18.06, Fall 2003, Problem Set 1

Due before 4PM on Wednesday September 10th, 2003, in the boxes in 2-106. No late homework will be accepted. There is one box for each recitation section. For full credit, please be sure to show and explain your work.

1. Exercise 7 from Section 1.2 on page 18.

2. Exercise 18 from Section 2.2 on page 44. (For the second part of the question, use for q and t the values that give infinitely many solutions.)

3. Solve the following system of equations by elimination and back substitution:

$$\begin{cases} x & +y & & = & 1 \\ 4x & +6y & +z & = & 4 \\ -2x & +2y & & = & -10 \end{cases}$$

   Let $Ax = b$ denote the above system. Write the three elimination matrices E_{21}, E_{31} and E_{32} that put A into triangular form U with $E_{32}E_{31}E_{21}A = U$.

   Compute $M = E_{32}E_{31}E_{21}$.

4. Consider the 3 elimination matrices of the previous exercise. Which of the products $E_{21}E_{31}$, $E_{21}E_{32}$ and $E_{31}E_{32}$ commute[1] and which do not? Explain. You actually do not need to compute the products to answer this question, you can simply reason about what elimination matrices do.

5. Consider $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$. Calculate $B = A^2 + A$. Is $AB = BA$?

6. Let A be a square matrix. Assume that a system of equations $Ax = b$ has a *unique* solution. Is it possible that the system of equations $Ax = c$ for some other right-hand-side c has either no solutions or infinitely many? Please explain your reasoning clearly. (Hint: think of what happens when you use elimination.)

[1] The product AB commutes (or the matrices A and B commute) if $AB = BA$.