Addressing Alternative Explanations: Multiple Regression

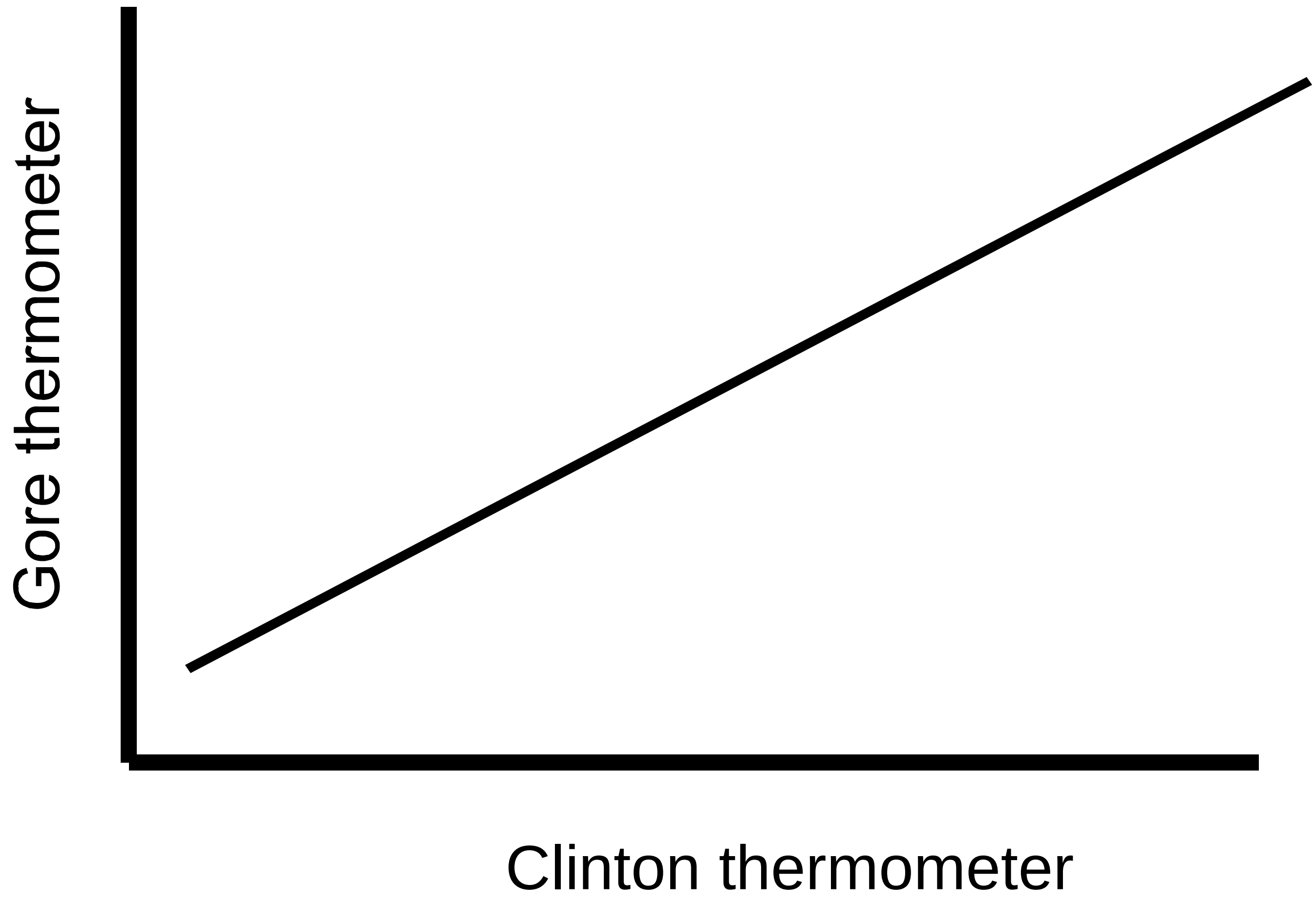
17.871

Spring 2013

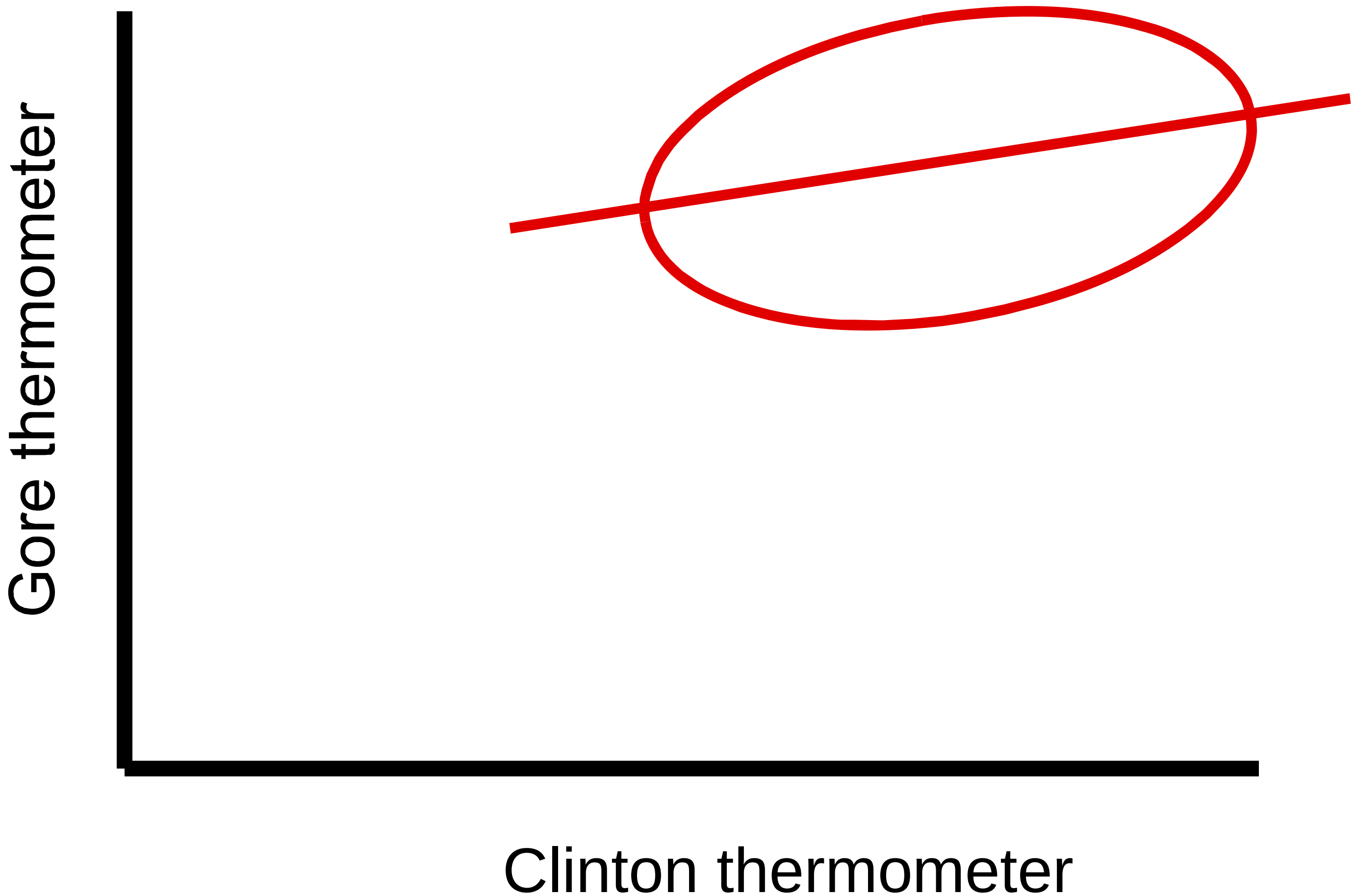
Did Clinton hurt Gore example

- Did Clinton hurt Gore in the 2000 election?
  - Treatment is not liking Bill Clinton

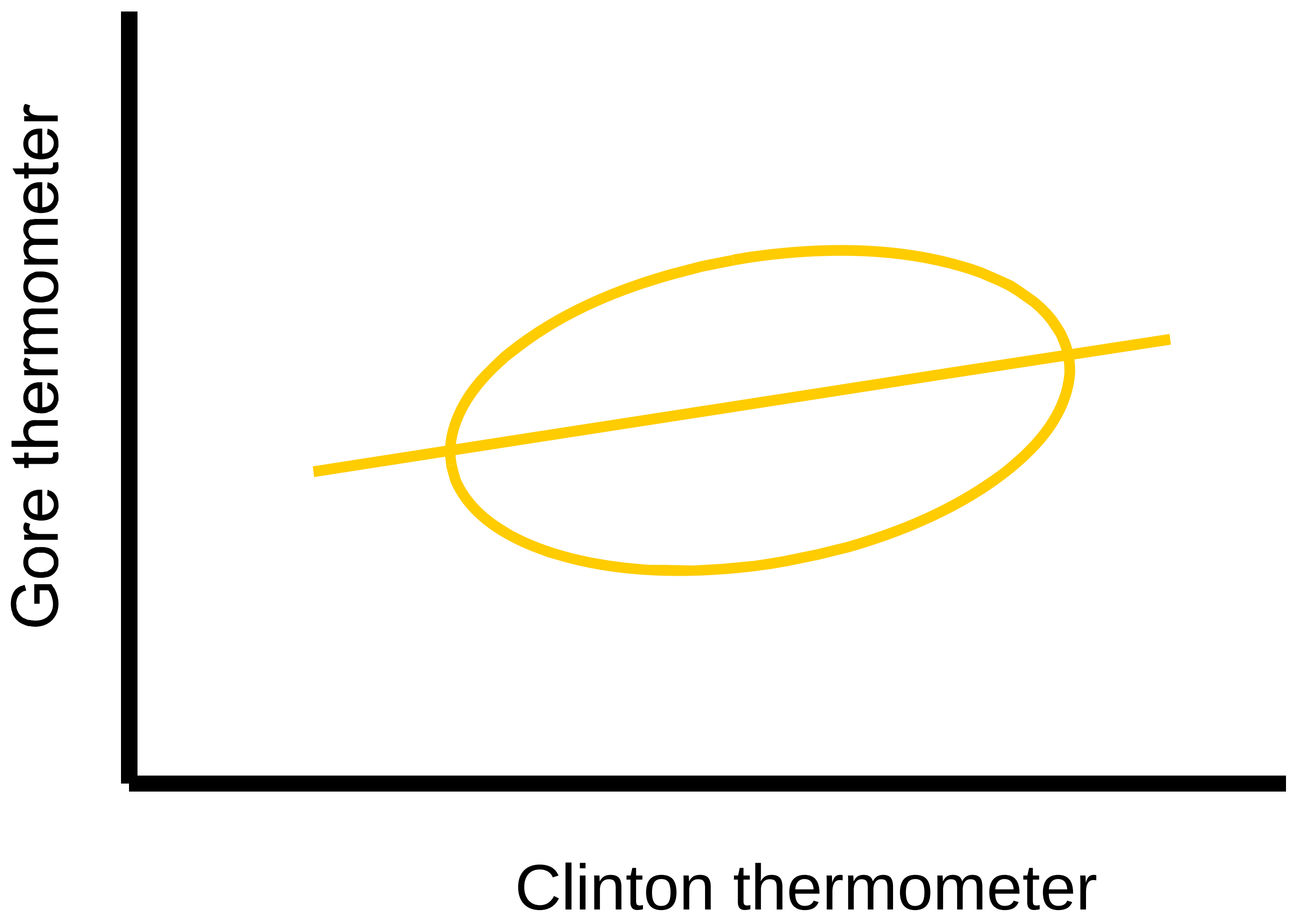
Bivariate regression of Gore thermometer on Clinton thermometer



