Your name is: ________________________________

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<th>Recitations</th>
<th>#</th>
<th>Time</th>
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<th>Instructor</th>
<th>Office</th>
<th>Phone</th>
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1 \textbf{(36 pts.)} Let $A$ be the square matrix

$$A = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} q_1^T + \begin{bmatrix} -1 \\ -1 \end{bmatrix} q_2^T,$$

where $q_1$ and $q_2$ are orthonormal vectors in $\mathbb{R}^3$. (12p)

(a) Find $x$ such that

$$Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(b) Choose $a$ such that the column space of $A$ has dimension 1. (8p)

(c) If

$$q_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } q_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and $a = 0$, solve

$$Ay = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

in the least squares sense. (16p)
2 (28 pts.) Let

\[ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}. \]

(a) By Gram-Schmidt, factor \( A \) into \( QR \) where \( Q \) is orthogonal and \( R \) is upper triangular. (16p such that 10p from \( Q \) and 6p from \( R \))

(b) Find the inverse of \( R \) and then give the inverse of \( A \) by using \( A = QR \). (12p such that 4p from \( R^{-1} \) and 4p from \( Q^{-1} \) and 4p from \( A^{-1} \))
(36 pts.) (a) Let $u$, $v$ and $w$ be linearly independent. How is the matrix $A$ with columns $u$, $v$, $w$ related to the matrix $B$ with columns $u + v$, $u - v$, $u - 2v + w$? Show that those three columns are linearly independent.

(12p)

(b) Using Cramer’s rule, find $b_3$ such that $x_3 = 0$ for the solution of

$$
\begin{bmatrix}
2 & 1 & -1 \\
1 & 1 & 1 \\
1 & -2 & -3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
=
\begin{bmatrix}
3 \\
1 \\
b_3
\end{bmatrix}.
$$

(12p)

(c) Using rules for the determinant (so do not compute it with any of the 3 formulas), show the steps and rules that lead to

$$
\begin{vmatrix}
1 & a & b + c \\
1 & b & c + a \\
1 & c & a + b
\end{vmatrix}
= 0
$$

(12p)