Too Much Calculus

Calculus I, Calculus II, Calculus III—what an imbalance in our teaching! All the rest of mathematics is overwhelmed by calculus. The next course might be differential equations (more derivatives), and the previous course is probably pre-calculus. I really think it is our job to adjust this balance, we cannot expect others to do it. We know the central role of linear algebra. It is much more than a random math course, its applications touch many more students than calculus. We are in a digital world now.

Since I am urging that we do something about our teaching which might mean persuading and even leading our colleagues, I should highlight the good that can come. When more students take linear algebra, the mathematics department is doing something right. Statistics and discrete mathematics are needed too. The chair (and even the dean) will approve. Most of all, we are doing something right for our students.

The feeling that "Linear Algebra is a good course" is communicated to the class. They won't be stars, but this is a course they can catch on to. The web can help (I depend on `web.mit.edu/18.06/www/`). When they know you are trying, the students try too. We can't ask for more.

May I express some thoughts about the structure of a linear algebra course. It could be organized by the equations it solves, or by the ideas and the algorithms that solve them. Those have to fit together, and we could concentrate on four equations:

$$Ax = b, \qquad A'Ax = A'b, \qquad Ax = \lambda x, \qquad du/dt = Au$$

An essential goal is to recognize the linear system in the applications. That is so important, just to identify the matrix A! Then come the ideas that help us to think about those equations:

> Subspaces and bases, projections and orthogonality, eigenvectors and eigenvalues

And the algorithms are central (not forgetting matrix multiplication itself):

$$Ax = \text{ combination of the columns; elimination and } A = LU; \text{ Gram-Schmidt.}$$

The key abstraction is a linear transformation. We know everything when we know what happens to a basis. I emphasize examples, others would prefer proofs, listen to the class whatever you do.

I come back to the main point, because it requires our effort and action to ensure a good course. We often find support, we often meet indifference. Other faculty have their own work to do, and even senior engineers may think of linear algebra as an incidental tool—not realizing how computing has changed things. In the end they will put the students first. They can be convinced when the first students see the light.

The reform of Calculus I, Calculus II, Calculus III must go beyond the presentation of those particular topics. They are important but not all-important. We need to present the mathematics that is most useful to the most students.