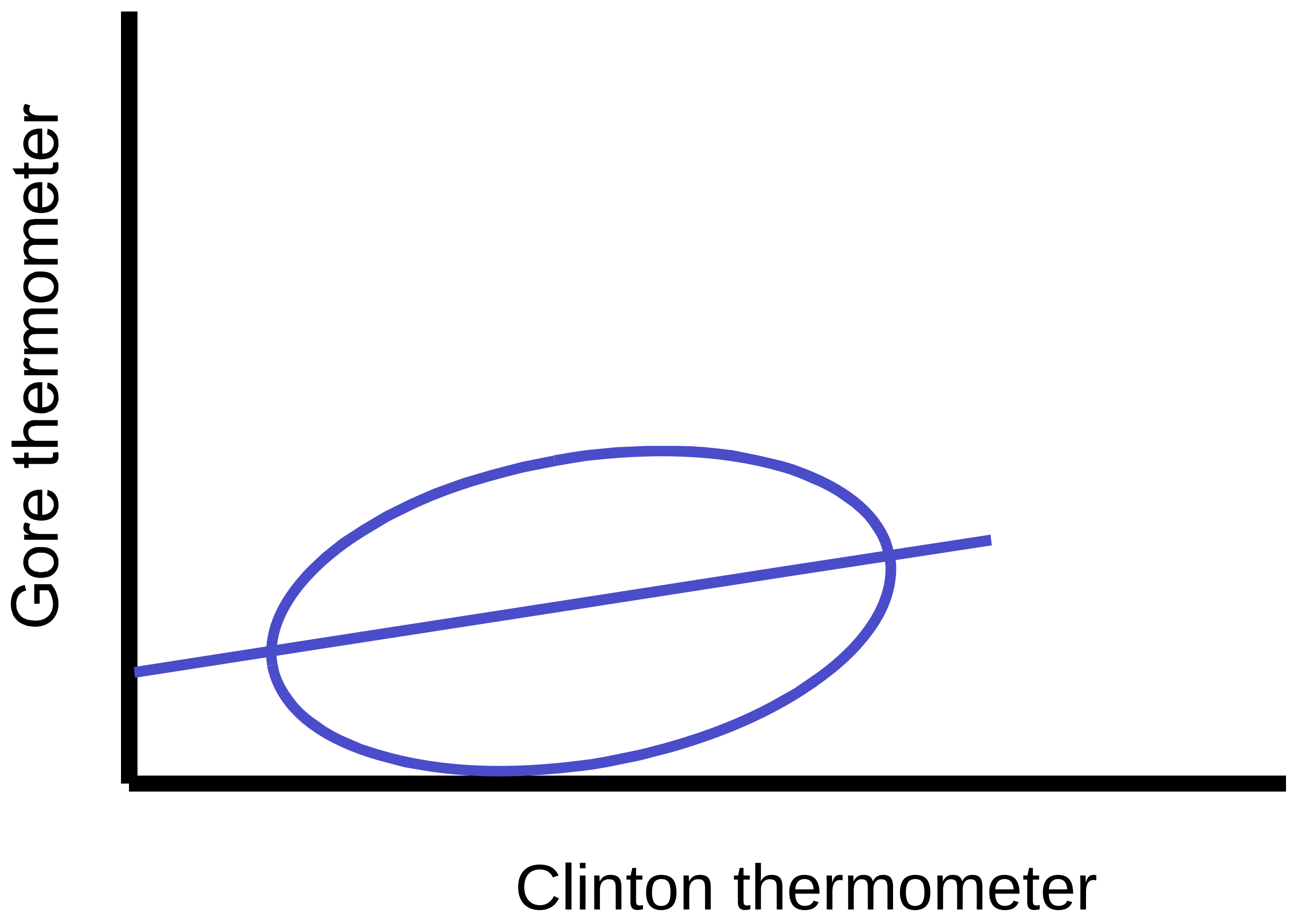
Democratic picture



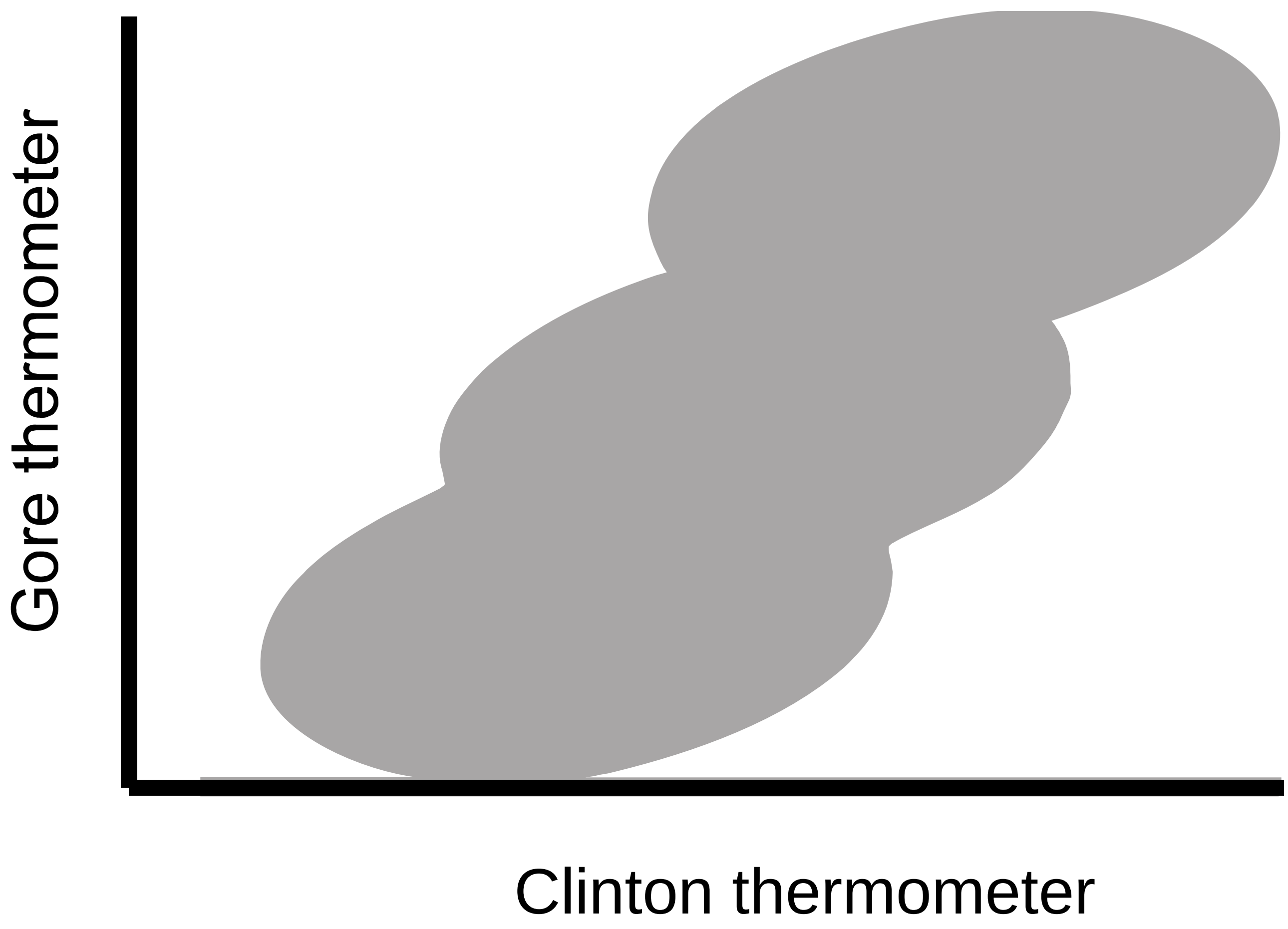
Independent picture



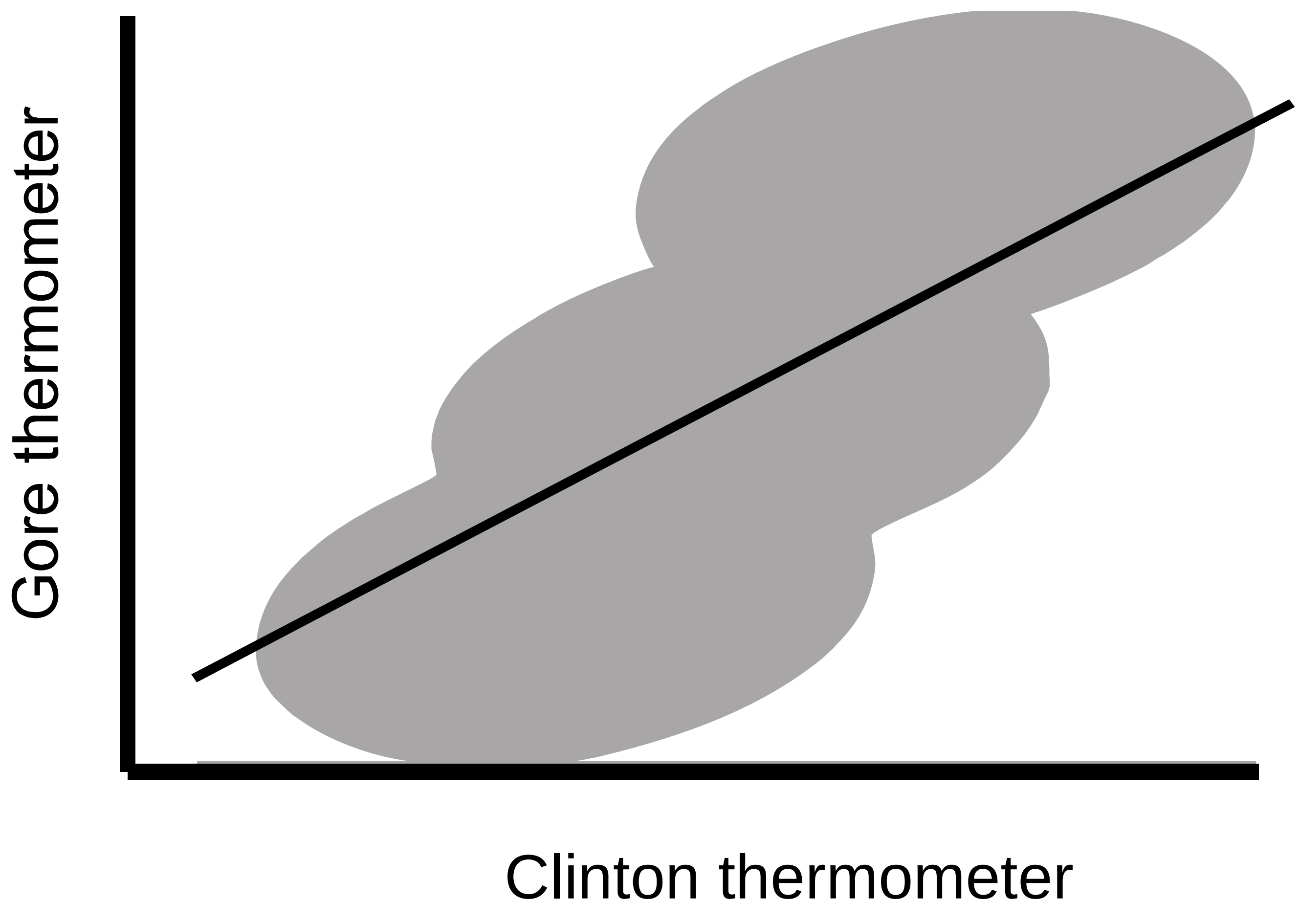
Republican picture



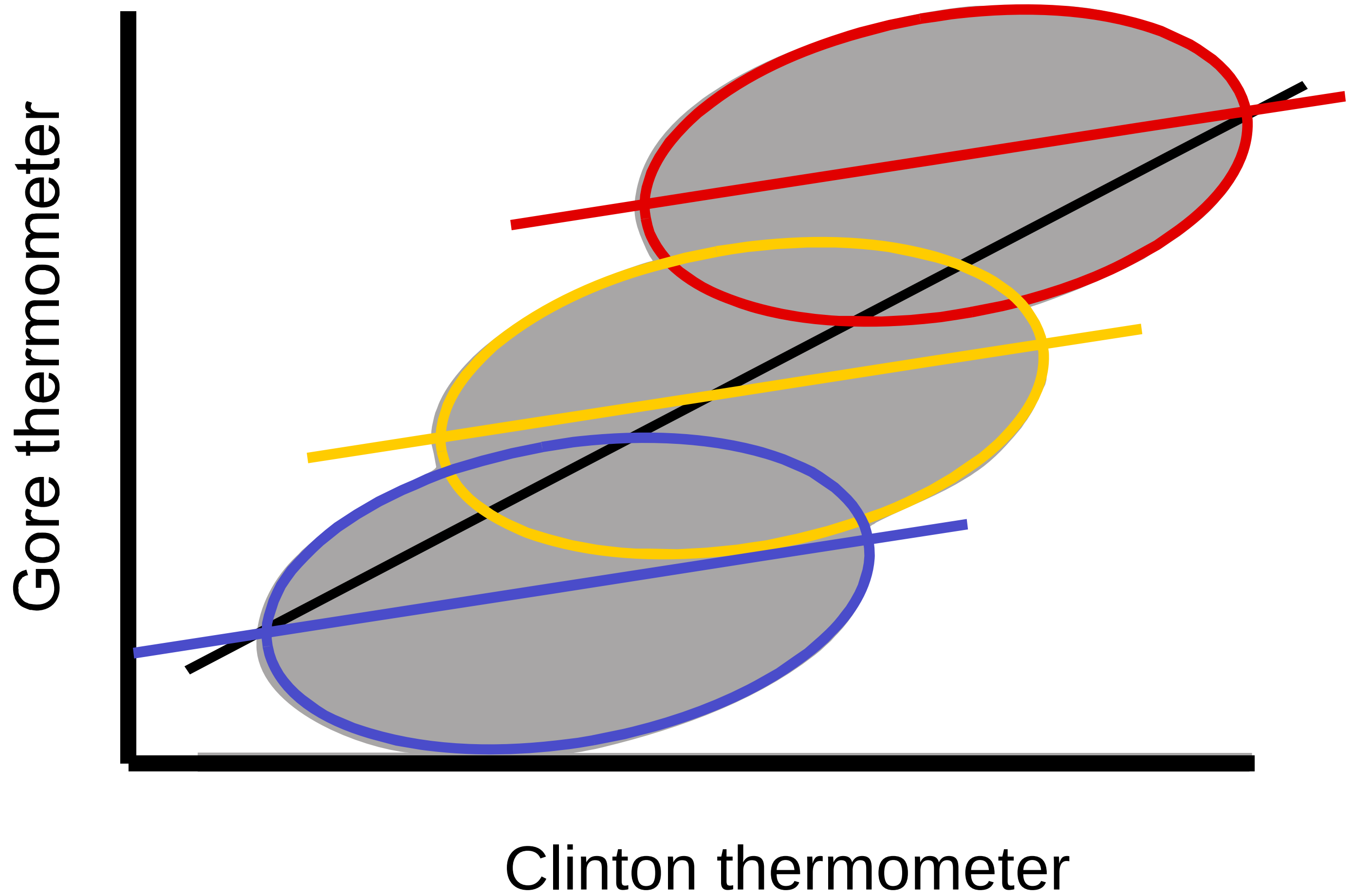
Combined data picture



Combined data picture with regression: bias!

