Gram-Schmidt orthogonalization —a nice example

I like the orthogonal columns because you can see their orthogonality quickly—and you can also see how they combine the columns of A (not using later columns! This is the Gram-Schmidt idea).

$$\begin{bmatrix} 1 & & & \\ -1 & 1 & & \\ & -1 & 1 & \\ & & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ -1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ & -1 & \frac{1}{3} & \frac{1}{4} \\ & & -1 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & & \\ & 1 & -\frac{2}{3} & \\ & & 1 & -\frac{3}{4} \\ & & & 1 \end{bmatrix}$$

If you want **orthonormal** columns, divide those orthogonal columns by their lengths $\sqrt{2}$, $\sqrt{3/2}$, $\sqrt{4/3}$, $\sqrt{1/4}$. To keep the equation right, multiply the rows of the last matrix by those same numbers.

Then you have $A = QR$. (This also means that $A^{\mathrm{T}}A = R^{\mathrm{T}}R$. That upper triangular Cholesky factor R is the same as $\sqrt{D}\,L^{\mathrm{T}}$ in the usual LDL^{T} factorization of $A^{\mathrm{T}}A$. The numbers in the square roots are the pivots of $A^{\mathrm{T}}A$.)