Problem 1 Wednesday 10/11

For each of these, find a matrix satisfying the conditions given or explain why none can exist.

(a) Column space contains \[\begin{bmatrix} -1 \\ 1 \\ 1 \\ 3 \\ 5 \end{bmatrix}\] and \[\begin{bmatrix} 2 \\ 4 \\ 1 \\ -2 \\ 1 \end{bmatrix}\], and nullspace contains \[\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\]

(b) Row space contains \[\begin{bmatrix} -1 \\ 1 \\ 1 \\ 3 \\ 5 \end{bmatrix}\] and \[\begin{bmatrix} 2 \\ 4 \\ 1 \\ -2 \\ 1 \end{bmatrix}\], and nullspace contains \[\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\]

(c) \[Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\] is solvable; \[A^T \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\] is zero.

(d) A nonzero matrix where every row is perpendicular to every column

(e) Rows sum to a row of zeros, and columns sum to \[\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}\]

Problem 2 Wednesday 10/11

Do Problem #12 from section 4.1 in your book.

Problem 3 Wednesday 10/11

Do Problem #26 from section 4.1 in your book.

Problem 4 Friday 10/13

Do Problem #2 from section 4.2 in your book. What is the permutation matrix \(P\)? What is the error \(e = b - p\)?

Problem 5 Friday 10/13

Do Problem #13 from section 4.2 in your book. Do this two different ways:
(a) geometrically, tell what subspace we’re projecting \(b\) orthogonally onto
(b) algebraically, calculate \(P = A(A^TA)^{-1}A^T\)

Problem 6 Friday 10/13

A subspace \(S\) has basis \(\{a, b, c\}\) where \(a = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}\), \(b = \begin{bmatrix} 5 \\ 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}\), \(c = \begin{bmatrix} 0 \\ 5 \\ 4 \\ 1 \\ -1 \end{bmatrix}\).

(a) What are the dot products \(a^Tb, a^Tc, b^Tc\)? Are the basis vectors orthogonal?

Now let’s compute a new basis \(\{\hat{a}, \hat{b}, \hat{c}\}\) for the same subspace. Start by letting \(\hat{a} = a\).

(b) Compute the projection \(P_1 b\) of \(b\) onto the line described by \(a\). What is the error \((b - P_1 b)\)? Call this error vector \(\hat{b}\).

(c) Compute the projection \(P_1 c\) of \(c\) onto the plane described by \(a\) and \(b\). What is the error \((c - P_1 c)\)? Call this error vector \(\hat{c}\). Does \(\hat{c}\) change if we project onto the plane with basis \(\hat{a}\) and \(\hat{b}\) instead? Why or why not?
(d) What are the dot products $\hat{a}^T \hat{b}$, $\hat{a}^T \hat{c}$, $\hat{b}^T \hat{c}$? Are the new basis vectors orthogonal? (This process for finding an orthogonal basis is called the “Gram-Schmidt Process” — the full version also scales each vector to “normalize” it to unit length.)

(e) Find the matrix $R$ relating the old basis and the new basis: $\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} \hat{a} & \hat{b} & \hat{c} \end{bmatrix} \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$

(f) Explain how you know $\{\hat{a}, \hat{b}, \hat{c}\}$ is a basis for $S$. (Don’t forget to show it both spans the subspace, and is linearly independent!)

**Problem 7 Monday 10/16**

Do Problem #17 from section 4.3 in your book.

**Problem 8 Monday 10/16**

Do Problem #27 from section 4.3 in your book.