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Grading

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Total: 
1 (20 pts.) Let $v = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(a) Show that $A = vv^T$ is a symmetric $3 \times 3$ matrix. What is the rank of $A$?

(b) Find the projection matrix $P$ onto the nullspace of $A$.

(c) Calculate $PA$ and $AP$.

(d) Show that for any symmetric $n \times n$ matrix $B$, and the projection matrix $Q$ onto the nullspace of $G$, we have $QA = AQ = 0$. 


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Given 4 points \((t, b) = (0, 1), (1, 2), (2, 2)\) and \((3, 3)\).

(a) Use the least square methods to find a line \(b = C + Dt\) which best fits the four points.

(b) Find equations \textbf{\textit{(do not solve)}} for the coefficients \(C', D', E'\) in \(b = C' + D't + E't^2\), the parabola which best fits the four points above.
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3 (16 pts.) Fill in the blanks below.

(a) The nullspace of $AB$ contains the nullspace of ____________.

(b) Let $P$ be the projection matrix to the column space of a matrix $A$, then $I - P$ is the projection matrix to ____________.

(c) The projection of the vector \[
\begin{pmatrix}
1 \\
3 \\
1 \\
\end{pmatrix}
\] onto the nullspace of the matrix
\[
\begin{pmatrix}
1 & 2 & 3 \\
0 & 2 & 4 \\
0 & 0 & 4 \\
\end{pmatrix}
\]
is ____________.

(d) Suppose $A$ is an $m \times n$ matrix, and the row space of $A$ is $n$ dimensional, then its nullspace is ____________ dimensional.

(e) Let $\hat{x}$ be the least-squares solution to $Ax = b$. Then $b - A\hat{x}$ is orthogonal to the ____________ of $A$.

(f) Let $A$ be the $m \times n$ (edge-node) incidence matrix of a connected graph with $n$ vertices and $m$ edges. Then the left nullspace of $A$ has dimension ____________.

(g) Suppose $A$ is a $5 \times 5$ matrix, and $\det(A) = 2$. Then $\det(2AA^T) =$ ____________.

(h) If $A$ is an arbitrary orthogonal matrix, the possible values of $\det(A)$ are ____________.
4 (16 pts.) (a) Compute the determinant of the matrix 
\[
\begin{pmatrix}
0 & a & b \\
-a & 0 & c \\
-b & -c & 0
\end{pmatrix}.
\]

(b) Suppose $A$ is any antisymmetric $n \times n$ matrix, and $n$ is odd. Find the determinant of $A$. 
Let $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{pmatrix}$.

(a) Do Gram-Schmidt, but without normalizing lengths, to get an orthogonal but not orthonormal basis $B$ for $C(A)$.

(b) Compute the (4-dimensional) volume of the 2 parallelepipeds with edges given by the columns of $A$ and the columns of $B$.

(c) For an arbitrary $n \times n$ matrix $A$, write down this “unnormalized” Gram-Schmidt process in terms of $A$ multiplied by a sequence of matrices (similar to the QR decomposition), and use it to prove that this process preserves the volume.

(d) Show that for any orthogonal matrix $B$, $B^T B$ is a diagonal matrix, then use this to derive that $\det(B)$ equals the product of the lengths of its column vectors.
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