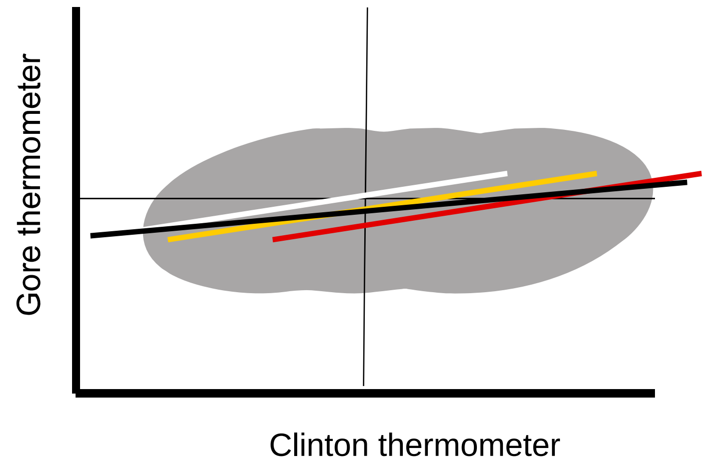


Combined data picture with "true" regression lines overlaid

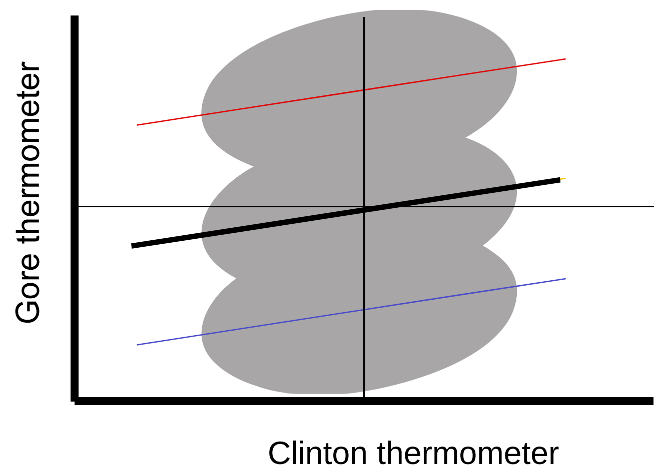


Tempting yet wrong normalizations

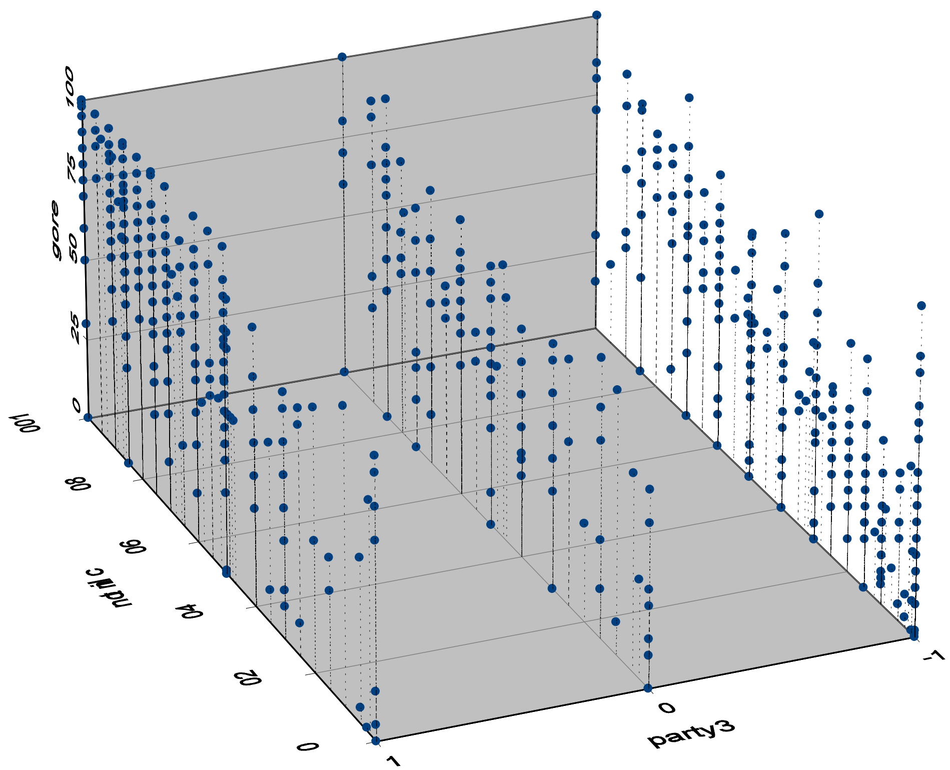
Subtract the Gore therm. from the avg. Gore therm. score



