How are the properties of a matrix reflected in its eigenvalues and eigenvectors? This question is fundamental throughout Chapter 6. A table that organizes the key facts may be helpful. For each class of matrices, here are the special properties of the eigenvalues λ_i and eigenvectors $\boldsymbol{x}_i$.

Symmetric: $A^{\mathrm{T}} = A$	real λ's	orthogonal $\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_j = 0$
Orthogonal: $Q^{\mathrm{T}} = Q^{-1}$	all $\lvert\lambda\rvert = 1$	orthogonal $\overline{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{x}_j = 0$
Skew-symmetric: $A^{\mathrm{T}} = -A$	imaginary λ's	orthogonal $\overline{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{x}_j = 0$
Complex Hermitian: $\overline{A}^{\mathrm{T}} = A$	real λ's	orthogonal $\overline{\boldsymbol{x}}_i^{\mathrm{T}}\boldsymbol{x}_j = 0$
Positive Definite: $\boldsymbol{x}^{\mathrm{T}}A\boldsymbol{x} > 0$	all $\lambda > 0$	orthogonal
Markov: $m_{ij} > 0, \sum_{i=1}^{n} m_{ij} = 1$	$\lambda_{\max} = 1$	steady state $\boldsymbol{x} > 0$
Similar: $B = M^{-1}AM$	$\lambda(B) = \lambda(A)$	$\boldsymbol{x}(B) = M^{-1}\boldsymbol{x}(A)$
Projection: $P = P^2 = P^{\mathrm{T}}$	$\lambda = 1;\ 0$	column space; nullspace
Reflection: $I - 2\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}$	$\lambda = -1;\ 1, .., 1$	$\boldsymbol{u}; \boldsymbol{u}^{\perp}$
Rank One: $\boldsymbol{u}\boldsymbol{v}^{\mathrm{T}}$	$\lambda = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{u};\ 0, .., 0$	$\boldsymbol{u};\ \boldsymbol{v}^{\perp}$
Inverse: A^{-1}	$1/\lambda(A)$	eigenvectors of A
Shift: $A + cI$	$\lambda(A) + c$	eigenvectors of A
Stable Powers: $A^n \to 0$	all $\lvert\lambda\rvert < 1$	
Stable Exponential: $e^{At} \to 0$	all $Re\ \lambda < 0$	
Cyclic Permutation: $P(1, .., n) = (2, .., n, 1)$	$\lambda_k = e^{2\pi ik/n}$	$\boldsymbol{x}_k = (1, \lambda_k, \ldots, \lambda_k^{n-1})$
Tridiagonal: $-1, 2, -1$ on diagonals	$\lambda_k = 2 - 2\cos\dfrac{k\pi}{n+1}$	$\boldsymbol{x}_k = \left(\sin\dfrac{k\pi}{n+1}, \sin\dfrac{2k\pi}{n+1}, \ldots\right)$
Diagonalizable: $S\Lambda S^{-1}$	diagonal of Λ	columns of S are independent
Symmetric: $Q\Lambda Q^{\mathrm{T}}$	diagonal of Λ (real)	columns of Q are orthonormal
Jordan: $J = M^{-1}AM$	diagonal of J	each block gives $\boldsymbol{x} = (0, .., 1, .., 0)$
Square: $A = U\Sigma V^{\mathrm{T}}$	$\mathrm{rank}(A) = \mathrm{rank}(\Sigma)$	eigenvectors of $A^{\mathrm{T}}A, AA^{\mathrm{T}}$ in V, U