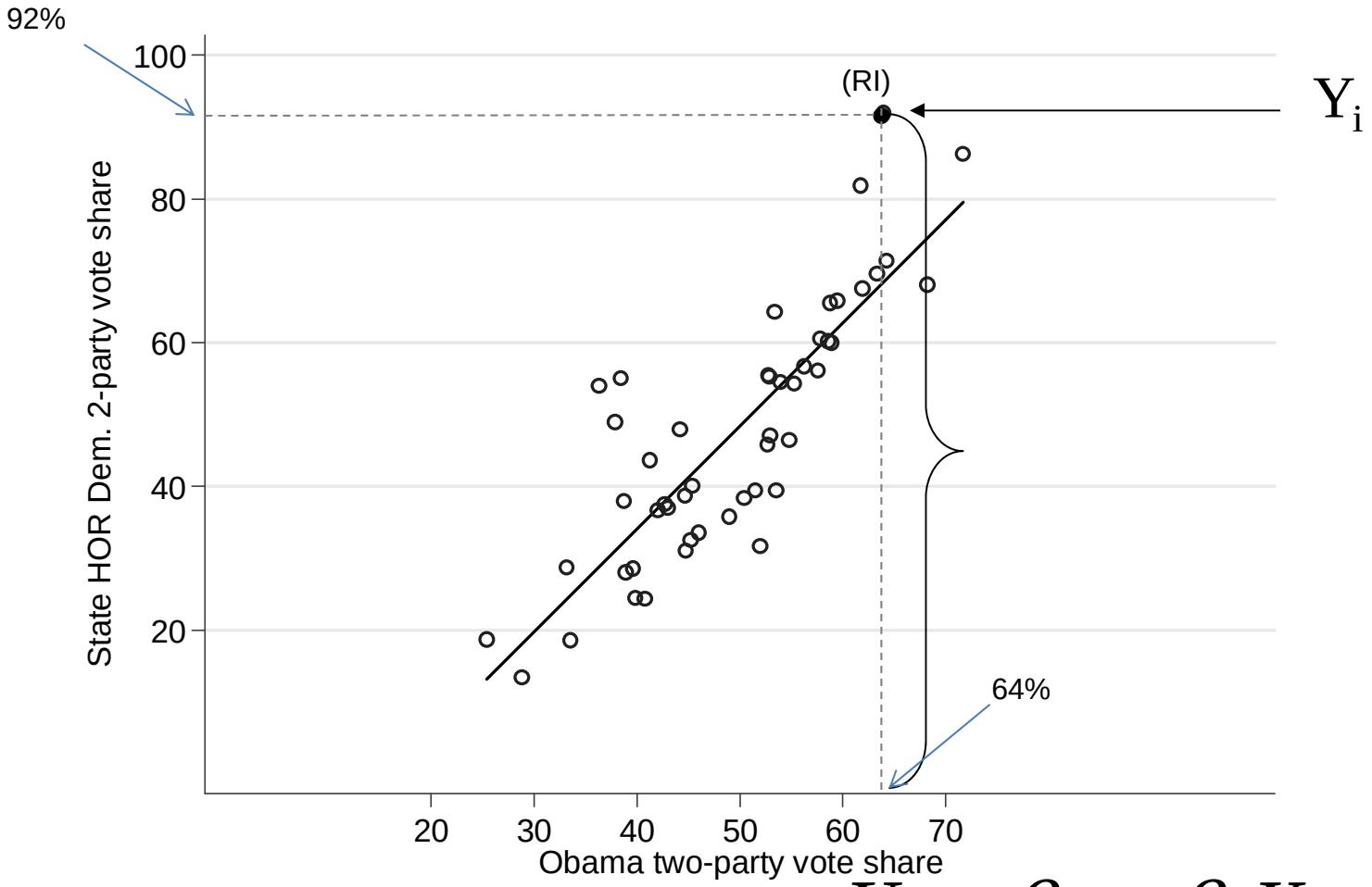
$$\beta_0 = -23.3$$

$$\beta_1 = 1.43$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How did we get that line?

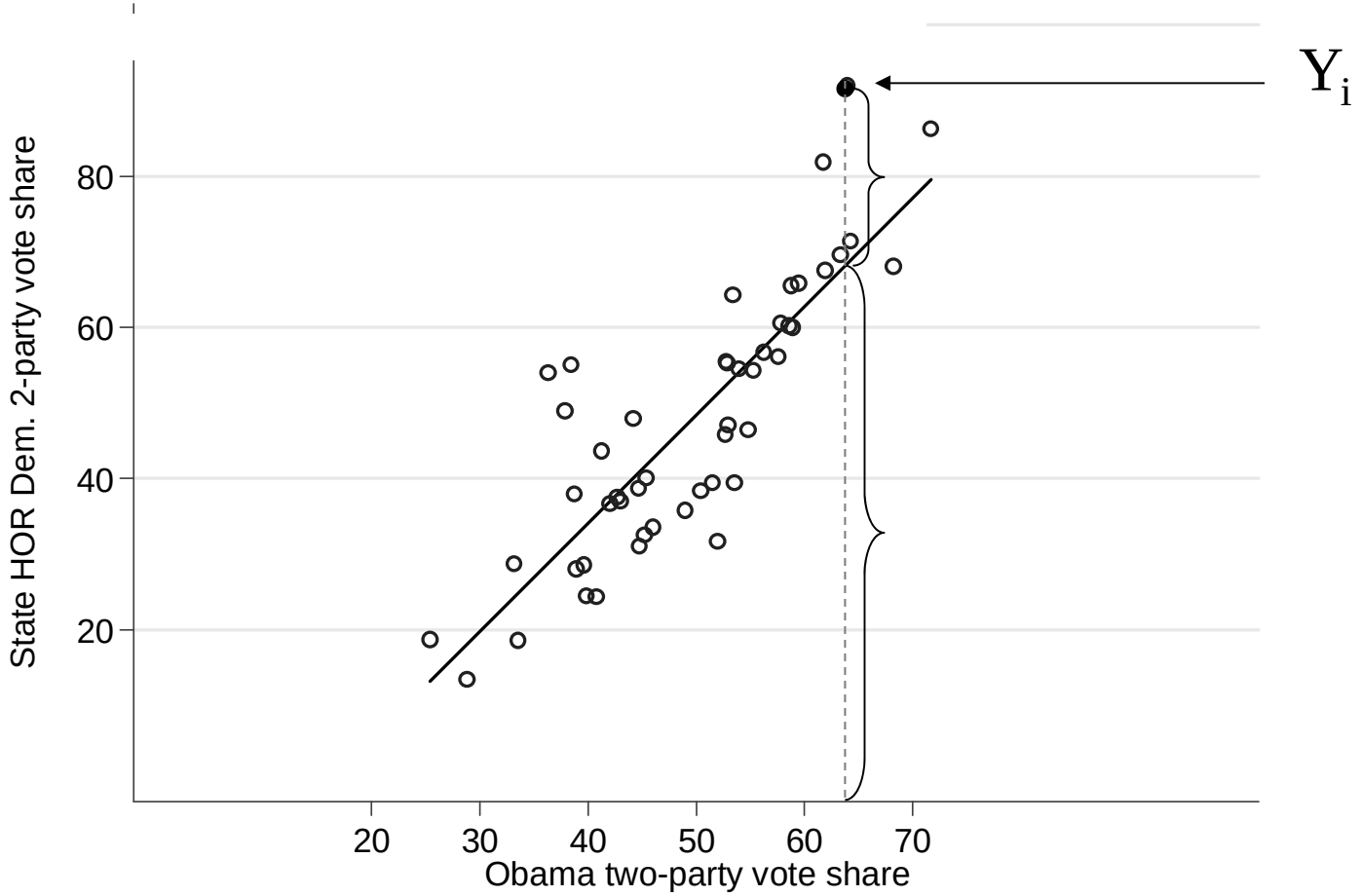
1. Pick a value of Y_i



$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

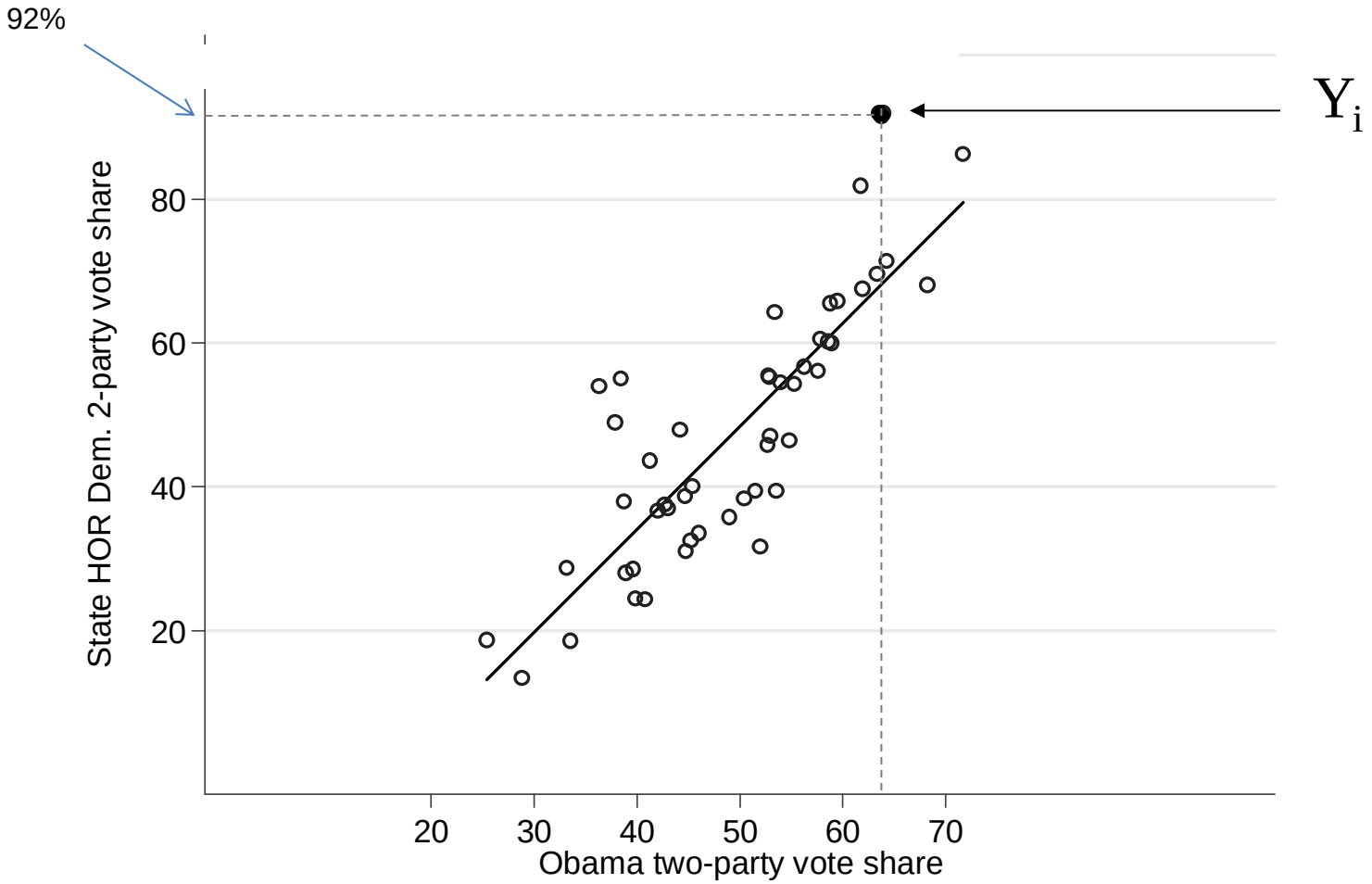
How did we get that line?

2. Decompose Y_i into two parts



$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How did we get that line?
3. Label the points