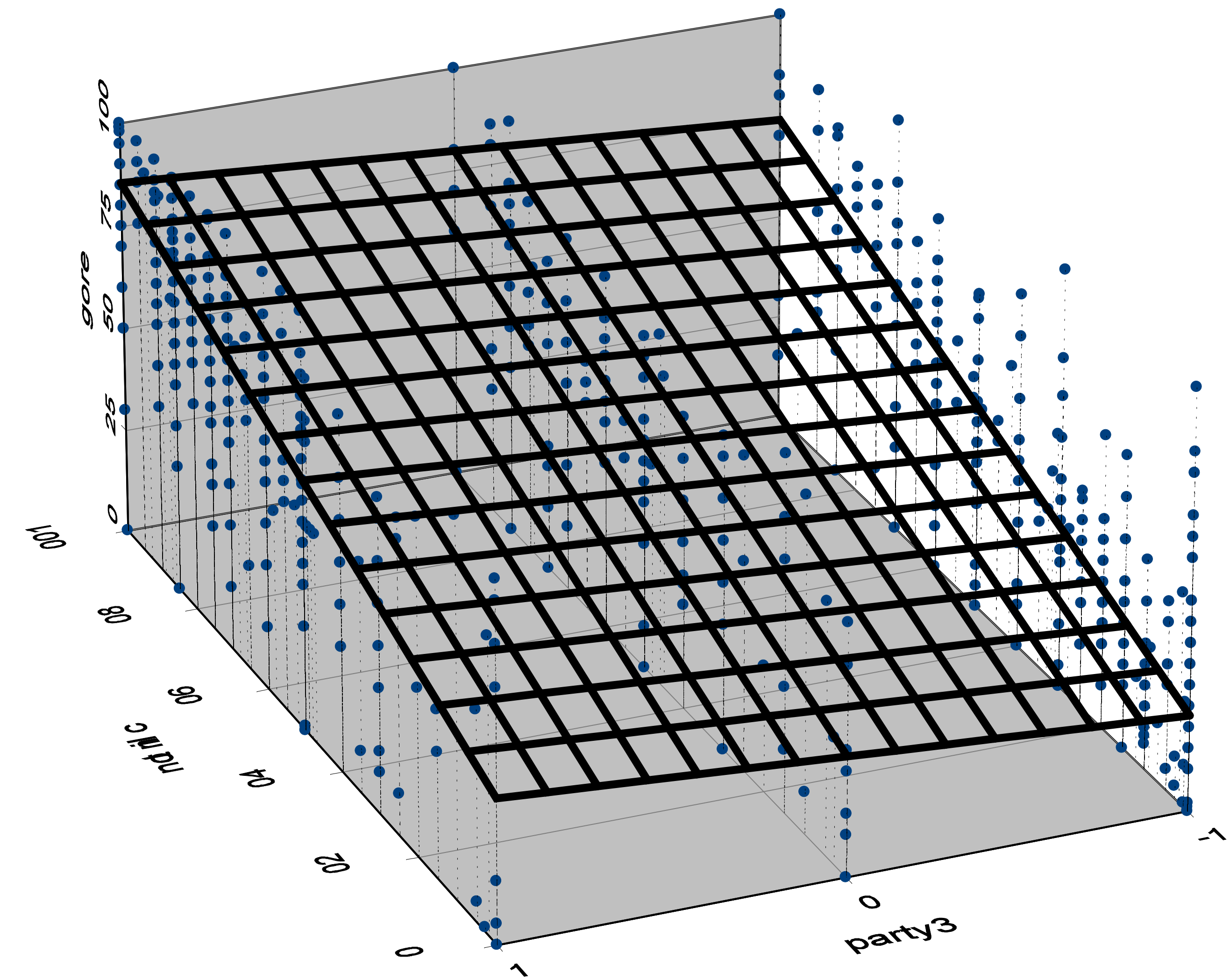
Subtract the Clinton therm. from the avg. Clinton therm. score



