Course 18.06, Fall 2002: Homework 1

Due 4:00PM, Wednesday September 11, in the boxes at 2-106.
There is one box for each recitation section.
For full credit, be sure to show and explain your work.
Circle your answers.
Please staple Part I in front of Part II.

Part I

1A If \((a,b)\) is a multiple of \((c,d)\) prove that \((a,c)\) is a multiple of \((b,d)\). You can assume that \(a, b, c, d\) are not equal to zero.

1B This system has no solution, because the three planes in the row picture don’t pass through a point. No combination of the three columns produces \(b\):

\[
\begin{align*}
    x + 3y + 5z &= 4 \\
    x + 2y - 3z &= 5 \\
    2x + 5y + 2z &= 8
\end{align*}
\]

\[
\begin{bmatrix}
    1 & 3 & 5 \\
    1 & 2 & -3 \\
    2 & 5 & 2
\end{bmatrix}\begin{bmatrix}
x \\y \\z
\end{bmatrix} = \begin{bmatrix}
    4 \\5 \\8
\end{bmatrix} = b
\]

(1) Multiply the equations by 1,1, \(-1\) and add to show that these planes don’t meet at a point. Are any two of the planes parallel? What are the equations of planes parallel to \(x + 3y + 5z = 4\)?

(2) Take the dot product of each column (and also \(b\)) with \(z = (1,1,\text{-}1)\). Why can’t a combination of the three columns equal \(b\)?

(3) Find three right side vectors \(b^*\) and \(b^{**}\) and \(b^{***}\) that \(do\) allow solutions.

1C Three of these matrices are invertible, and three are singular. Find the inverse when it exists. Give reasons for noninvertibility (too few pivots, nonzero solution to \(Ax = 0\)) for the other three. The matrices \(A, B, C, D, E, F\) are

\[
\begin{bmatrix}
    4 & 3 \\
    8 & 6
\end{bmatrix} \quad \begin{bmatrix}
    4 & 3 \\
    8 & 7
\end{bmatrix} \quad \begin{bmatrix}
    6 & 6 \\
    6 & 0
\end{bmatrix} \quad \begin{bmatrix}
    6 & 6 \\
    6 & 6
\end{bmatrix} \quad \begin{bmatrix}
    1 & 0 & 0 \\
    1 & 1 & 0
\end{bmatrix} \quad \begin{bmatrix}
    1 & 1 & 1 \\
    1 & 1 & 0
\end{bmatrix}.
\]

Part II

These are questions from the textbook.

Section 2.1: 19
Section 2.2: 12, 18, 25
Section 2.3: 1, 2, 21, 25
Section 2.4: 35
Section 2.5: 7