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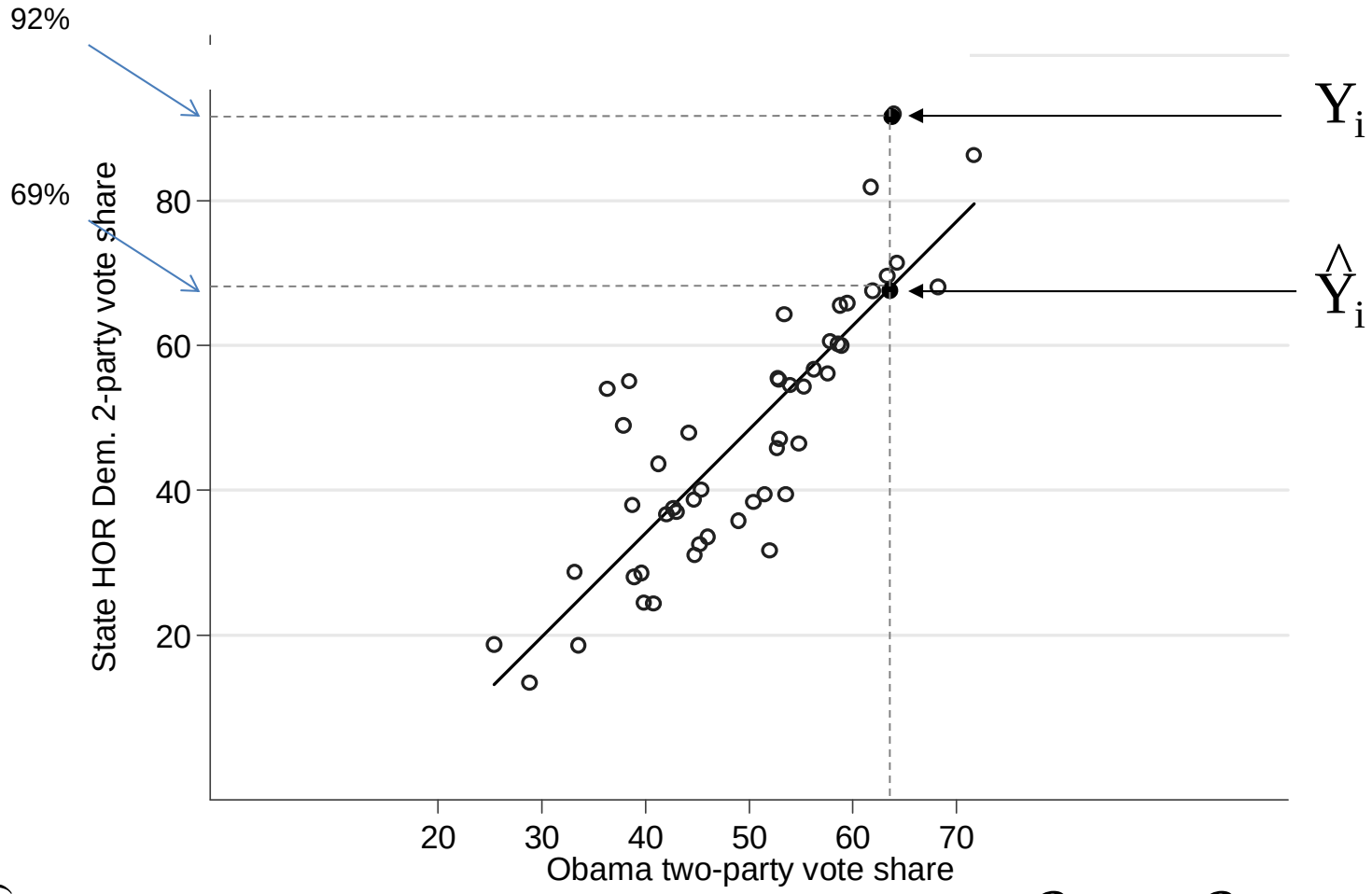
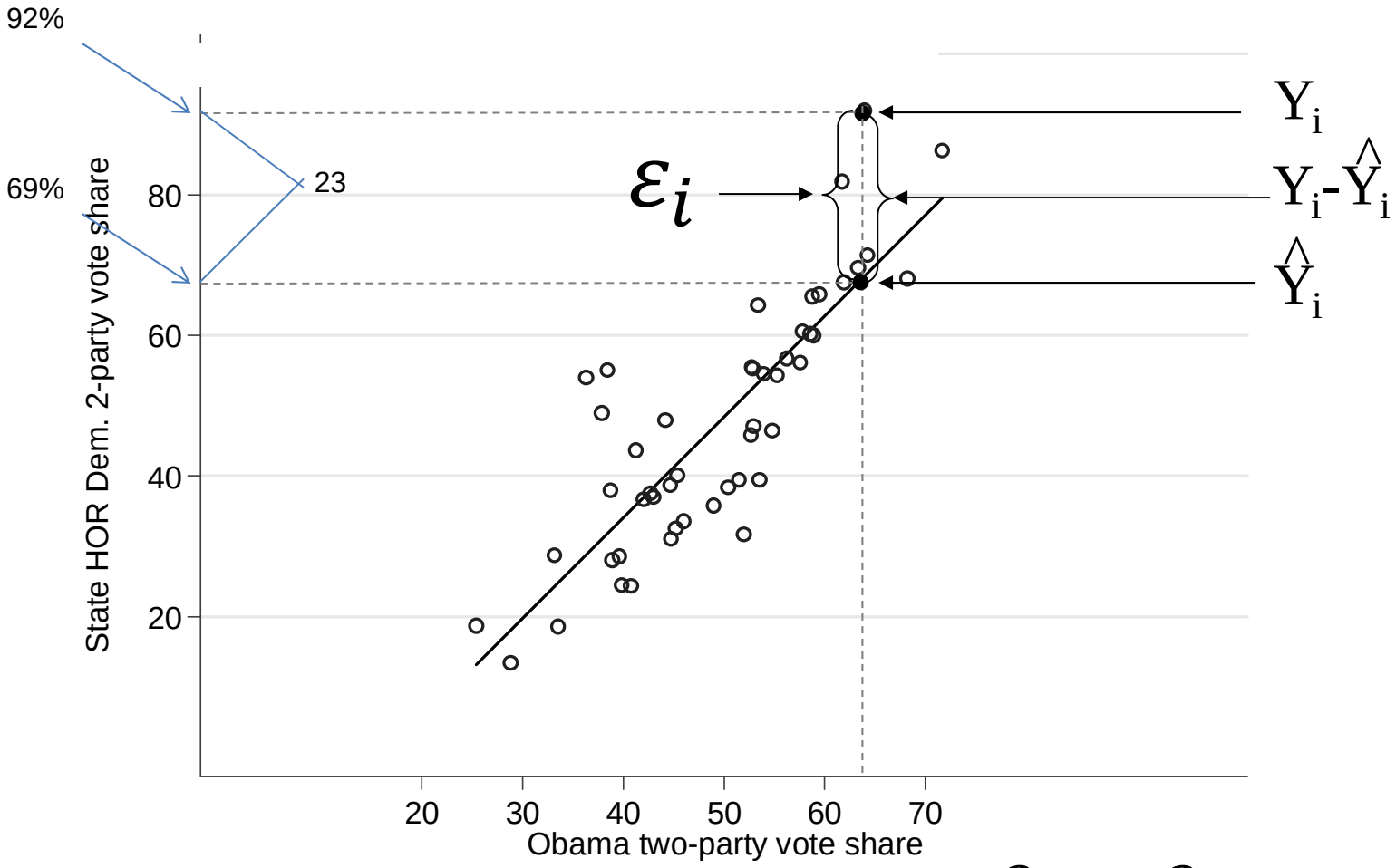


$$\hat{Y}_i = \beta_0 + \beta_1 X_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

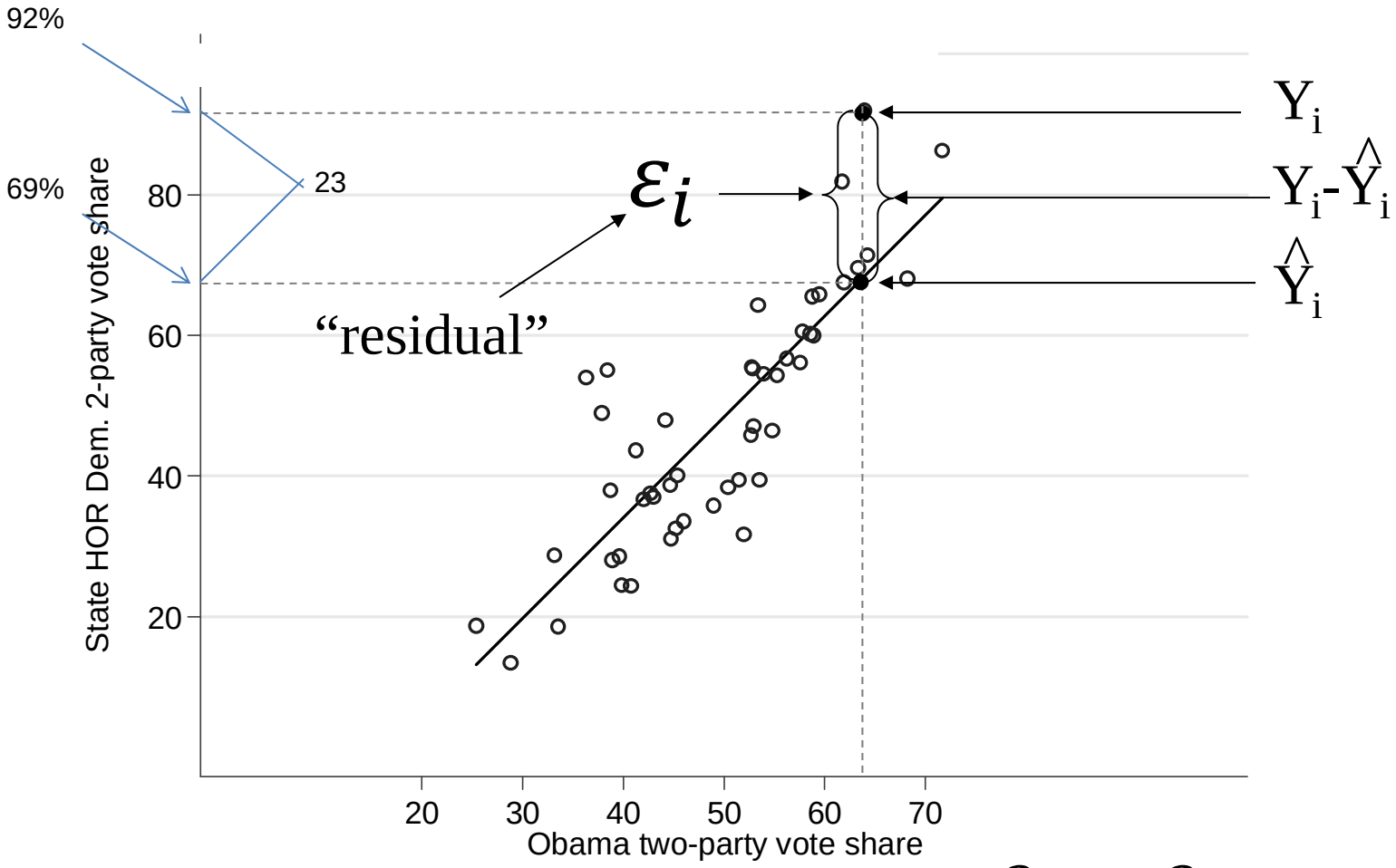
How did we get that line?
3. Label the points



$$\varepsilon_i = Y_i - \beta_0 - \beta_1 X_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How did we get that line?
3. Label the points



$$\varepsilon_i = Y_i - \beta_0 - \beta_1 X_i$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

What is ε_i? (sometimes u_i)

- Wrong functional form
- Measurement error
- Stochastic component in Y
- Unmeasured influences on Y

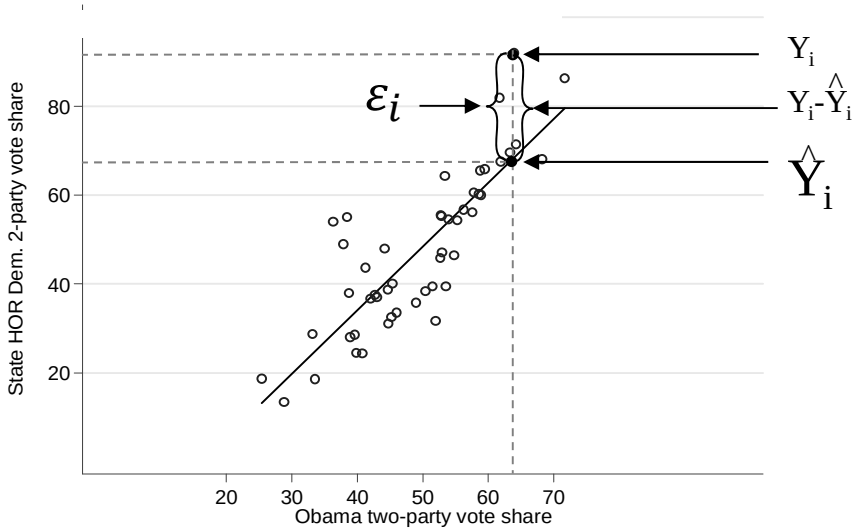
$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

The Method of Least Squares

Pick β_0 and β_1 to minimize $\sum_{i=1}^{n} \varepsilon_i^2$

$$\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 \text{ or}$$

$$\sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)^2$$



Solve for $\frac{\partial \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)^2}{\partial \beta_1} = 0$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (\overline{Y} - Y_i)(\overline{X} - X_i)}{\sum_{i=1}^{n} (\overline{X} - X_i)^2}$$

Remember…

$$\frac{\sum_{i=1}^{n}(\overline{X}-X_i)^2}{n}=Var(X)$$

$$\hat{\beta}_1=\frac{\sum_{i=1}^{n}(\overline{Y}-Y_i)(\overline{X}-X_i)}{\sum_{i=1}^{n}(\overline{X}-X_i)^2}$$

New idea…

$$\frac{\sum_{i=1}^{n}(\overline{X}-X_i)(\overline{Y}-Y_i)}{n}=Cov(X,Y)$$

$$\hat{\beta}_1=\frac{\sum_{i=1}^{n}(\overline{Y}-Y_i)(\overline{X}-X_i)}{\sum_{i=1}^{n}(\overline{X}-X_i)^2}$$

Therefore

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (\bar{Y} - Y_i)(\bar{X} - X_i)}{\sum_{i=1}^{n} (\bar{X} - X_i)^2} =$$

$$\frac{\text{cov}(X, Y)}{\text{var}(X)}$$

Remember this for the problem set!

Regression commands in STATA

- `reg` *`depvar expvars`*
  - E.g., `reg y x`
  - E.g., `reg hpct ppct`
- `corr depvar expvar,cov`

- Making predictions from regression lines
  - `predict` *`newvar`*
  - `predict` *`newvar,`* `resid`
    - *newvar* will now equal ε_i

Drawing linear regression plot in Stata

```
twoway (lfit hpct ppct) (scatter hpct ppct)
```

OR

```
lfit hpct ppct  || scatter hpct ppct
```

State legislature example

```
. reg hpct ppct

      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  1,    47) =  112.73
       Model |  10808.7878     1  10808.7878            Prob > F      =  0.0000
    Residual |  4506.44332    47  95.8817727            R-squared     =  0.7058
-------------+------------------------------           Adj R-squared =  0.6995
       Total |  15315.2312    48  319.067316            Root MSE      =  9.7919

------------------------------------------------------------------------------
        hpct |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        ppct |   1.433516    .135015    10.62   0.000     1.161901    1.705131
       _cons |  -23.25307   6.809819    -3.41   0.001    -36.95266   -9.553483
------------------------------------------------------------------------------
```