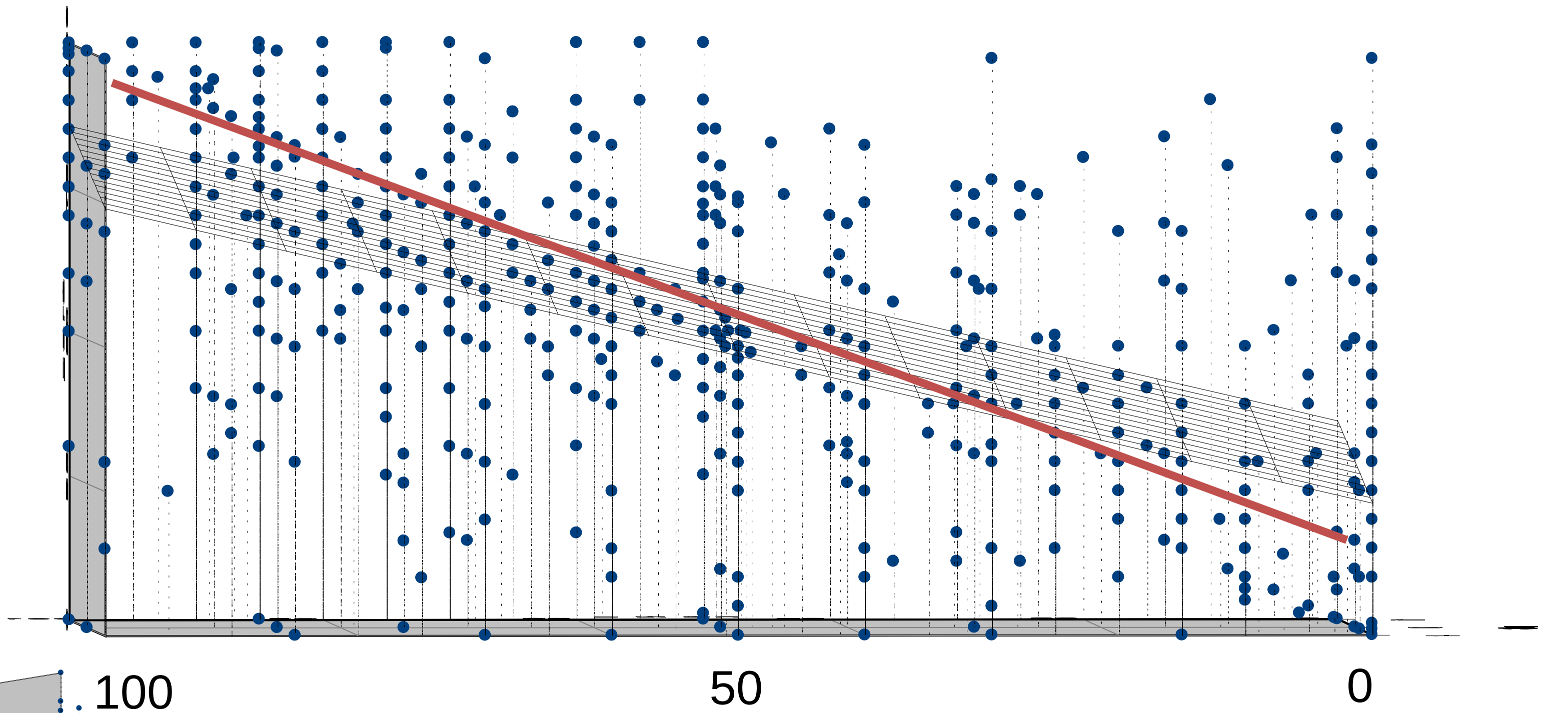
3D Relationship

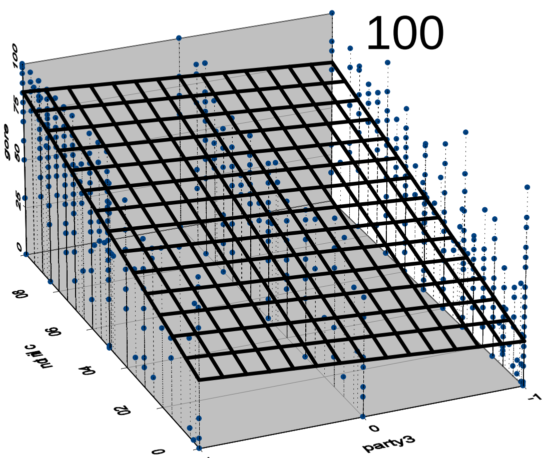


3D Linear Relationship

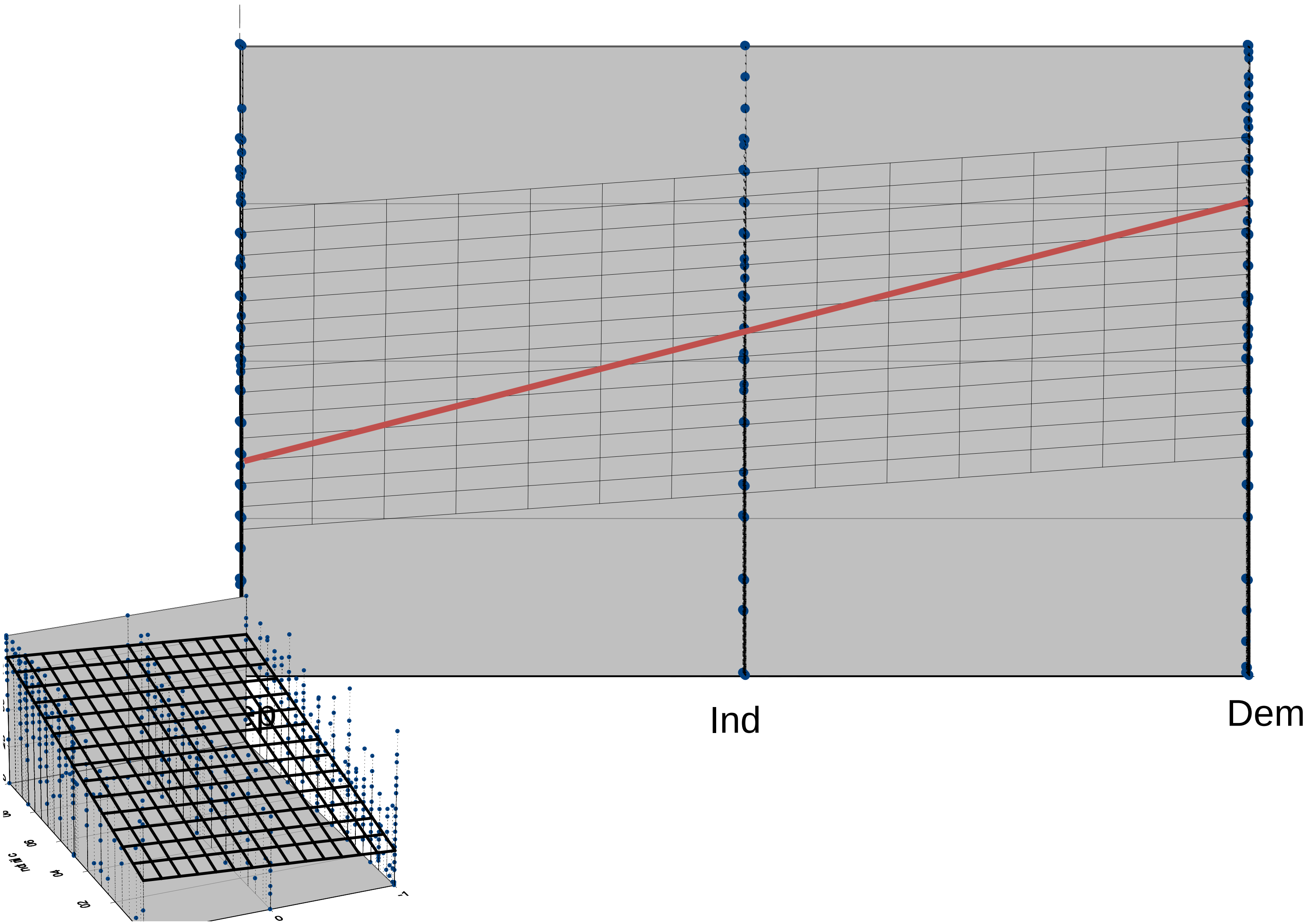


3D Relationship: Clinton





3D Relationship: party



The Linear Relationship between Three Variables

Gore thermometer

Clinton thermometer

Party ID

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i$$

The method of least squares (again)

Pick β_0, β_1, and β_2 to minimize

$$\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \text{ or}$$

$$\sum_{i=1}^{n}(Y_i - \beta_0 - \beta_1 X_i - \beta_2 X_2)^2$$

The Slope Coefficients

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(\overline{Y} - Y_i)(\overline{X}_1 - X_{1,i})}{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})^2} - \hat{\beta}_2 \frac{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})(\overline{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})^2} \text{ and}$$

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(\overline{Y} - Y_i)(\overline{X}_2 - X_{1,i})}{\sum_{i=1}^{n}(\overline{X}_2 - X_{2,i})^2} - \hat{\beta}_1 \frac{\sum_{i=1}^{n}(\overline{X}_1 - X_{1,i})(\overline{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\overline{X}_2 - X_{2,i})^2}$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}_1 - \hat{\beta}_2 \overline{X}_2$$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Slope Coefficients More Simply

$$\hat{\beta}_1 = \frac{\operatorname{cov}(X_1, Y)}{\operatorname{var}(X_1)} - \hat{\beta}_2 \frac{\operatorname{cov}(X_1, X_2)}{\operatorname{var}(X_1)} \text{ and}$$

$$\hat{\beta}_2 = \frac{\operatorname{cov}(X_2, Y)}{\operatorname{var}(X_2)} - \hat{\beta}_1 \frac{\operatorname{cov}(X_1, X_2)}{\operatorname{var}(X_2)}$$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Matrix form

y_1		1	$x_{1,1}$	$x_{2,1}$	…	$x_{k,1}$
y_2		1	$x_{1,2}$	$x_{2,2}$	…	$x_{k,2}$
…		1	…	…	…	…
y_n		1	$x_{1,n}$	$x_{2,n}$	…	$x_{k,n}$

$$\beta = (X'X)^{-1} X'y$$

Multivariate slope coefficients

Clinton effect (on Gore) in bivariate (*B*) regression

Bivariate estimate: $\hat{\beta}_1^B = \dfrac{\operatorname{cov}(X_1, Y)}{\operatorname{var}(X_1)}$ vs.

Are Gore and Party ID related?

Multivariate estimate: $\hat{\beta}_1^M = \dfrac{\operatorname{cov}(X_1, Y)}{\operatorname{var}(X_1)} - \hat{\beta}_2^M \dfrac{\operatorname{cov}(X_1, X_2)}{\operatorname{var}(X_1)}$

Clinton effect (on Gore) in multivariate (*M*) regression

Are Clinton and Party ID related?

When does $\hat{\beta}_1^B = \hat{\beta}_1^M$? Obviously, when $\hat{\beta}_2^M \dfrac{\operatorname{cov}(X_1, X_2)}{\operatorname{var}(X_1)} = 0$

X_1 is Clinton thermometer, X_2 is PID, and Y is Gore thermometer

The Output

```
. reg gore clinton party3

      Source |       SS       df       MS              Number of obs =    1745
-------------+------------------------------           F(  2,  1742) = 1048.04
       Model |  629261.91     2  314630.955           Prob > F      =  0.0000
    Residual |  522964.934  1742  300.209492           R-squared     =  0.5461
-------------+------------------------------           Adj R-squared =  0.5456
       Total |  1152226.84  1744   660.68053           Root MSE      =  17.327

------------------------------------------------------------------------------
        gore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     clinton |   .5122875   .0175952    29.12   0.000     .4777776    .5467975
      party3 |   5.770523   .5594846    10.31   0.000     4.673191    6.867856
       _cons |    28.6299   1.025472    27.92   0.000     26.61862    30.64119
------------------------------------------------------------------------------
```

Interpretation of `clinton` effect: *Holding constant party identification*, a one-point increase in the Clinton feeling thermometer is associated with a .51 increase in the Gore thermometer.

Separate regressions

	(1)	(2)	(3)
Intercept	23.1	55.9	28.6
Clinton	0.62	--	0.51
Party	--	15.7	5.8

$$\hat{\beta}_1 = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} \text{ and}$$

$$\hat{\beta}_2 = \frac{\text{cov}(X_2, Y)}{\text{var}(X_2)} - \hat{\beta}_1 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_2)}$$

Why did the Clinton Coefficient change from 0.62 to 0.51

```
. corr gore clinton party, cov
(obs=1745)

             |     gore  clinton   party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993  883.182
      party3 |  13.7008   16.905    .8735
```

The Calculations

$$\hat{\beta}_1^B = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} = \frac{549.993}{883.182} = 0.6227$$

$$\hat{\beta}_1^M = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} - \hat{\beta}_2^M \frac{\text{cov}(clinton, party)}{\text{var}(clinton)}$$

$$= \frac{549.993}{883.182} - 5.7705 \frac{16.905}{883.182}$$

$$= 0.6227 - 0.1105$$

$$= 0.5122$$

```
. corr gore clinton party,cov
(obs=1745)

             |     gore  clinton   party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993  883.182
      party3 |  13.7008   16.905    .8735
```

Another way of thinking about this

Rewrite

$$\hat{\beta}_1^M = \frac{\operatorname{cov}(gore, clinton)}{\operatorname{var}(clinton)} - \hat{\beta}_2^M \frac{\operatorname{cov}(clinton, party)}{\operatorname{var}(clinton)}$$

as

$$\frac{\operatorname{cov}(gore, clinton)}{\operatorname{var}(clinton)} = \hat{\beta}_1^M + \hat{\beta}_2^M \frac{\operatorname{cov}(clinton, party)}{\operatorname{var}(clinton)}$$

Total effect = Direct effect + indirect effect

The Total Effect of the Clinton thermometer on the Gore thermometer (.61) can be Broken down into a direct effect of .51, plus an indirect effect (though party) of .11

Graphical way of thinking about this

Total effect

$$\text{Clinton feeling thermometer} \xrightarrow{\hat{\beta}_1^B} \text{Gore feeling thermometer}$$

Graphical way of thinking about this

Total effect

$\hat{\beta}_1^B$

Clinton feeling thermometer $\longrightarrow$ Gore feeling thermometer

Can be broken down into:

Direct effect

$\hat{\beta}_1^M$

Clinton feeling thermometer $\dashrightarrow$ Gore feeling thermometer

$$\frac{\mathrm{cov}(gore, clinton)}{\mathrm{var}(clinton)} = \gamma_{2,1}$$

Party ID

$\hat{\beta}_2^M$

Drinking and Greek Life Example

- Why is there a correlation between living in a fraternity/sorority house and drinking?
  - Greek organizations often emphasize social gatherings that have alcohol. The effect is being in the Greek organization itself, not the house.
  - There's something about the House environment itself.

Dependent variable: Times Drinking in Past 30 Days

C8. When did you last have a drink (that is more than just a few sips)?

- ◯ I have never had a drink ⟶ Skip to **C22** (page 10)
- ◯ Not in the past year ⟶ Skip to **C22** (page 10)
- ◯ More than 30 days ago, but in the past year ⟶ Skip to **C17** (page 8)
- ◯ More than a week ago, but in the past 30 days ⟶ Go to **C9**
- ◯ Within the last week ⟶ Go to **C9**

C9. On how many occasions have you had a drink of alcohol in the <u>past 30 days</u>? (Choose one answer.)

