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1 (32 pts.) Suppose $A$ is the tridiagonal matrix

$$
A = \begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 3 & -1 \\
0 & 0 & -1 & 0
\end{bmatrix}
$$

(a) Carry out the row elimination to find the upper triangular factor $U$. (10)

(b) What matrix $L$ yields $A = LU$? (6)

(c) Solve $Ax = b$ with

$$
b = \begin{bmatrix}
-1 \\
2 \\
-2 \\
0
\end{bmatrix}.
$$

All components of the solution $x$ happen to be 0’s or 1’s. What linear combination of the columns of $A$ produces $b$? (10)

(d) If you change the entry $A_{4,4} = 0$ in the right lower-corner of $A$ to $A_{4,4} = \_\_\_\_$ the matrix becomes singular. (Hint: look at pivots) (6)
2 (36 pts.)

(a) Suppose $A^n = 0$. Show that $(I - A)^{-1} = I + A + A^2 + \cdots + A^{n-1}$. (10)

(b) Assume $A$ and $B$ are commuting matrices (that is, $AB = BA$). If they both are also nonsingular, show that $A^{-1}$ and $B^{-1}$ commute. (10)

(c) Which are true and which false. (Give a good reason!!!)

Let $A$ be an $m$-by-$n$ matrix. Then $Ax = 0$ has always a non-zero solution if

(i) $\text{rank}(A) < m$ (5)
(ii) $\text{rank}(A) < n$ (5)
(iii) $m = n$ and $A^2 = 0$ (6)
3 (32 pts.) Suppose after elimination on a matrix $A$ we reach its row reduced echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the null space matrix of $A$. (10)

(b) What is the null space of $A^T$? (6)

(c) What is the rank of 2-by-9 block matrix $[A \ A \ A]$? (6)

(d) Find a complete solution to $Rx = d$ with $d = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (10)