State legislature example

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      Source |       SS       df       MS              Number of obs =      49
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------------------------------------------------------------------------------
```

Interpretation: a one percentage point increase in Obama's vote leads to a 1.43 percentage point increase in the Democratic composition of the lower house in the state legislature

State legislature example

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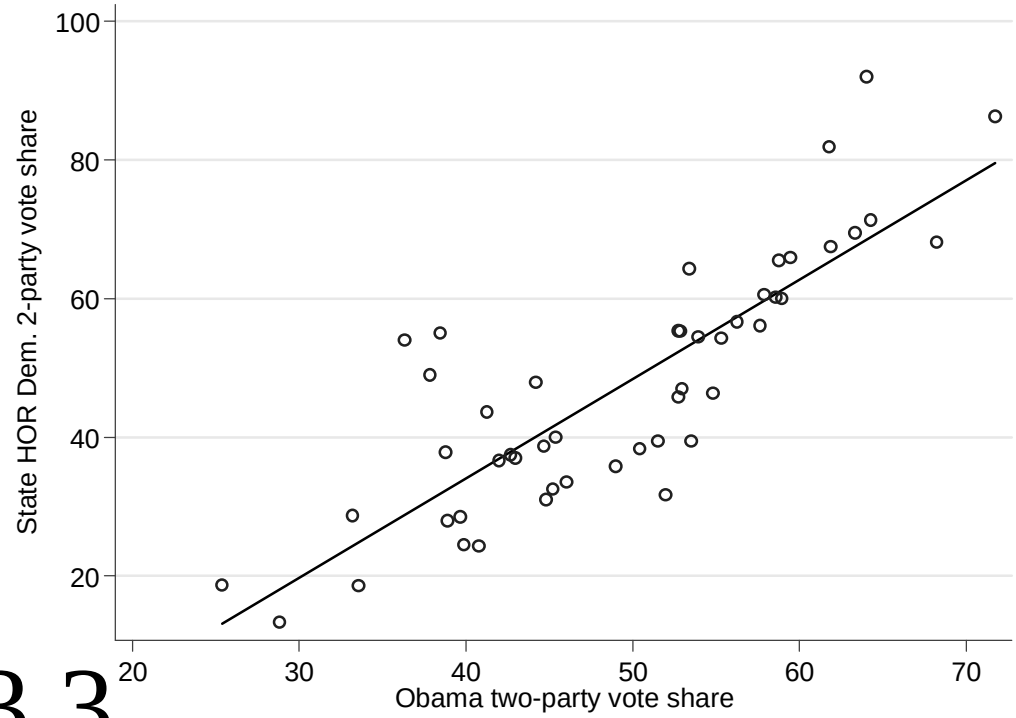
       _cons |  -23.25307   6.809819    -3.41   0.001    -36.95266   -9.553483
------------------------------------------------------------------------------
```

Always include interpretation in your presentations and papers

Interpretation: a one percentage point increase in Obama's vote leads to a 1.43 percentage point increase in the Democratic composition of the lower house in the state legislature

Linear Relationship between Support for Obama and Democrats in State Legislatures



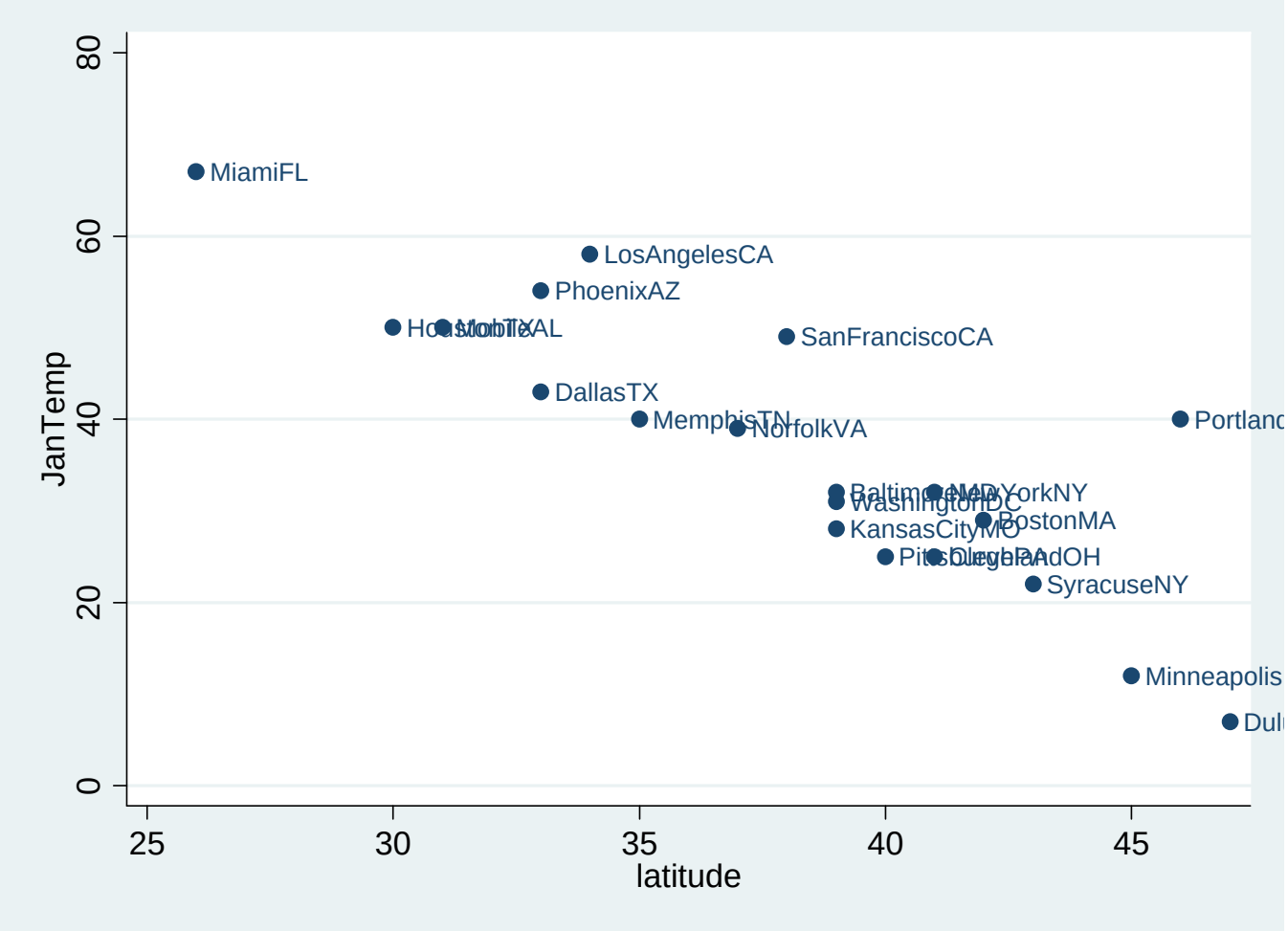
$$\beta_0 = -23.3$$

$$\beta_1 = 1.43$$

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

More regression examples

Temperature and Latitude




```
scatter JanTemp latitude, mlabel(city)
```

```
. reg jantemp latitude

      Source |       SS       df       MS              Number of obs =      20
-------------+------------------------------           F(  1,    18) =   49.34
       Model |  3250.72219     1  3250.72219           Prob > F      =  0.0000
    Residual |  1185.82781    18  65.8793228           R-squared     =  0.7327
-------------+------------------------------           Adj R-squared =  0.7179
       Total |     4436.55    19  233.502632           Root MSE      =  8.1166

------------------------------------------------------------------------------
     jantemp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    latitude |  -2.341428   .3333232    -7.02   0.000    -3.041714   -1.641142
       _cons |   125.5072   12.77915     9.82   0.000     98.65921    152.3552
------------------------------------------------------------------------------
```

Interpretation: a one point increase in latitude is associated with a 2.3 decrease in average temperature (in Fahrenheit).

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

How to add a regression line:

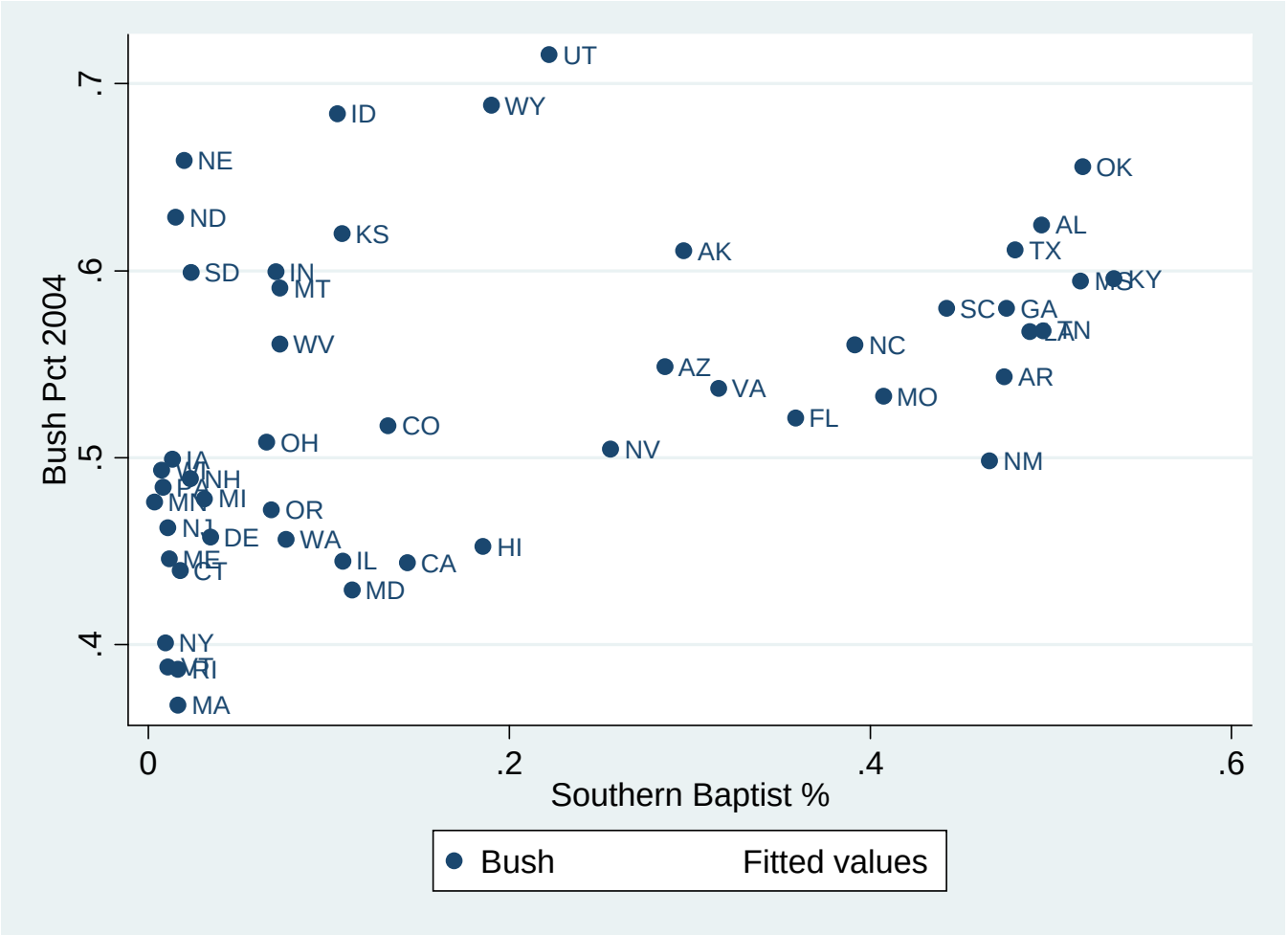
Stata command: `lfit`


```
scatter JanTemp latitude, mlabel(city) || lfit JanTemp latitude
```

or often better

```
scatter JanTemp latitude, mlabel(city) m(i) || lfit JanTemp latitude
```

Bush vote and Southern Baptists



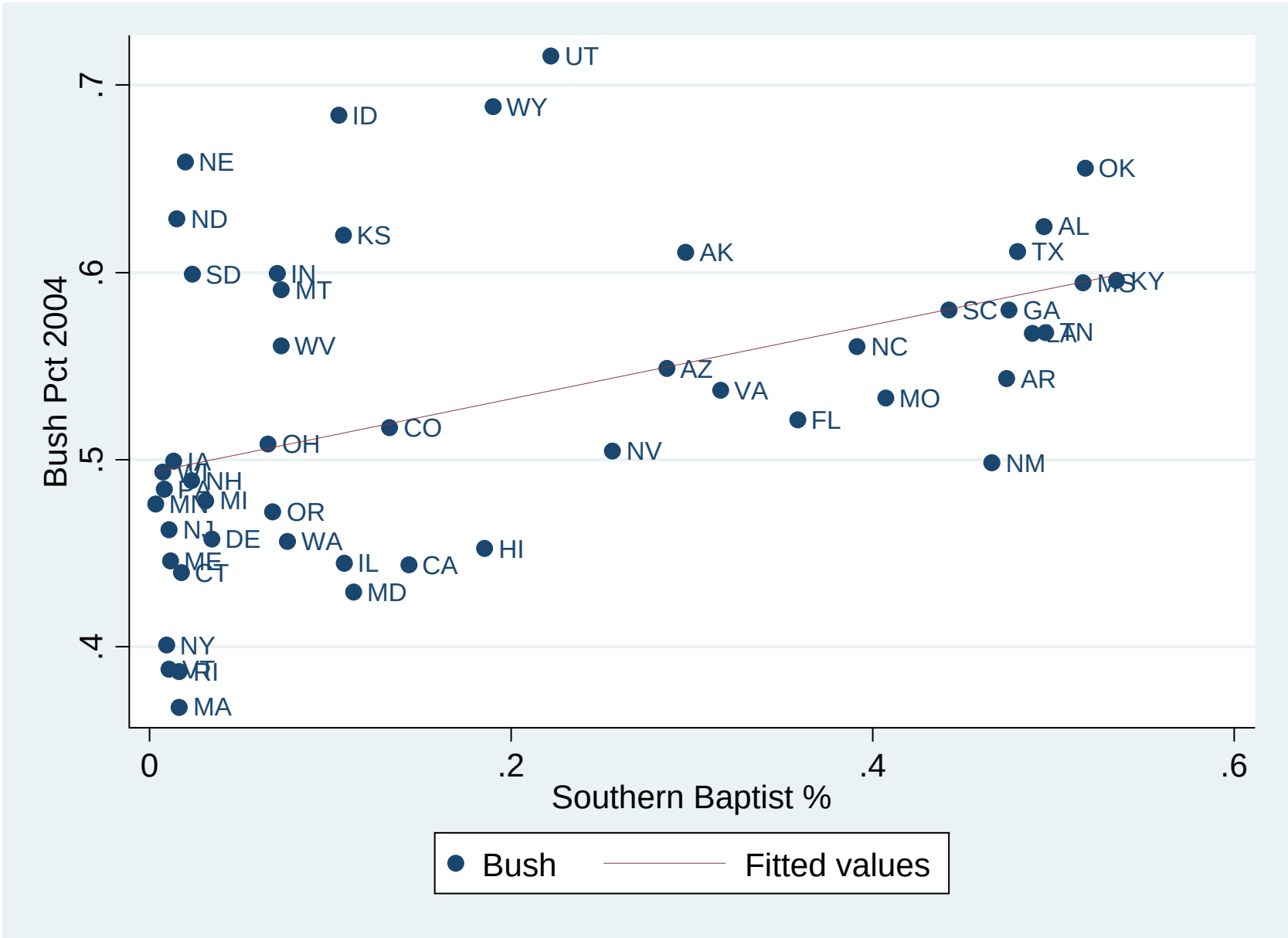
```
. reg bush sbc_mpct [aw=votes]
(sum of wgt is   1.2207e+08)

      Source |       SS       df       MS              Number of obs =      50
-------------+------------------------------           F(  1,    48) =   40.18
       Model |  .118925068     1  .118925068           Prob > F      =  0.0000
    Residual |  .142084951    48  .002960103           R-squared     =  0.4556
-------------+------------------------------           Adj R-squared =  0.4443
       Total |  .261010018    49  .005326735           Root MSE      =  .05441

------------------------------------------------------------------------------
        bush |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    sbc_mpct |    .261779   .0413001     6.34   0.000     .1787395    .3448185
       _cons |   .4563507   .0112155    40.69   0.000     .4338004    .4789011
------------------------------------------------------------------------------
```

Coefficient interpretation:

- A one percentage point increase in Baptist percentage is associated with a .26 percentage point increase in Bush vote share at the state level.