- 1 ◯ Did not drink in the last 30 days
- 2 ◯ 1 to 2 occasions
- 3 ◯ 3 to 5 occasions
- 4 ◯ 6 to 9 occasions
- 5 ◯ 10 to 19 occasions
- 6 ◯ 20 to 39 occasions
- 7 ◯ 40 or more occasions

```
. infix age 10-11 residence 16 greek 24 screen 102
timespast30 103 howmuchpast30 104 gpa 278-279 studying 281
timeshs 325 howmuchhs 326 socializing 283 stwgt_99 475-493
weight99 494-512 using da3818.dat,clear
(14138 observations read)

. recode  timespast30 timeshs (1=0) (2=1.5) (3=4) (4=7.5)
(5=14.5) (6=29.5) (7=45)
(timespast30: 6571 changes made)
(timeshs: 10272 changes made)

. replace timespast30=0 if screen<=3
(4631 real changes made)
```

```
. tab timespast30

timespast30 |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      4,652       33.37       33.37
        1.5 |      2,737       19.64       53.01
          4 |      2,653       19.03       72.04
        7.5 |      1,854       13.30       85.34
       14.5 |      1,648       11.82       97.17
       29.5 |        350        2.51       99.68
         45 |         45        0.32      100.00
------------+-----------------------------------
      Total |     13,939      100.00
```

Key explanatory variables

- Live in fraternity/sorority house
  - Indicator variable (dummy variable)
  - Coded 1 if live in, 0 otherwise
- Member of fraternity/sorority
  - Indicator variable (dummy variable)
  - Coded 1 if member, 0 otherwise

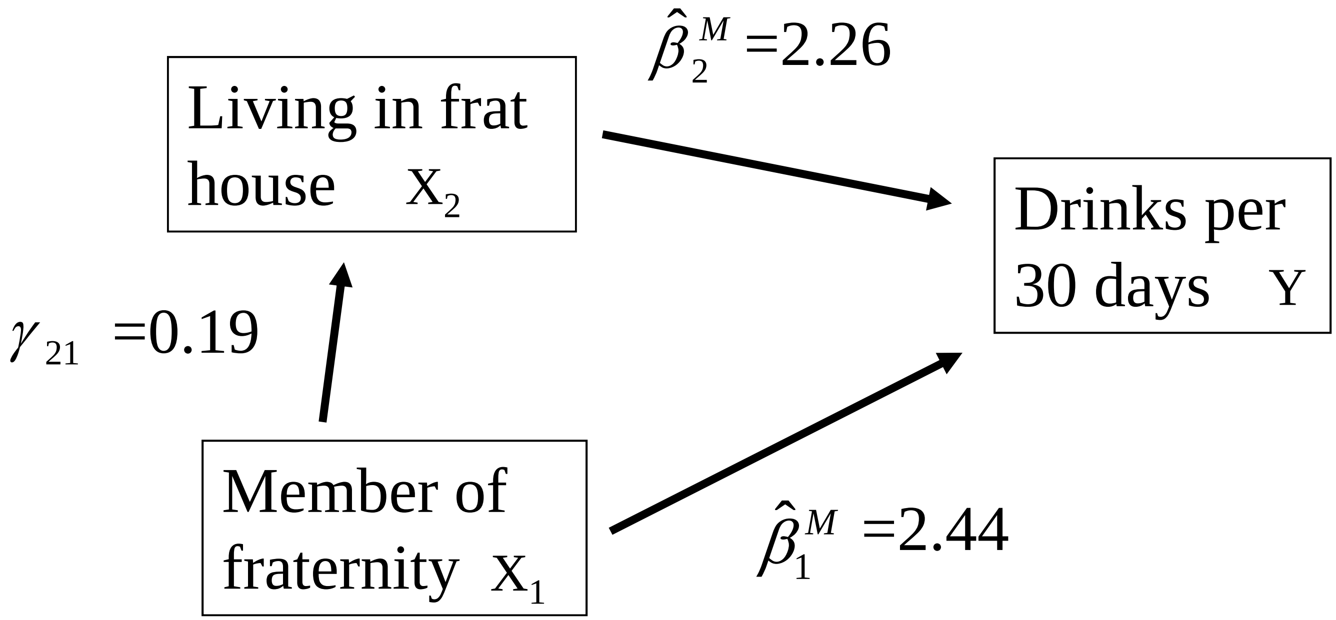
Three Regressions

Dependent variable: number of times drinking in past 30 days			
Live in frat/sor house (indicator variable)	4.44 (0.35)	---	2.26 (0.38)
Member of frat/sor (indicator variable)	---	2.88 (0.16)	2.44 (0.18)
Intercept	4.54 (0.56)	4.27 (0.059)	4.27 (0.059)
S.E.R.	6.49	6.44	6.44
R^2	.011	.023	.025
N	13,876	13,876	13,876

What is the substantive interpretation of the coefficients?

Note: Standard errors in parentheses. Corr. Between living in frat/sor house and being a member of a Greek organization is .42

The Picture



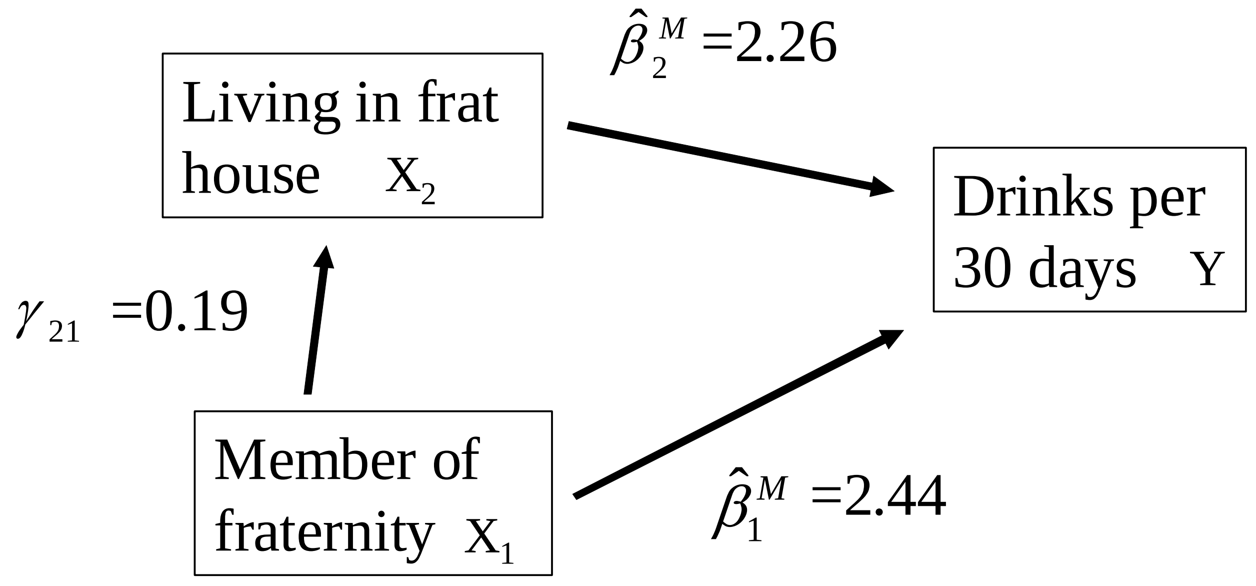
Remember that:

$$\hat{\beta}_1^B = 2.88$$

Accounting for the total effect

$$\hat{\beta}_1^B = \hat{\beta}_1^M + \hat{\beta}_2^M \gamma_{21}$$

Total effect = Direct effect + indirect effect



Accounting for the effects of frat house living and Greek membership on drinking

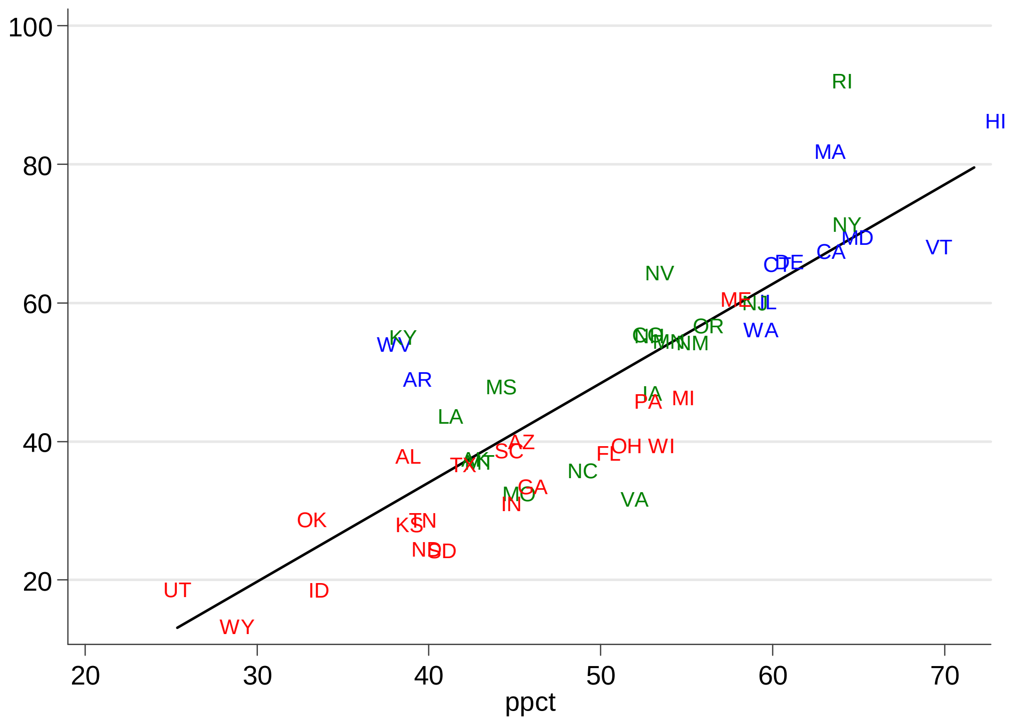
From bivariate regressions

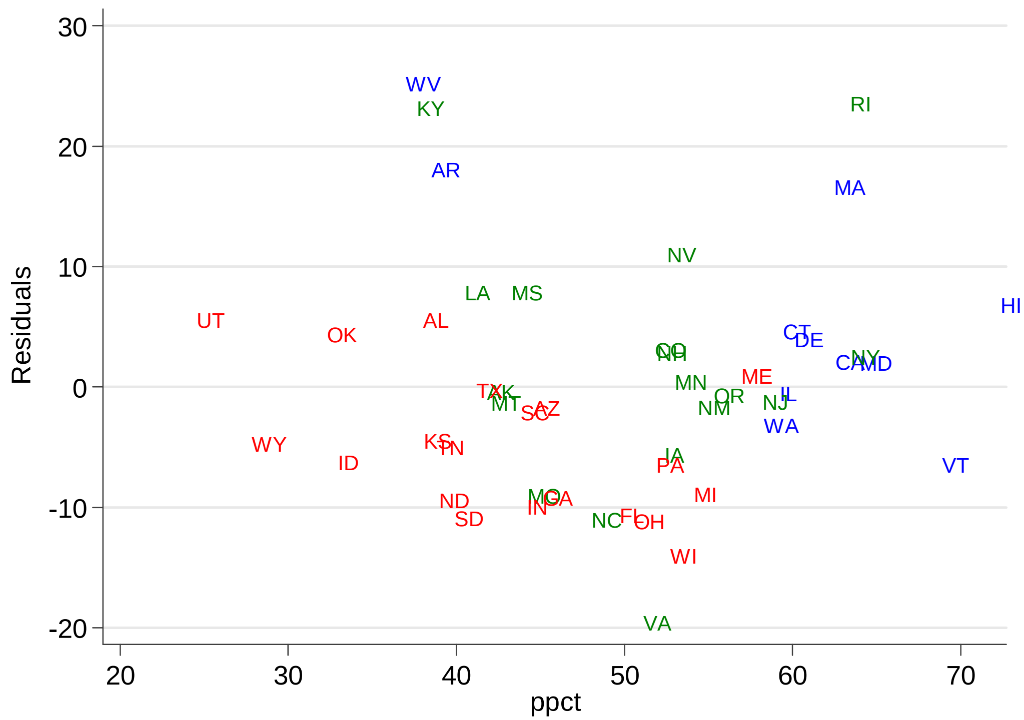
From multiple regressions

From accounting identity: T=D+I

Effect	Total	Direct	Indirect
Member of Greek org.	2.88	2.44 (85%)	0.44 (15%)
Live in frat/ sor. house	4.44	2.26 (51%)	2.18 (49%)