Bush Pct 2004
.4
.5
.6
.7
0
.2
.4
.6
Southern Baptist %
UT
ID
WY
NE
OK
ND
KS
AK
AL
TX
SD
IN
MT
MS
KY
SC
GA
WV
NC
AZ
AR
VA
MO
FL
CO
OH
NV
NM
NH
MI
OR
DE
WA
HI
IL
CA
CT
MD
NY
MA
Bush
Fitted values

When the dependent variable is binary

- You can run a linear regression (though the graph will look weird)
- There are other techniques you can use, which are “more correct,” but difficult to interpret directly
  - Logit
  - Probit
- For now, use linear regression, not logit and probit

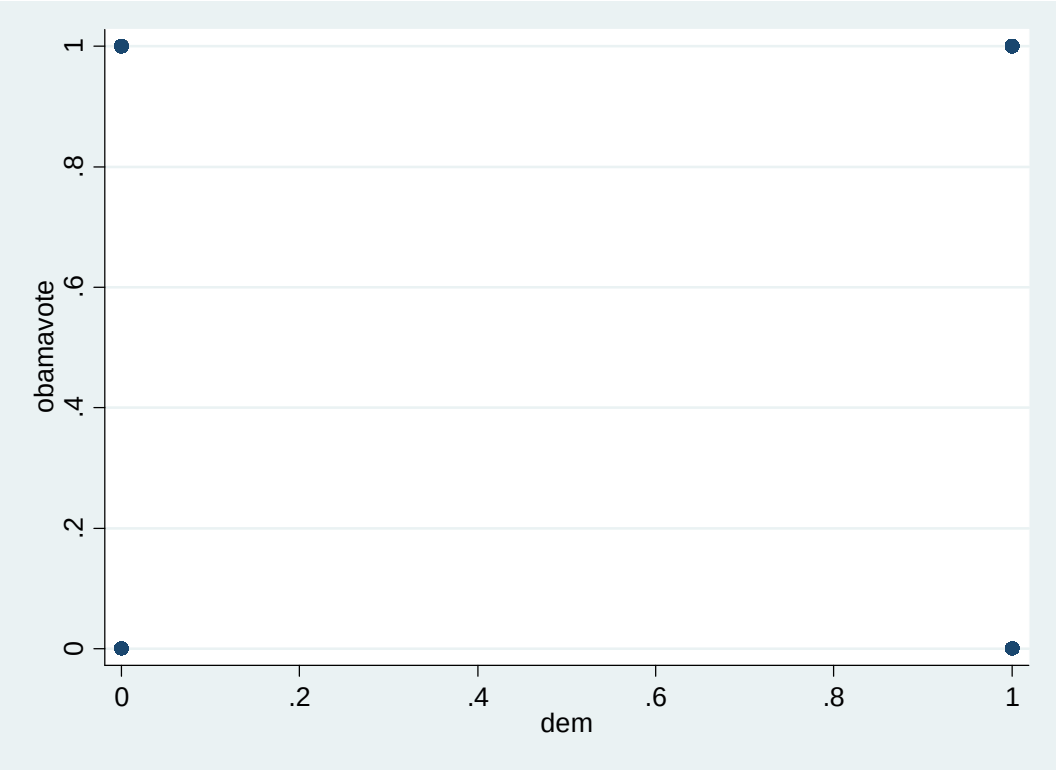
Example: Vote for Obama and Party ID

```
. table dem ,c(mean obamavote) row

---------------------------
      dem | mean(obamav~e)
----------+----------------
        0 |       .0439405
        1 |       .9514394
          |
    Total |       .5424054
---------------------------
```

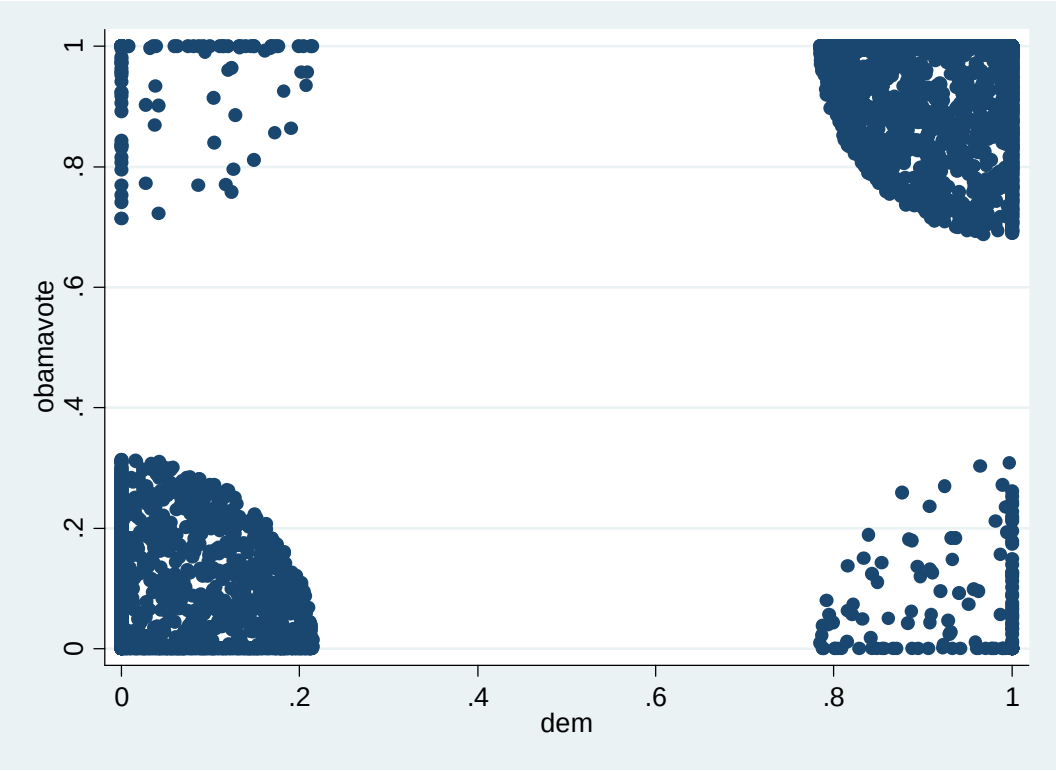
Source: 2012 SPAE, unweighted results

Why this is a case where you don't want to do a scatterplot




```
scatter obamavote dem
```

It’s a little better with the “jitter” command




```
scatter obamavote dem, jitter(50)
```

```
. reg obamavote dem

      Source |       SS       df       MS              Number of obs =    6261
-------------+------------------------------           F(  1,  6259) =28798.56
       Model |  1276.54962     1  1276.54962           Prob > F      =  0.0000
    Residual |  277.441757  6259   .04432685           R-squared     =  0.8215
-------------+------------------------------           Adj R-squared =  0.8214
       Total |  1553.99138  6260  .248241434           Root MSE      =  .21054

------------------------------------------------------------------------------
   obamavote |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

obamavote = 0.0439405 + 0.9074989*dem + e

```
. table dem ,c(mean obamavote) row

-------------------------
      dem | mean(obamav~e)
----------+--------------
        0 |       .0439405
        1 |       .9514394
          |
    Total |       .5424054
-------------------------
```

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-------------+----------------------------------------------------------------
         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

obamavote = 0.0439405 + 0.9074989*0 = 0.0439405

```
. table dem ,c(mean obamavote) row

-------------------------
      dem | mean(obamav~e)
----------+--------------
        0 |       .0439405
        1 |       .9514394
          |
    Total |       .5424054
-------------------------
```

When the respondent is a Republican, dem = 0

```
. reg obamavote dem

      Source |       SS       df       MS              Number of obs =    6261
-------------+------------------------------           F(  1,  6259) =28798.56
       Model |  1276.54962     1  1276.54962           Prob > F      =  0.0000
    Residual |  277.441757  6259   .04432685           R-squared     =  0.8215
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------------------------------------------------------------------------------
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         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

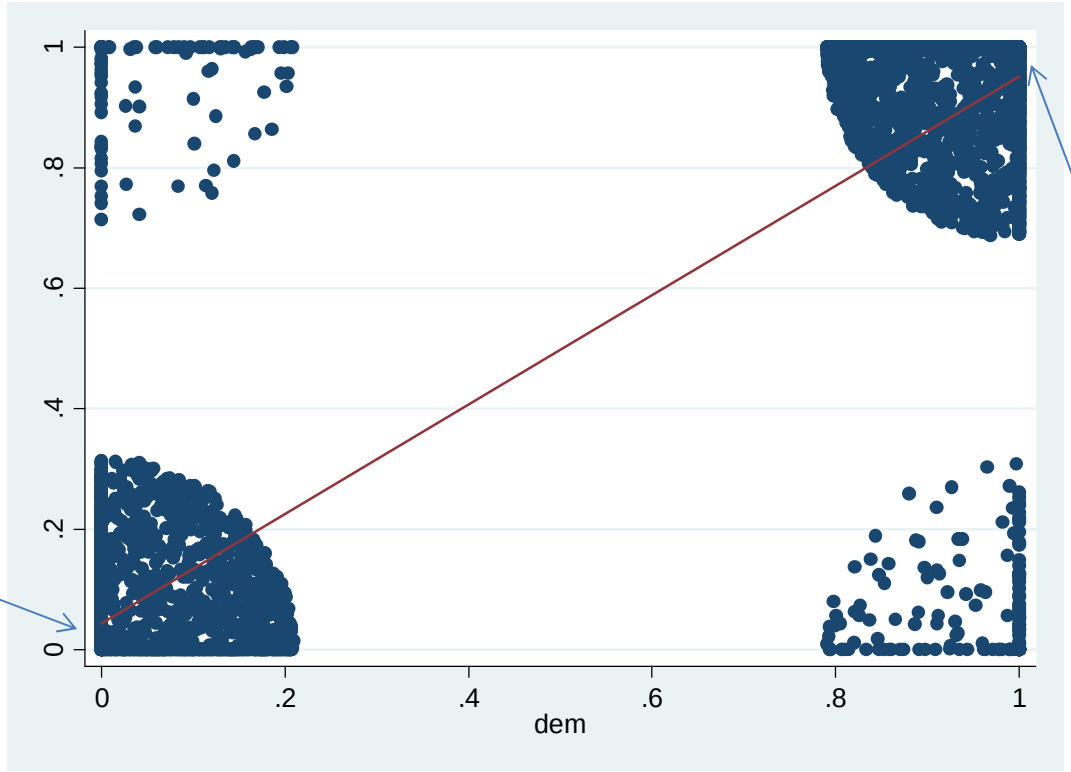
obamavote = 0.0439405 + 0.9074989*1 = 0.9514394

```
. table dem ,c(mean obamavote) row

-------------------------
      dem | mean(obamav~e)
----------+--------------
        0 |       .0439405
        1 |       .9514394
          |
    Total |       .5424054
-------------------------
```

When the respondent is a Democrat, dem = 1

Geometric interpretation



.0439405+.9074989
=.9514394

.0439405

obamavote	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
dem	**.9074989**	.0053476	169.70	0.000	.8970157	.9179821
_cons	**.0439405**	.0039633	11.09	0.000	.0361711	.0517099

```
. reg obamavote dem

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-------------+------------------------------           Adj R-squared =  0.8214
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------------------------------------------------------------------------------
   obamavote |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

Coefficient interpretation:

- Republicans have a 4.4% probability of voting for Obama
- A one point increase in Democrat-ness increases the probability of voting for Obama by 90.7 percentage points

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         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

A better coefficient interpretation:

- Republicans have a 4.4% probability of voting for Obama
- Democrats are 90.7 percentage points more likely to vote for Obama (NOT 90.7 **percent** more likely…)

```
. reg obamavote dem

      Source |       SS       df       MS              Number of obs =    6261
-------------+------------------------------           F(  1,  6259) =28798.56
       Model |  1276.54962     1  1276.54962           Prob > F      =  0.0000
    Residual |  277.441757  6259   .04432685           R-squared     =  0.8215
-------------+------------------------------           Adj R-squared =  0.8214
       Total |  1553.99138  6260  .248241434           Root MSE      =  .21054

------------------------------------------------------------------------------
   obamavote |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         dem |   .9074989   .0053476   169.70   0.000     .8970157    .9179821
       _cons |   .0439405   .0039633    11.09   0.000     .0361711    .0517099
------------------------------------------------------------------------------
```

Another coefficient interpretation:

- Republicans have a 4.4% probability of voting for Obama
- As a voter moves from being a Republican to being a Democrats, she becomes 90.7 percentage points more likely to vote for Obama (NOT 90.7 **percent** more likely…)

Additional regression in bivariate relationship topics

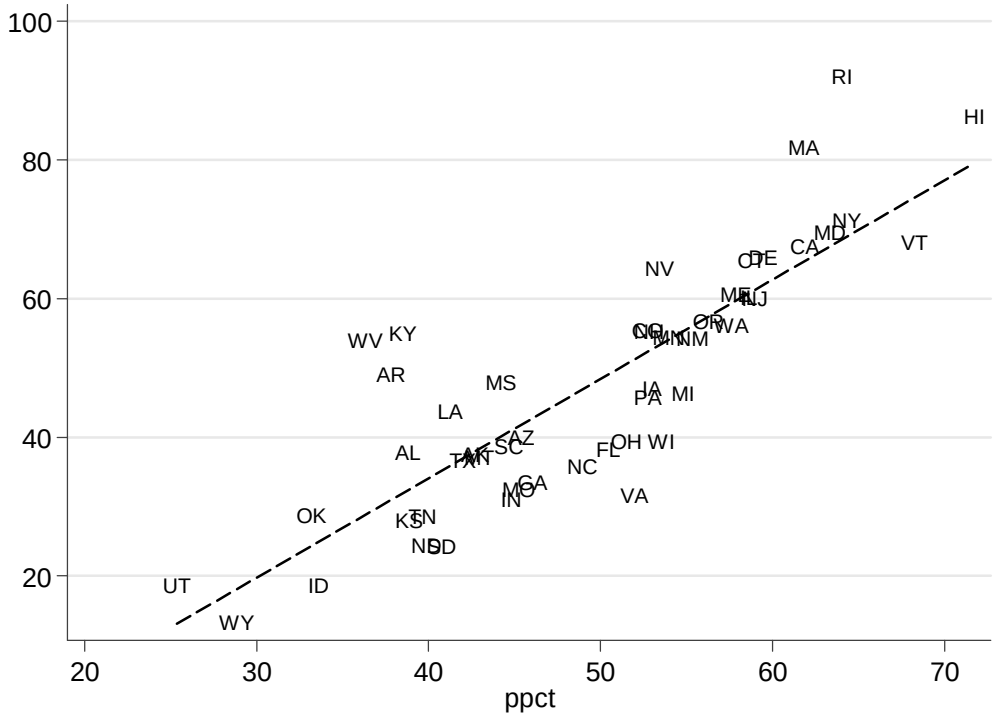
- Residuals
- Comparing coefficients
- Functional form
- Goodness of fit (R^2 and SER)
- Correlation
- Using the appropriate graph/table

Residuals

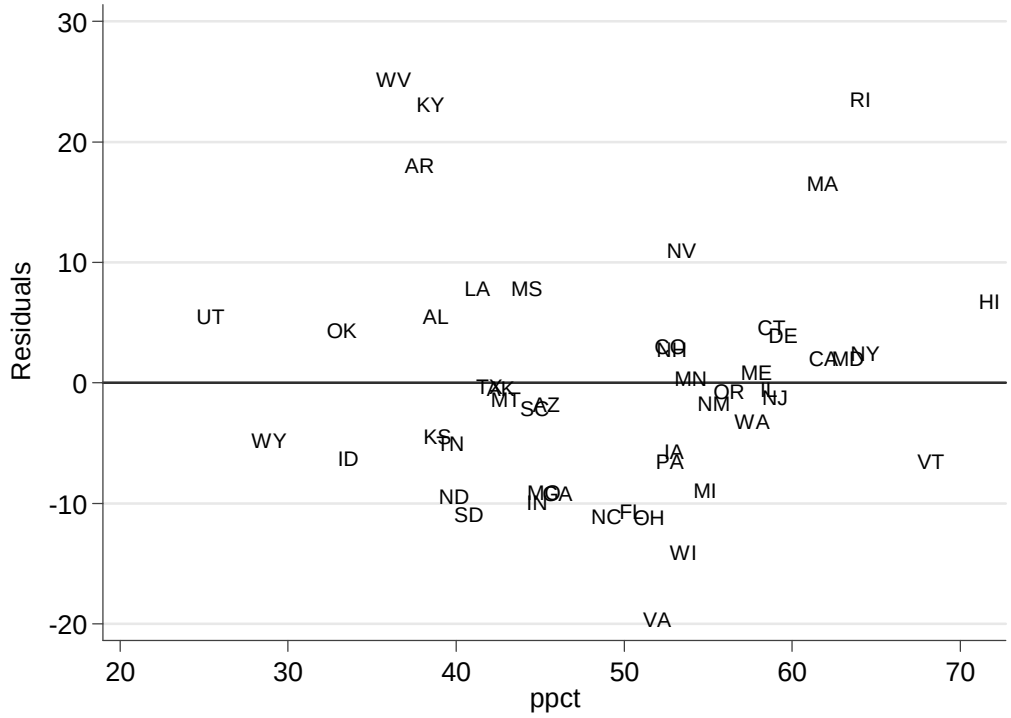
Residuals

$$e_i = Y_i - B_0 - B_1 X_i$$

Residuals are valuable for picking out outliers and interesting cases




```
twoway (scatter hpct ppct, msymbol(none) mlabel(stabbr) mlabposition(0)) (lfit hpct
ppct), legend(off) scheme(Tufte)
```



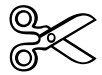

```
. predict ry,resid
(1 missing value generated)

. twoway (scatter ry ppct,msymbol(none) mlabel(stabbr) mlabposition(0)),legend(off)
scheme(Tufte) yline(0)
```

```
. gsort -ry

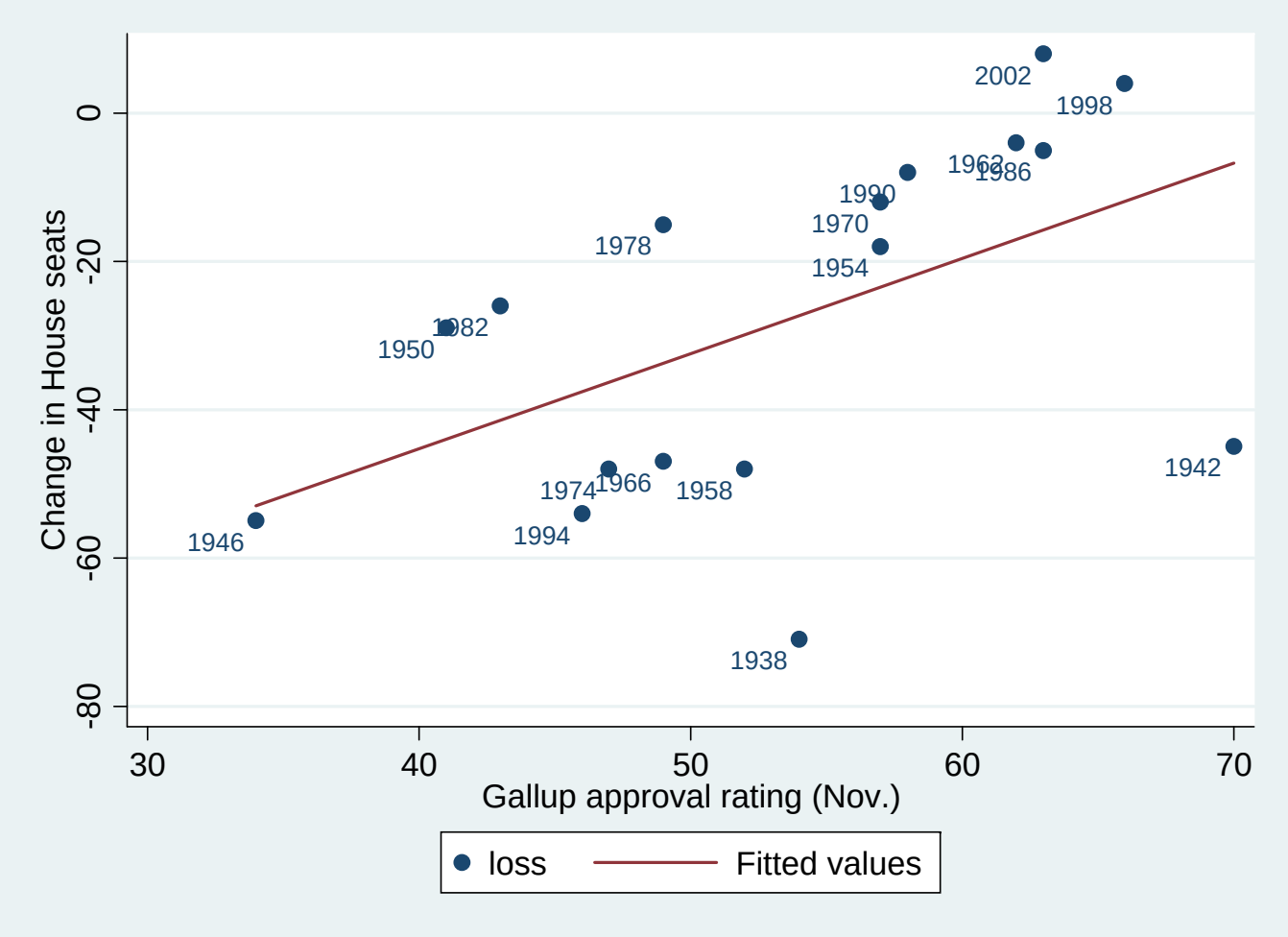
. list state ry
```

	state	ry
1.	West Virginia	25.17959
2.	Rhode Island	23.48404
3.	Kentucky	23.12402
4.	Arkansas	18.00081
5.	Massachusetts	16.55731
6.	Nevada	10.97821
7.	Mississippi	7.828268
8.	Louisiana	7.805305
9.	Hawaii	6.73896
10.	Utah	5.545974



46.	North Carolina	-11.10709
47.	Ohio	-11.19955
48.	Wisconsin	-14.06958
49.	Virginia	-19.61035
50.	Nebraska	.

Use residuals to diagnose potential problems



```
. reg loss gallup

      Source |       SS       df       MS              Number of obs =      17
-------------+------------------------------           F(  1,    15) =    5.70
       Model |  2493.96962     1  2493.96962           Prob > F      =  0.0306
    Residual |  6564.50097    15  437.633398           R-squared     =  0.2753
-------------+------------------------------           Adj R-squared =  0.2270
       Total |  9058.47059    16  566.154412           Root MSE      =   20.92

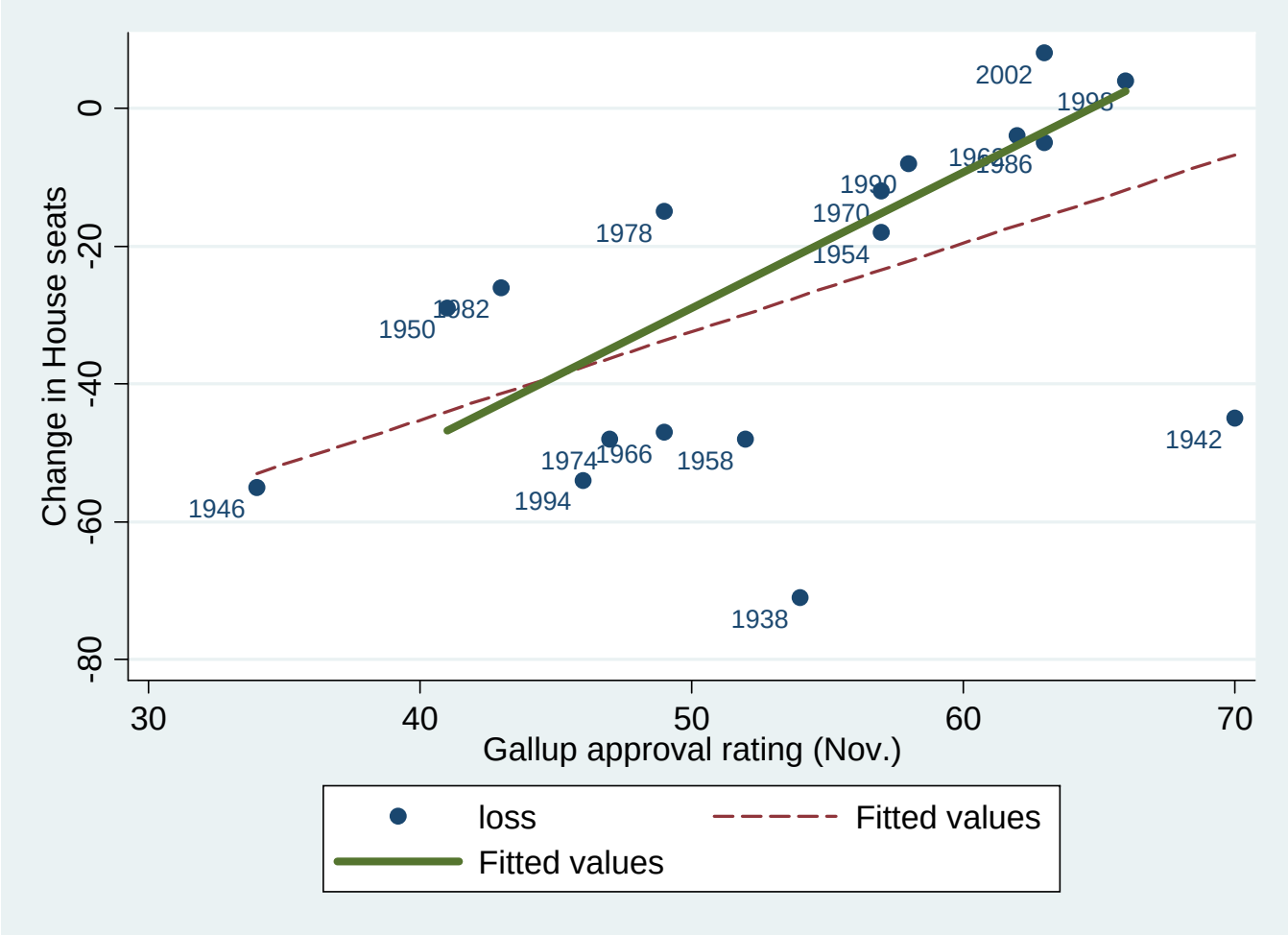
------------------------------------------------------------------------------
       Seats |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      gallup |   1.283411     .53762     2.39   0.031     .1375011    2.429321
       _cons |  -96.59926   29.25347    -3.30   0.005    -158.9516   -34.24697
------------------------------------------------------------------------------


 . reg loss gallup if year>1946

      Source |       SS       df       MS              Number of obs =      14
-------------+------------------------------           F(  1,    12) =   17.53
       Model |  3332.58872     1  3332.58872           Prob > F      =  0.0013
    Residual |  2280.83985    12  190.069988           R-squared     =  0.5937
-------------+------------------------------           Adj R-squared =  0.5598
       Total |  5613.42857    13  431.802198           Root MSE      =  13.787

------------------------------------------------------------------------------
       seats |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      gallup |    1.96812   .4700211     4.19   0.001     .9440315    2.992208
       _cons |  -127.4281   25.54753    -4.99   0.000    -183.0914   -71.76486
------------------------------------------------------------------------------
```