Return to the state legislative example





Residuals
ppct
30
20
10
0
-10
-20
20
30
40
50
60
70
WV
KY
RI
AR
MA
NV
LA
MS
HI
UT
OK
AL
CT
DE
CO
NH
NY
CA
MD
ME
MN
TX
AK
MT
AZ
SC
OR
NM
NJ
IL
WA
WY
ID
KS
TN
IA
PA
VT
MI
ND
MO
GA
IN
SD
NC
FL
OH
WI
VA

```
. list state ry after10 in 1/10

     +-------------------------------------+
     |         state          ry   after10 |
     |-------------------------------------|
  1. | West Virginia    25.17959         1 |
  2. |  Rhode Island    23.48404         0 |
  3. |      Kentucky    23.12402         0 |
  4. |      Arkansas    18.00081         1 |
  5. | Massachusetts    16.55731         1 |
     |-------------------------------------|
  6. |        Nevada    10.97821         0 |
  7. |   Mississippi    7.828268         0 |
  8. |     Louisiana    7.805305         0 |
  9. |        Hawaii     6.73896         1 |
 10. |          Utah    5.545974        -1 |
     +-------------------------------------+
```

```
+-------------------------------------+
     |          state          ry   after10 |
     |---------------------------------------|
 41. |        Georgia   -9.231257        -1 |
 42. |   North Dakota   -9.460065        -1 |
 43. |        Indiana   -9.967912        -1 |
 44. |        Florida   -10.72338        -1 |
 45. |   South Dakota   -10.92215        -1 |
     |---------------------------------------|
 46. | North Carolina   -11.10709         0 |
 47. |           Ohio   -11.19955        -1 |
 48. |      Wisconsin   -14.06958        -1 |
 49. |       Virginia   -19.61035         0 |
     +---------------------------------------+
```

	(1)	(2)	(3)
Obama vote	1.43 (0.14)	---	1.09 (0.14)
Dem. state	---	16.33 (2.33)	8.25 (1.82)
Intercept	-23.25 (6.81)	50.51 (1.85)	-4.93 (7.01)
N	49	49	49
S.E.R.	9.79	12.62	8.23
R^2	.71	.51	.80

Implications of for model-building

- Q: When do you decide whether to “control for” another variable?
  - A1: When excluding another variable(s) would lead to a biased estimate of the effect you are interested in
    - The omitted variable is correlated with the independent variable of interest **and**
    - The omitted variable is also related (statistically) to the dependent variable.*
  - A2: When theory or the question tell you to (e.g. the role of race in American politics)
  - A3: to deal with efficiency (covered after spring break)

*If you don’t do this, you commit **omitted variables bias**

Standardized regression

- Used to try and judge which variables are "more important" in a multiple regression
- Other standardizations are possible (e.g., putting all variables into a 0,1 interval)
- Less informative than regressing on raw values or the 0,1 interval, but is useful to know about because it is so common.

The idea

- Transform **every** variable according to the following formula:

$$newvar = \frac{oldvar - \overline{oldvar}}{\sigma_{oldvar}}$$

- Do the regression on these “z-scores”
  - The intercept drops away
  - In bivariate regression, the standardized coefficient is equal to the correlation coefficient
  - The coefficients are sometimes called “BETA” coefficients (very confusingly)

Example: Knowledge of party control of Congress

- Who knows which party controls the House?
- Variables:
  - **know_reps:** =1 if the R knows the House is controlled by the Reps, 0 otherwise
  - **lfaminc** = (recoded) family income, in thousands
  - **educ** = education completed, in years (recoded from categorical variable

Summary statistics

```
. summ know_reps lf educ_new [aw=v102]

    Variable |     Obs      Weight        Mean   Std. Dev.       Min        Max
-------------+-----------------------------------------------------------------
   know_reps |     842  842.842355    .6431059   .4793679          0          1
     lfaminc |     878  896.217452    4.066185   1.069957   1.609438   6.907755
    educ_new |    1000        1000    13.27775   2.483685          8         18
```

Comparison of regular regression and standardized regression

```
. reg know_reps lf educ_new [aw=v102],beta
(sum of wgt is   7.5562e+02)

      Source |       SS       df       MS              Number of obs =     734
-------------+------------------------------           F(  2,   731) =   31.39
       Model |  13.2004642     2  6.60023208           Prob > F      =  0.0000
    Residual |    153.7241   731  .210292886           R-squared     =  0.0791
-------------+------------------------------           Adj R-squared =  0.0766
       Total |  166.924564   733  .227727918           Root MSE      =  .45858

------------------------------------------------------------------------------
   know_reps |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
     lfaminc |    .046186   .0158845     2.91   0.004                   .10494
    educ_new |   .0469317   .0069827     6.72   0.000                 .2425739
       _cons |  -.1633859   .1046256    -1.56   0.119                        .
------------------------------------------------------------------------------
```

Comparison of 0-1 normalization and standardized regression

```
. reg know_reps lfaminc01 educ_new01 [aw=v102],beta
(sum of wgt is   7.5562e+02)

      Source |       SS       df       MS              Number of obs =     734
-------------+------------------------------           F(  2,   731) =   31.39
       Model |  13.2004641     2  6.60023203           Prob > F      =  0.0000
    Residual |    153.7241   731  .210292886           R-squared     =  0.0791
-------------+------------------------------           Adj R-squared =  0.0766
       Total |  166.924564   733  .227727918           Root MSE      =  .45858

------------------------------------------------------------------------------
   know_reps |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
   lfaminc01 |   .2447083   .0841609     2.91   0.004                   .10494
  educ_new01 |   .4693169   .0698272     6.72   0.000                 .2425739
       _cons |   .2864012   .0516328     5.55   0.000                        .
------------------------------------------------------------------------------
```

(Multi)collinearity

- Collinearity is when two or more independent variables are highly correlated
  – More precisely: when one independent variable is a linear combination of the other independent variables
- Effects:
  – Coefficients of the affected variables may will be unstable
  – Standard errors (for after spring break) will be inflated
  – BUT, the overall predictive power of the regression will not be affected

Simplest example: Sex

- Say I have a variable named **female**, coded 1 if the respondent is female, 0 if the respondent is male
- A second variable named male coded 1 if the respondent is male, 0 if the respondent is female is ***collinear*** with **female** because

male = 1-female

Without the redundant variable

```
. reg approve female  [aw=v102]
(sum of wgt is   9.5361e+02)

      Source |       SS       df       MS              Number of obs =     977
-------------+------------------------------           F(  1,   975) =    5.70
       Model |  8.42953813     1  8.42953813           Prob > F      =  0.0172
    Residual |  1442.49523   975  1.47948229           R-squared     =  0.0058
-------------+------------------------------           Adj R-squared =  0.0048
       Total |  1450.92477   976  1.48660325           Root MSE      =  1.2163

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .1858039   .0778409     2.39   0.017     .0330489    .3385589
       _cons |   2.292257   .0555348    41.28   0.000     2.183275    2.401238
------------------------------------------------------------------------------
```

With the redundant variable

```
. reg approve female  male [aw=v102]
(sum of wgt is   9.5361e+02)
note: male omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     977
-------------+------------------------------           F(  1,   975) =    5.70
       Model |  8.42953813     1  8.42953813           Prob > F      =  0.0172
    Residual |  1442.49523   975  1.47948229           R-squared     =  0.0058
-------------+------------------------------           Adj R-squared =  0.0048
       Total |  1450.92477   976  1.48660325           Root MSE      =  1.2163

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .1858039   .0778409     2.39   0.017     .0330489    .3385589
        male |          0  (omitted)
       _cons |   2.292257   .0555348    41.28   0.000     2.183275    2.401238
------------------------------------------------------------------------------
```

With a fake collinear version of female

```
. gen female2=female

. replace female2=0 if female==1&_n<=50
(15 real changes made)


. tab female female2

           |        female2
    female |         0          1 |     Total
-----------+----------------------+----------
         0 |       461          0 |       461
         1 |        15        524 |       539
-----------+----------------------+----------
     Total |       476        524 |     1,000


. corr female female2
(obs=1000)

             |   female  female2
-------------+------------------
      female |   1.0000
     female2 |   0.9703   1.0000
```

With a fake collinear version of female

```
. reg approve_o female female2 [aw=v102]
(sum of wgt is   9.5361e+02)

      Source |       SS       df       MS              Number of obs =     977
-------------+------------------------------           F(  2,   974) =    6.41
       Model |  18.8520935     2  9.42604675           Prob > F      =  0.0017
    Residual |  1432.07268   974  1.47030049           R-squared     =  0.0130
-------------+------------------------------           Adj R-squared =  0.0110
       Total |  1450.92477   976  1.48660325           Root MSE      =  1.2126

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |    1.03613   .3286674     3.15   0.002     .3911523    1.681108
     female2 |   -.874974   .3286329    -2.66   0.008    -1.519884   -.2300639
       _cons |   2.292257   .0553622    41.40   0.000     2.183614      2.4009
------------------------------------------------------------------------------
```

Multiple dummy variables

- Dummy variables are how we deal with categorical independent variables
  - Race (white, black, Hispanic, Asian-Amer., other)
  - Alliance (NATO, Warsaw Pact, other)
  - Religion (Christian, Jewish, Muslim, Hindu, other)
- To code dummy variables, create a new variable for each category
  - In the regression, exclude one of the variables (usually the most numerous) --- **remember the lecture on collinearity**