```
scatter loss gallup, mlabel(year) || lfit loss gallup || lfit loss gallup if year >1946
```



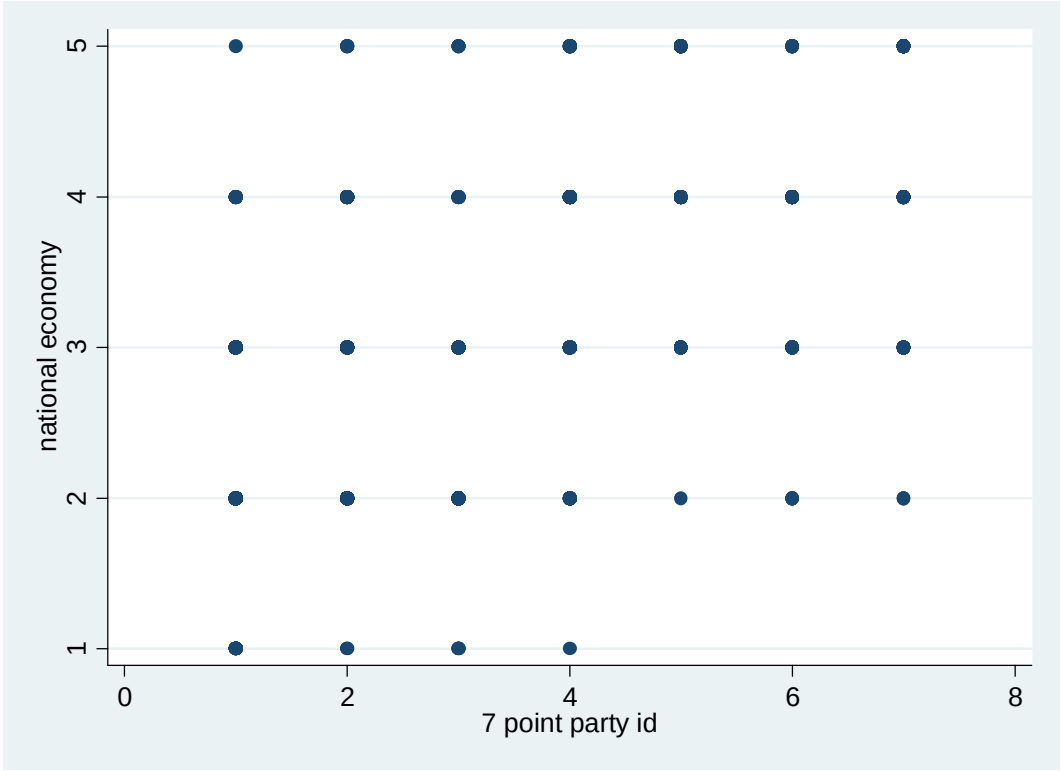
How to recode and interpret 0-1 variables

- Example: relationship between partisanship and saying the economy has gotten better in the past year

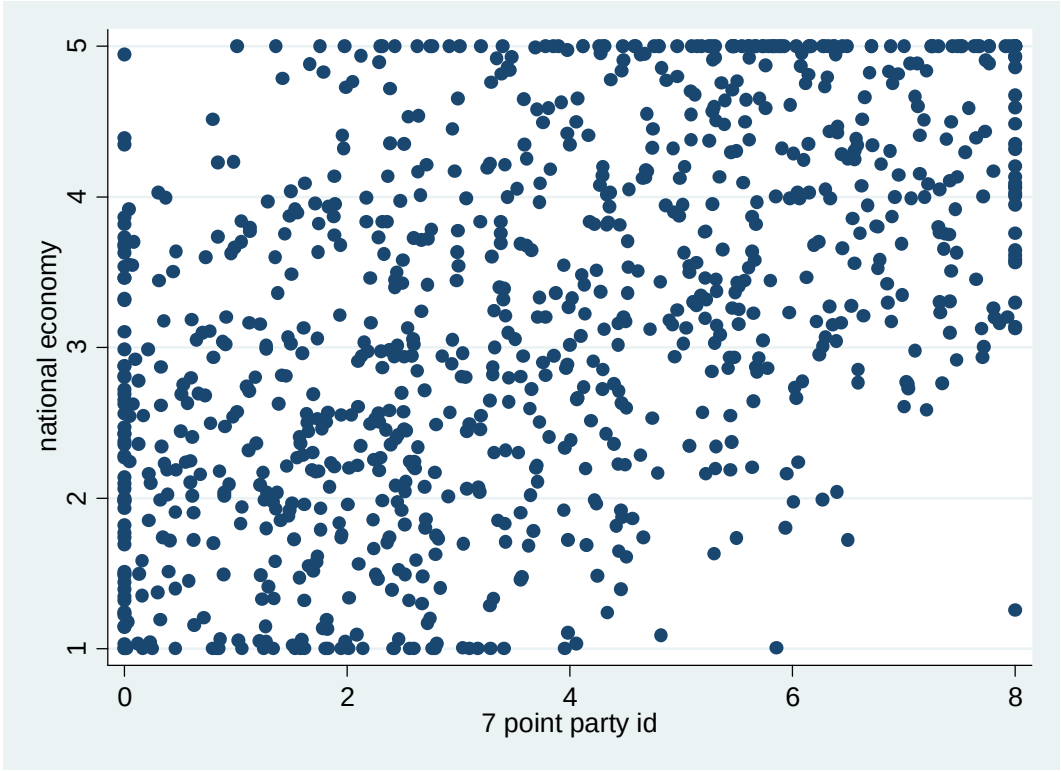
```
. tab pid7 cc302,row
```

7 point party id	Better gotten mu	gotten be	national economy stayed ab	gotten wo	Worse gotten mu	Total
strong democrat	21	146	65	12	1	245
	8.57	59.59	26.53	4.90	0.41	100.00
not very strong democ	2	45	29	26	5	107
	1.87	42.06	27.10	24.30	4.67	100.00
lean democrat	4	65	29	11	4	113
	3.54	57.52	25.66	9.73	3.54	100.00
independent	1	16	29	39	17	102
	0.98	15.69	28.43	38.24	16.67	100.00
lean republican	0	1	27	39	39	106
	0.00	0.94	25.47	36.79	36.79	100.00
not very strong repub	0	6	35	41	15	97
	0.00	6.19	36.08	42.27	15.46	100.00
strong republican	0	3	21	80	62	166
	0.00	1.81	12.65	48.19	37.35	100.00
Total	28	282	235	248	143	936
	2.99	30.13	25.11	26.50	15.28	100.00

scatter cc302 pid7



```
scatter cc302 pid7,jitter(50)
```



How to recode variables to 0-1 scale

- Party ID example: pid7 in CCES 2012
- Usually varies from
  - 1 (strong Democrat)
  - to 7 (strong Republican)
  - 8 is “unsure,” and needs to be recoded to missing (“.”)
- Stata code?
  - `gen newpid7 = (7-pid7)/(7-1)`

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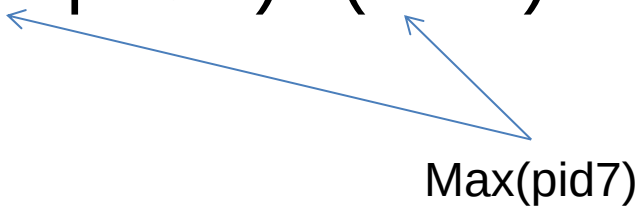

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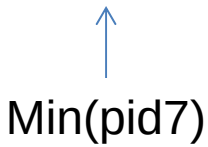


Regression interpretation with 0-1 scale

- Continue with pid7 example
  - `regress econworse newpid7` (both recoded to 0-1 scales)*
  - pid7 coefficient: intercept = .18, slope = .48
  - Interpretation?
    - The average strong Democrat rates the **weakness** of the national economy as a .18 on a one-point national economic weakness scale
    - Shifting from being a strong Democrat to a strong Republican corresponds with a .48 increase in evaluations of the national economy as being weak (on the one-point national economic weakness scale)

*econworse originally coded so that 1 = gotten much better, 5 = gotten much worse

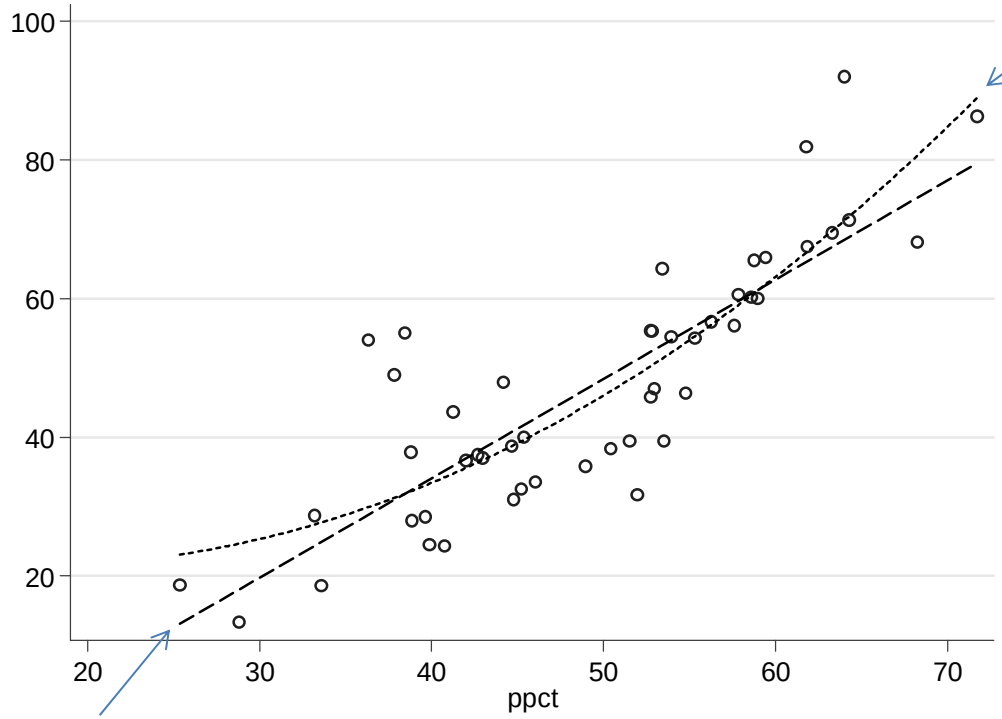
Functional Form

About the Functional Form

- Linear in the variables *vs.* linear in the parameters
  - $Y = a + bX + e$ (linear in both)
  - $Y = a + bX + cX^2 + e$ (linear in parms.)
  - $Y = a + X^b + e$ (linear in variables, not parms.)
- Regression must be linear in parameters

The Linear and Curvilinear Relationship between Dems. in State Legislatures & Obama Vote

$Y=28.0-0.77X+0.023X^2$



Y = -23.3+1.43*X

```
scatter hpct ppct || lfit hpct ppct || qfit hpct
ppct,scheme(Tufte) legend(off)
```

```
. gen ppct2=ppct^2

. reg hpct ppct ppct2

      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  2,    46) =   62.86
       Model |  11212.5662     2  5606.28309           Prob > F      =  0.0000
    Residual |  4102.66499    46  89.1883693           R-squared     =  0.7321
-------------+------------------------------           Adj R-squared =  0.7205
       Total |  15315.2312    48  319.067316           Root MSE      =   9.444

------------------------------------------------------------------------------
        hpct |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        ppct |  -.7652575   1.041559    -0.73   0.466    -2.861808    1.331293
       ppct2 |   .0225134   .0105809     2.13   0.039     .0012151    .0438116
       _cons |   28.00989   24.97192     1.12   0.268    -22.25598    78.27576
------------------------------------------------------------------------------
```

Log transformations (see Tufte, ch. 3)

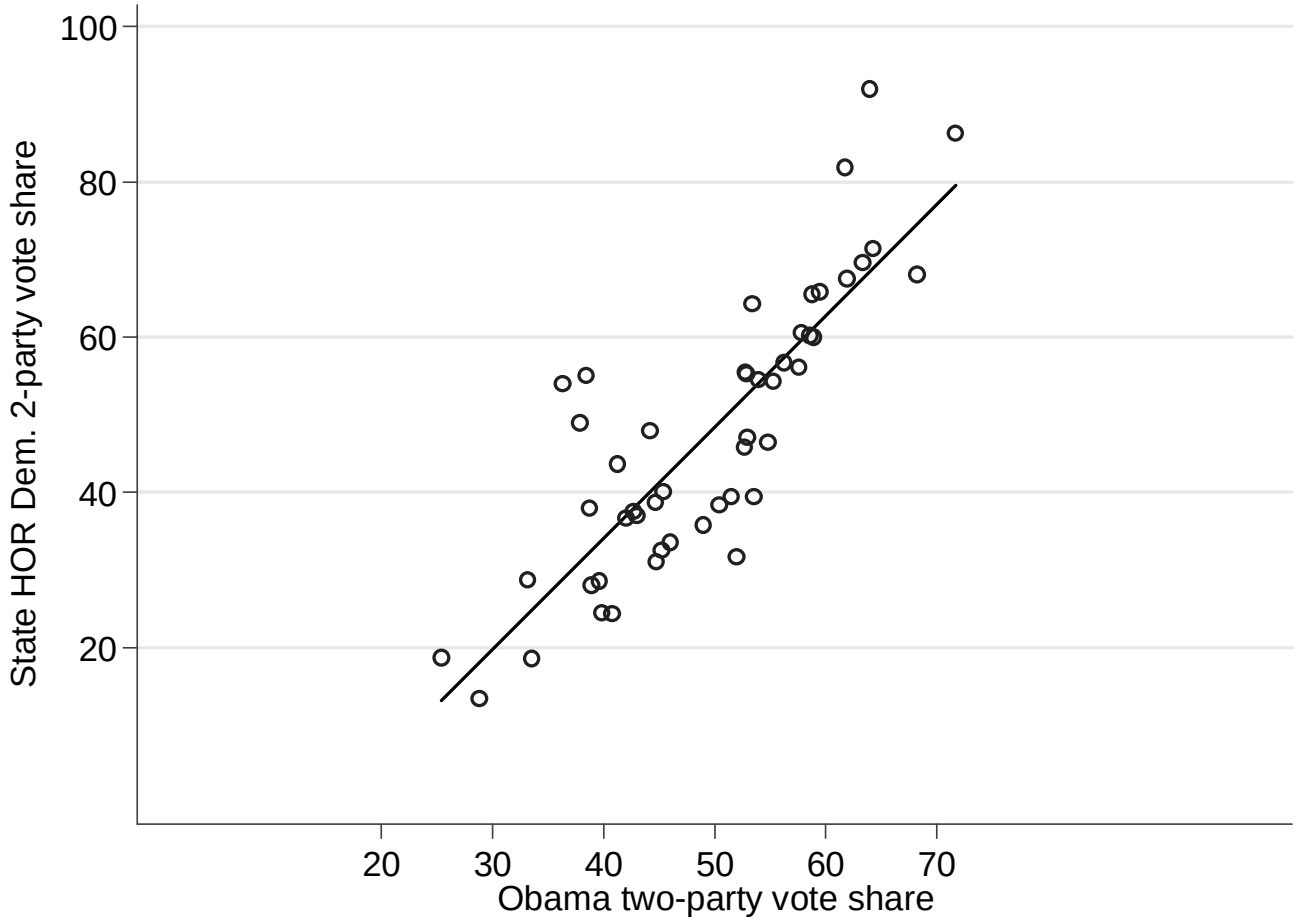
$Y = a + bX + e$	b = dY/dX, or b = the unit change in Y given a unit change in X	Typical case
$Y = a + b \ln X + e$	b = dY/(dX/X), or b = the unit change in Y given a % change in X	Log explanatory variable
$\ln Y = a + bX + e$	b = (dY/Y)/dX, or b = the % change in Y given a unit change in X	Log dependent variable
$\ln Y = a + b \ln X + e$	b = (dY/Y)/(dX/X), or b = the % change in Y given a % change in X (elasticity)	Economic production

Goodness of regression fit

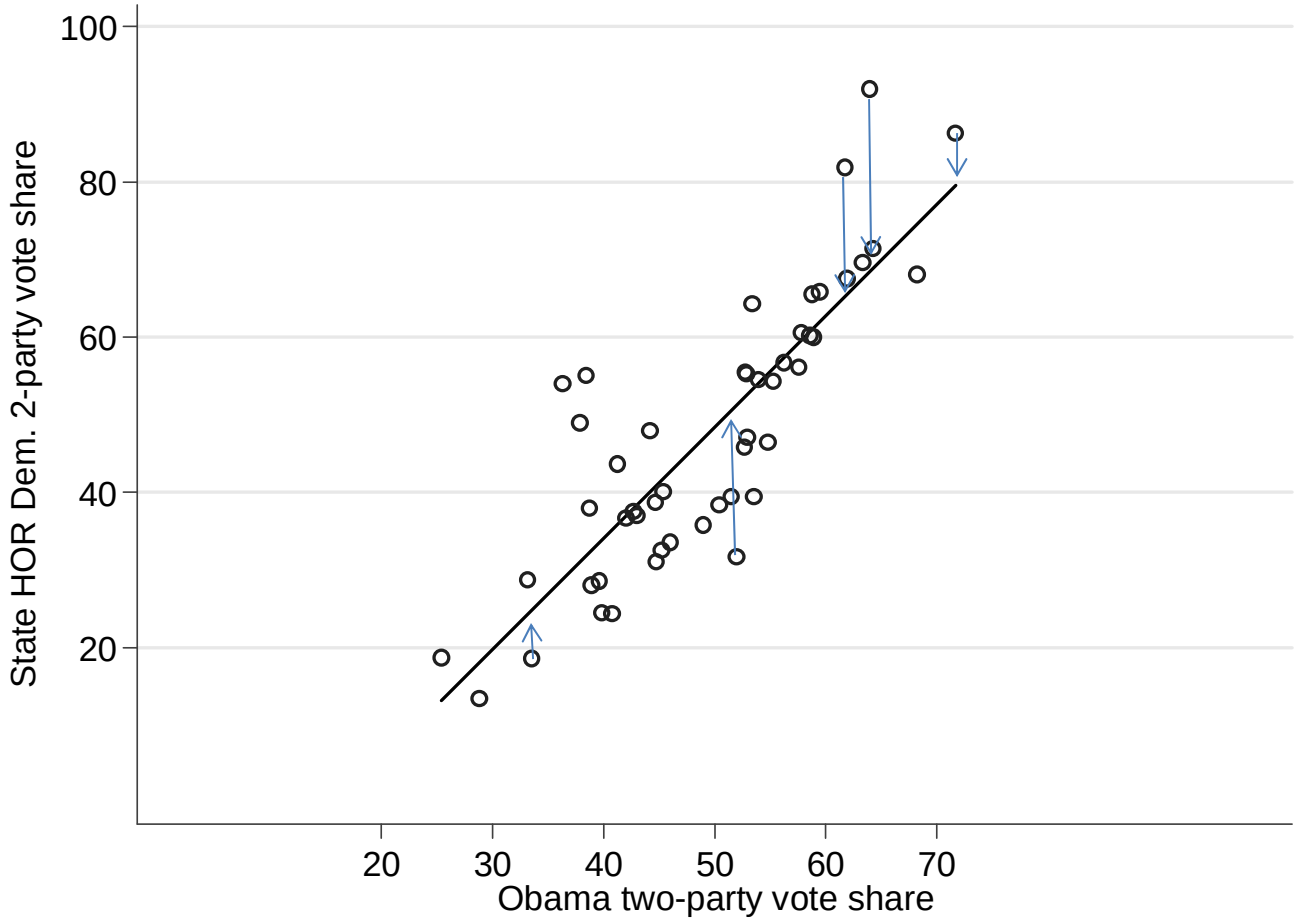
How “good” is the fitted line?

- Goodness-of-fit is often not relevant to research
- Goodness-of-fit receives too much emphasis
- Focus on
  - Substantive interpretation of coefficients (most important)
  - Statistical significance of coefficients (less important)
    - Confidence interval
    - Standard error of a coefficient
    - *t*-statistic: *coeff./s.e.*
- Nevertheless, you should know about
  - Standard Error of the Regression (SER)
    - Standard Error of the Estimate (SEE)
    - Also called Regrettably called Root Mean Squared Error (Root MSE) in Stata
  - R-squared (R^2)
    - Often not informative, use sparingly

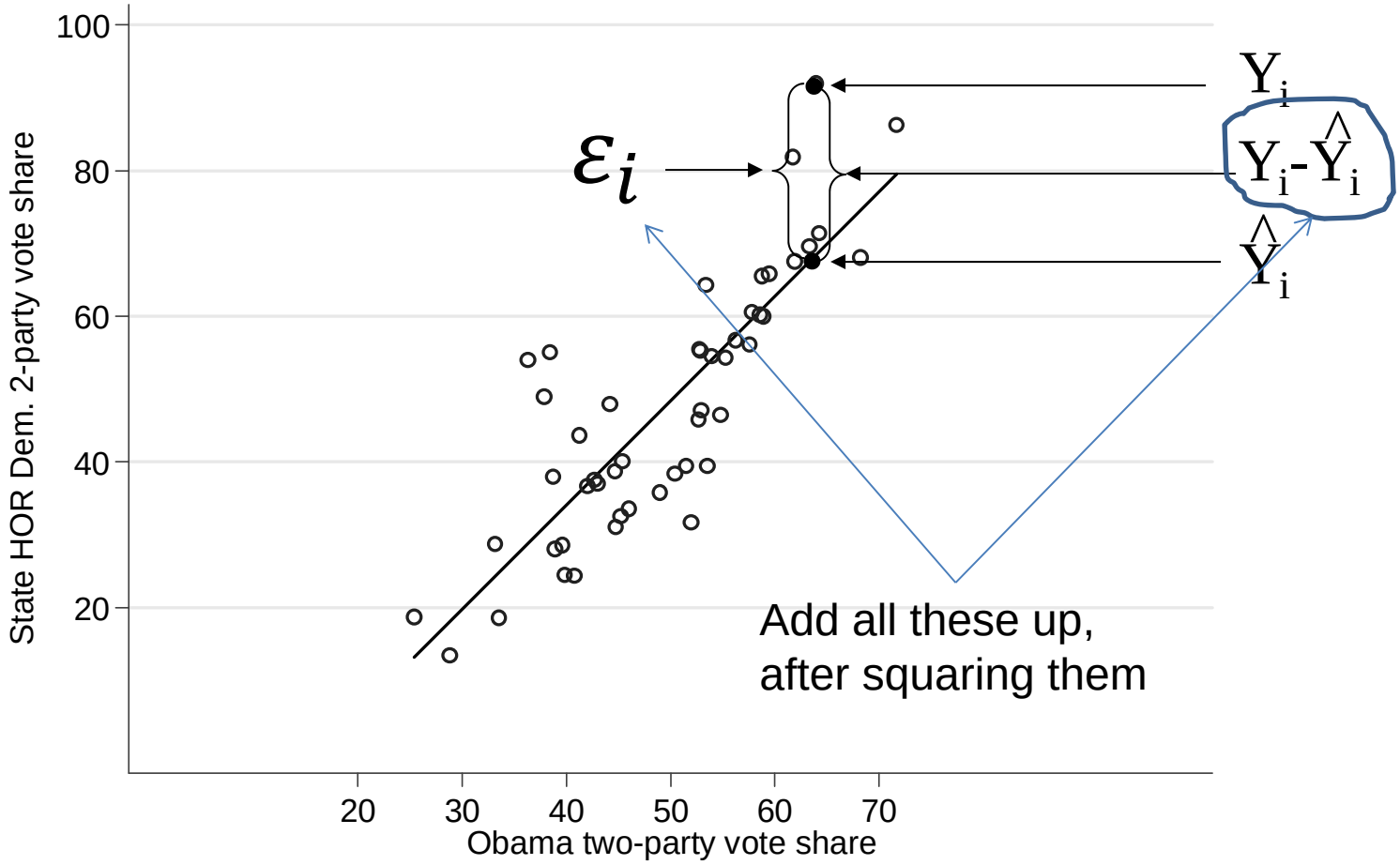
Standard error of the regression line



Standard error of the regression line



Standard error of the regression line



Standard Error of the Regression (SER)

- or Standard Error of the Estimate
- or Root Mean Squared Error (Root MSE)

$$\sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{d.f.}}$$

d.f. equals n minus the number of estimate coefficients (*B*s).
In bivariate regression case, *d.f.* = n-2.

SER interpretation called “Root MSE” in Stata

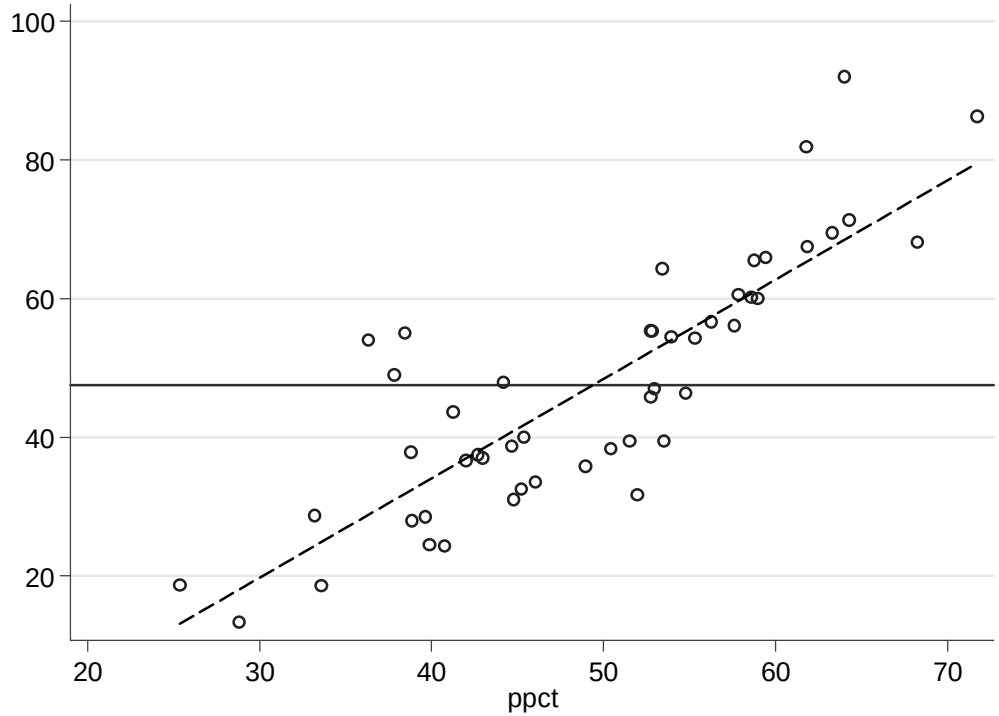
- On average, in-sample predictions will be off the mark by about one standard error of the regression

```
. reg hpct ppct

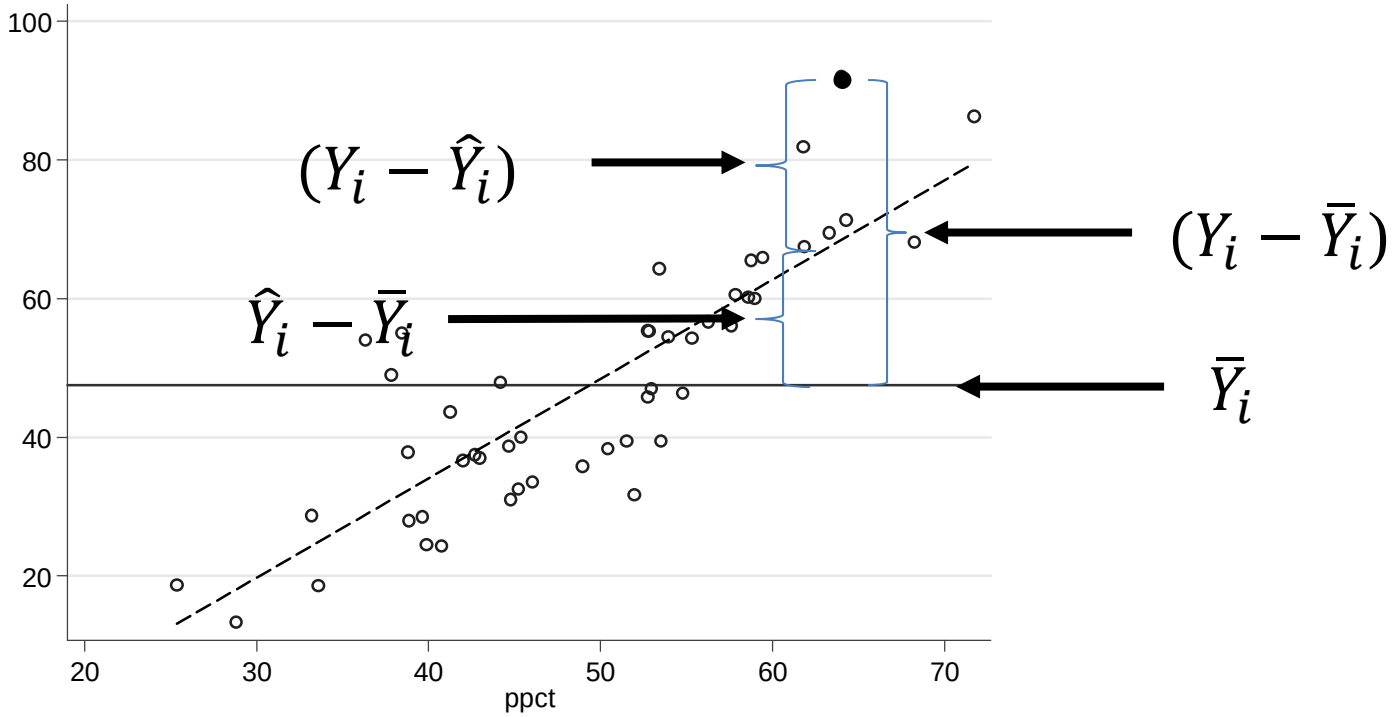
      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  1,    47) =  112.73
       Model |  10808.7878     1  10808.7878           Prob > F      =  0.0000
    Residual |  4506.44332    47  95.8817727           R-squared     =  0.7058
-------------+------------------------------           Adj R-squared =  0.6995
       Total |  15315.2312    48  319.067316           Root MSE      =  9.7919