Example: predicting Obama approval by race (white is omitted category)

```
. gen white=race==1
. gen black=race==2
. gen hisp=race==3
. gen other_race=1-white-black-hisp
. reg approve_o black hisp other_race [aw=v102]
(sum of wgt is   9.5361e+02)

      Source |       SS       df       MS              Number of obs =     977
-------------+------------------------------           F(  3,   973) =   33.34
       Model |  135.231871     3  45.0772903           Prob > F      =  0.0000
    Residual |   1315.6929   973  1.35220236           R-squared     =  0.0932
-------------+------------------------------           Adj R-squared =  0.0904
       Total |  1450.92477   976  1.48660325           Root MSE      =  1.1628

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       black |   1.097946   .1125408     9.76   0.000     .8770956    1.318797
        hisp |   .4846481   .1577768     3.07   0.002     .1750261      .79427
  other_race |   .0429154   .2131171     0.20   0.840    -.3753068    .4611375
       _cons |   2.216973   .0420862    52.68   0.000     2.134383    2.299563
------------------------------------------------------------------------------

.
```

Example: predicting Obama approval by race (forgetting omitted category)

```
. reg approve_o white black hisp other_race [aw=v102]
(sum of wgt is   9.5361e+02)
note: other_race omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     977
-------------+------------------------------           F(  3,   973) =   33.34
       Model |  135.231871     3  45.0772903           Prob > F      =  0.0000
    Residual |   1315.6929   973  1.35220236           R-squared     =  0.0932
-------------+------------------------------           Adj R-squared =  0.0904
       Total |  1450.92477   976  1.48660325           Root MSE      =  1.1628

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       white |  -.0429154   .2131171    -0.20   0.840    -.4611375    .3753068
       black |   1.055031    .233542     4.52   0.000     .5967269    1.513335
        hisp |   .4417327   .2583988     1.71   0.088    -.0653504    .9488157
  other_race |          0  (omitted)
       _cons |   2.259888   .2089202    10.82   0.000     1.849902    2.669875
------------------------------------------------------------------------------
```

Interaction Terms

- Sometimes we think that one set of regression coefficients apply to one population and another set to another population
- Example: evaluations of Barak Obama
  - Assume ideology and party ID is the baseline factor leading to approval of Obama
  - We know that African-Americans approve of Obama more than whites, but is it:
    - African-Americans simply like Obama better, controlling for Ideology and party ID **or**
    - African-Americans weigh ideology and party ID differently?

```
. reg approve_obama liberal01 dem01 african_am [aw=v102]
(sum of wgt is   7.5675e+02)

      Source |       SS       df       MS              Number of obs =     781
-------------+------------------------------           F(  3,   777) =  333.79
       Model |  638.646662     3  212.882221           Prob > F      =  0.0000
    Residual |  495.553833   777  .637778421           R-squared     =  0.5631
-------------+------------------------------           Adj R-squared =  0.5614
       Total |  1134.20049   780   1.4541032           Root MSE      =  .79861

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   liberal01 |   1.662104   .1287268    12.91   0.000      1.40941    1.914797
       dem01 |   1.222213   .0929022    13.16   0.000     1.039844    1.404582
  african_am |   .5646431   .0907161     6.22   0.000     .3865655    .7427207
       _cons |   .8615467   .0569603    15.13   0.000     .7497324    .9733611
------------------------------------------------------------------------------
```

```
. reg approve_obama liberal01 dem01 african_am [aw=v102] if african_am==1
(sum of wgt is   1.0086e+02)
note: african_am omitted because of collinearity

------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   liberal01 |   .6966604   .3192221     2.18   0.031     .0634095    1.329911
       dem01 |   1.205652   .3477157     3.47   0.001     .5158776    1.895426
  african_am |          0  (omitted)
       _cons |   1.886045   .3181779     5.93   0.000     1.254866    2.517224
------------------------------------------------------------------------------

. reg approve_obama liberal01 dem01 african_am [aw=v102] if african_am==0
(sum of wgt is   6.5588e+02)
note: african_am omitted because of collinearity
------------------------------------------------------------------------------
approve_ob~a |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   liberal01 |   1.934799   .1409995    13.72   0.000     1.657948     2.21165
       dem01 |   1.118628   .0969695    11.54   0.000     .9282298    1.309027
  african_am |          0  (omitted)
       _cons |   .7890373   .0572171    13.79   0.000     .6766921    .9013826
------------------------------------------------------------------------------
```

- We can do this comparison all in one regression
- obama_approve=b_0 + b_1*african_am + b_2*liberal01 + b_3*dem01 + b_4*african_am×liberal01 + b_5*african_am×dem01

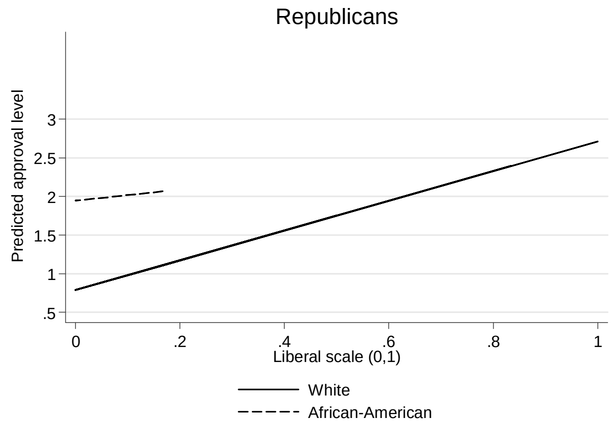
- Note, if the R is white, african_am = 0, therefore:
- obama_approve=b_0 + b_1*african_am + b_2*liberal01 + b_3*dem01 + b_4*african_am×liberal01 + b_5*african_am×dem01
- BECOMES
- obama_approve=b_0 + b_2*liberal01 + b_3*dem01 +

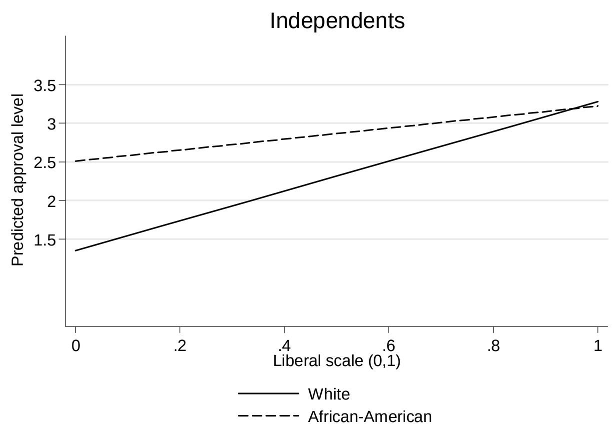
- Note, if the R is black, african_am = 1, therefore:
- obama_approve=b_0 + b_1*african_am + b_2*liberal01 + b_3*dem01 + b_4*african_am×liberal01 + b_5*african_am×dem01
- BECOMES
- obama_approve=(b_0 + b_1)+ (b_2+b_4)liberal01 + (b_3+ b_5)dem01

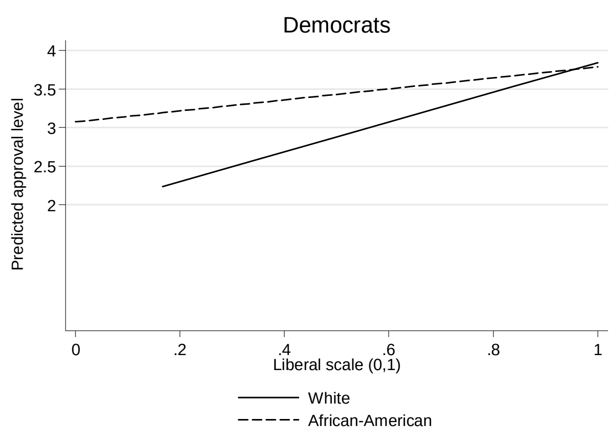
```
. gen african_amXliberal01=african_am*liberal01
(166 missing values generated)

. gen african_amXdem01=african_am*dem01
(180 missing values generated)

. reg approve_obama african_am liberal01 dem01 african_amXliberal01 african_amXdem01
[aw=v102]
(sum of wgt is   7.5675e+02)

      Source |       SS       df       MS              Number of obs =     781
-------------+------------------------------           F(  5,   775) =  207.71
       Model |  649.513362     5  129.902672           Prob > F      =  0.0000
    Residual |  484.687133   775  .625402752           R-squared     =  0.5727
-------------+------------------------------           Adj R-squared =  0.5699
       Total |  1134.20049   780   1.4541032           Root MSE      =  .79082

------------------------------------------------------------------------------------
       approve_obama |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------------+--------------------------------------------------------------
          african_am |   1.097008   .2732159     4.02   0.000     .5606767    1.633339
           liberal01 |   1.934799   .1456907    13.28   0.000     1.648803    2.220794
               dem01 |   1.118628   .1001958    11.16   0.000     .9219412    1.315316
african_amXliberal01 |  -1.238138   .3047051    -4.06   0.000    -1.836283   -.639993
    african_amXdem01 |   .0870236   .3082445     0.28   0.778    -.5180694    .6921166
               _cons |   .7890373   .0591208    13.35   0.000     .6729814    .9050932
------------------------------------------------------------------------------------
```

A more parsimonious model excludes the party interaction

```
. reg approve_obama african_am liberal01 dem01 african_amXliberal01 [aw=v102]
(sum of wgt is   7.5675e+02)

      Source |       SS       df       MS              Number of obs =     781
-------------+------------------------------           F(  4,   776) =  259.93
       Model |  649.463515     4  162.365879           Prob > F      =  0.0000
    Residual |   484.73698   776  .624661057           R-squared     =  0.5726
-------------+------------------------------           Adj R-squared =  0.5704
       Total |  1134.20049   780   1.4541032           Root MSE      =  .79036

--------------------------------------------------------------------------------------
       approve_obama |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------------+----------------------------------------------------------------
          african_am |   1.157717   .1684415     6.87   0.000     .8270618    1.488372
           liberal01 |   1.926111   .1423196    13.53   0.000     1.646734    2.205488
               dem01 |   1.127823   .0946985    11.91   0.000     .9419277    1.313719
african_amXliberal01 |  -1.213316    .291572    -4.16   0.000     -1.78568   -.6409531
               _cons |   .7884288   .0590465    13.35   0.000     .6725191    .9043386
--------------------------------------------------------------------------------------

. predict py if e(sample)
(option xb assumed; fitted values)
(219 missing values generated)
```


Republicans
Predicted approval level
.5
1
1.5
2
2.5
3
0
.2
.4
.6
.8
1
Liberal scale (0,1)
White
African-American


Independents
Predicted approval level
1.5
2
2.5
3
3.5
0
.2
.4
.6
.8
1
Liberal scale (0,1)
White
African-American


Democrats
Predicted approval level
2
2.5
3
3.5
4
0
.2
.4
.6
.8
1
Liberal scale (0,1)
White
African-American