------------------------------------------------------------------------------
        hpct |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        ppct |   1.433516    .135015    10.62   0.000     1.161901    1.705131
       _cons |  -23.25307   6.809819    -3.41   0.001    -36.95266   -9.553483
------------------------------------------------------------------------------
```

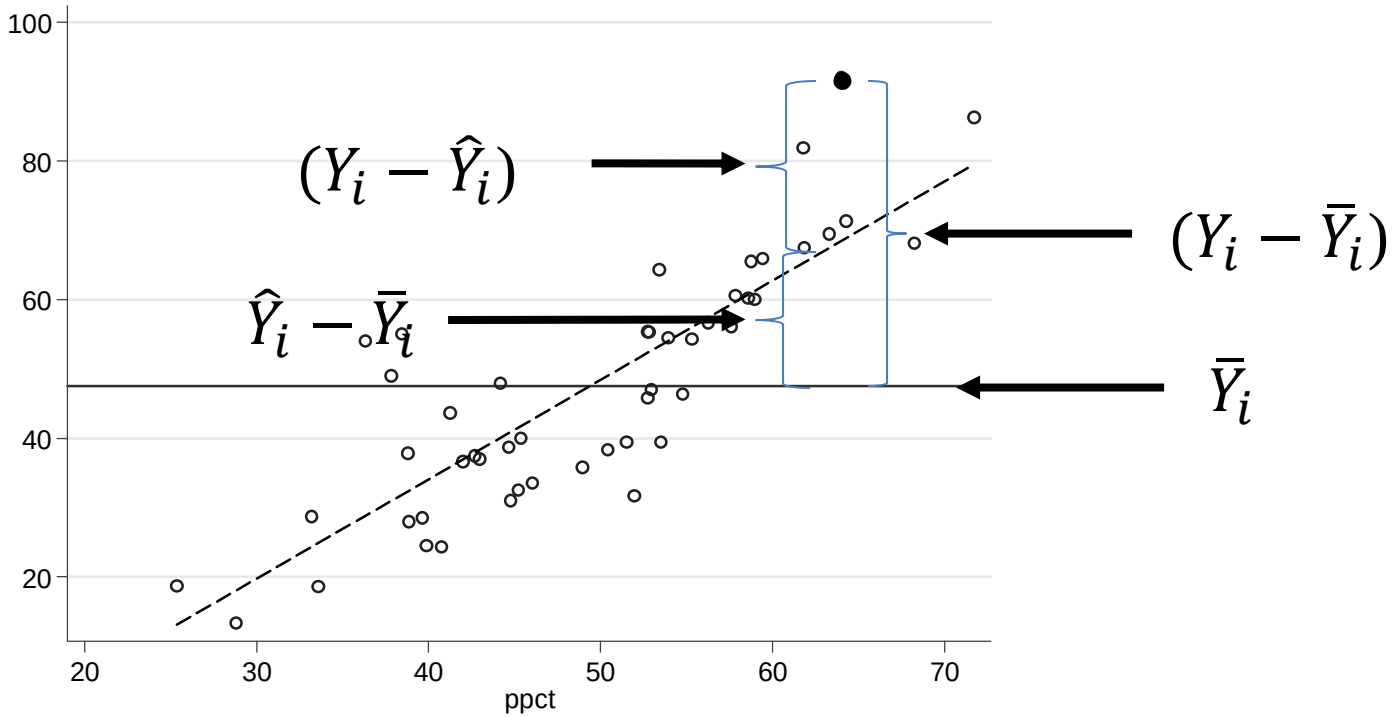
R^2: A less useful measure of fit



R^2: A less useful measure of fit



R^2: A less useful measure of fit

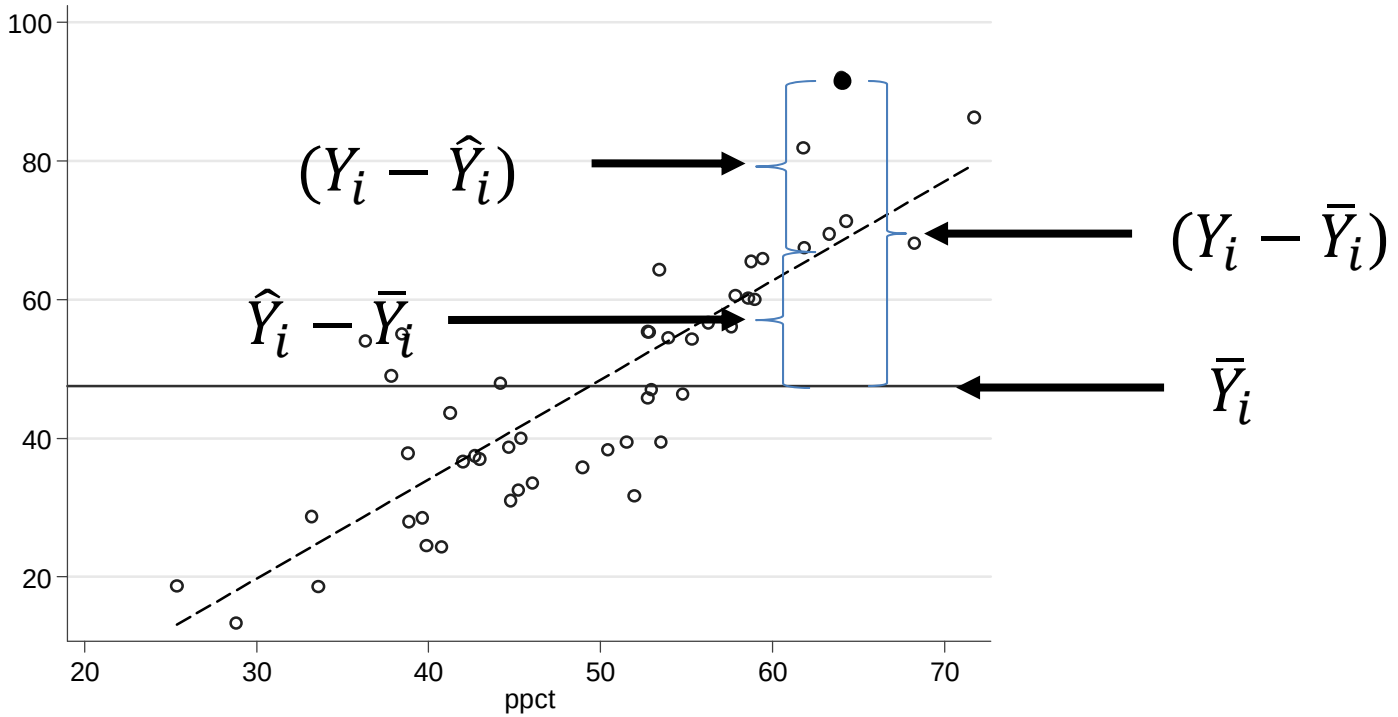


$$\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = "total\ sum\ of\ squares"$$

$$\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2 = "regression\ sum\ of\ squares"$$

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 = "residual\ sum\ of\ squares"$$

R^2: A less useful measure of fit



$$\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = \text{"}total\ sum\ of\ squares\text{"}$$

$$\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2 = \text{"}regression\ sum\ of\ squares\text{"}$$

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 = \text{"}residual\ sum\ of\ squares\text{"}$$

$$r^2 = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \quad \text{or}$$

pct. variance "explained"

Interpreting SER (Root MSE) and R^2

```
. reg hpct ppct

      Source |       SS       df       MS              Number of obs =      49
-------------+------------------------------           F(  1,    47) =  112.73
       Model |  10808.7878     1  10808.7878            Prob > F      =  0.0000
    Residual |  4506.44332    47  95.8817727            R-squared     =  0.7058
-------------+------------------------------           Adj R-squared =  0.6995
       Total |  15315.2312    48  319.067316            Root MSE      =  9.7919

------------------------------------------------------------------------------
        hpct |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        ppct |   1.433516    .135015    10.62   0.000     1.161901    1.705131
       _cons |  -23.25307   6.809819    -3.41   0.001    -36.95266   -9.553483
------------------------------------------------------------------------------
```

Interpreting SER (Root MSE):

- On average, in-sample predictions about the share of a legislature that is Democratic will be off the mark by about 9.8 **percentage points.**

Interpreting R^2

- Regression model explains about 71% of the variation in the Democratic share of legislative seats.

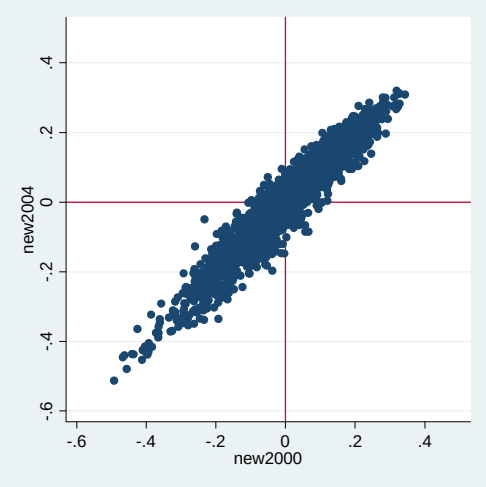
Correlation

Correlation

$$Corr(x,y) = \frac{Cov(x,y)}{\sigma_x \sigma_y} = r$$

$\text{Corr}(\text{BushPct}_{00},\text{BushPct}_{04}) = 0.96 =$

$$\frac{0.014858}{\sqrt{0.01499} \times \sqrt{0.01605}} \approx .96$$



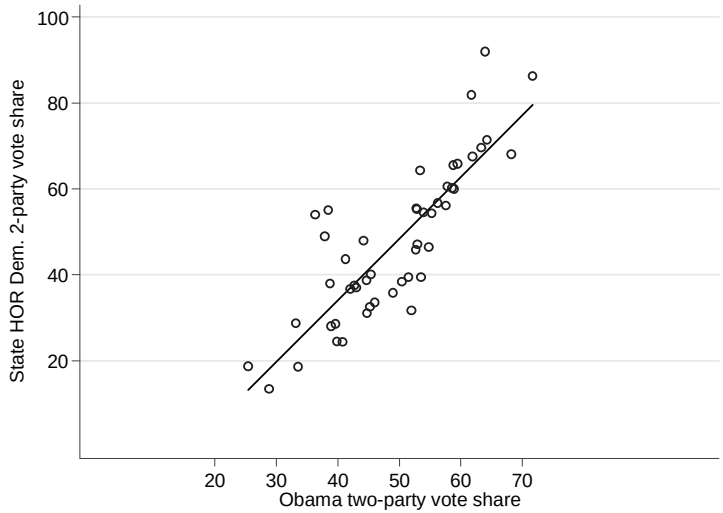
- Measures how closely data points fall along the line
- Varies between -1 and 1

(compare with Tufte p. 102)

Warning: Don’t correlate often!

- Correlation only measures linear relationship
- Correlation is sensitive to variance
- Correlation usually doesn’t measure a theoretically interesting quantity
- Same criticisms apply to R^2, which is the squared correlation between predictions and data points.
- Instead, focus on regression coefficients (slopes)

Sum-up

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-------------+----------------------------------------------------------------
        ppct |   1.433516    .135015    10.62   0.000     1.161901    1.705131
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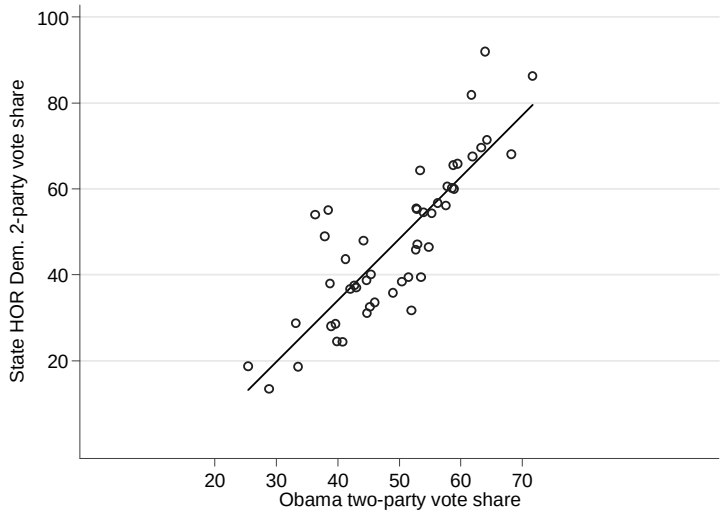
------------------------------------------------------------------------------
```

Sum-up




```
. reg hpct ppct
```

Source	SS	df	MS
Model	10808.7878	1	10808.7878
Residual	4506.44332	47	95.8817727
Total	15315.2312	48	319.067316

Number of obs	**49**
F(1, 47)	112.73
Prob > F	0.0000
R-squared	**0.7058**
Adj R-squared	0.6995
Root MSE	**9.7919**

hpct	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
ppct	**1.433516**	.135015	10.62	0.000	1.161901	1.705131
_cons	**-23.25307**	6.809819	-3.41	0.001	-36.95266	-9.553483