Fall 2012 18.06 final solutions
(questions ≥ 3, excluding 5, 7)

3. a) If $Av = \lambda v$ then $\lambda^2 = 6\lambda$, so $\lambda = 0$ or 6.

Sum evals $=$ trace $A = 12$, so evals must be $6, 6, 0$.

b) Basis for nullspace (dim 1); so just find a nullspace vector):

$$\left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\}.$$

Column space of a symmetric matrix is orthogonal to its nullspace, so a basis is e.g.

$$\left\{ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}.$$

c) $M = \frac{1}{6} A$ is symmetric, a projection, singular, Markov and none of the others.

4. a) Elimination:

$$G \Rightarrow \begin{pmatrix} -1 & 1 & 1 & 1 \\ 0 & -1-i & 0 & 1+i \\ 0 & 0 & 0 & 0 \\ 0 & 1+i & 0 & -1-i \end{pmatrix}$$

2nd row = − 4th row, so now clear that rank $G = 2$.

b) Note that $\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \in$ null G, so the constant function

$$x(t) = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$ works, since $\frac{d}{dt} x(t) = G x(t) = 0$.

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5. Already present in pdf.

6.

a) $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$ has $\lambda_1 = \lambda_2 = 0$.

b) $\sigma_1^2 + \sigma_2^2 + \dots + \sigma_{10}^2$ = sum of squares of entries of matrix.

All 100 entries square to 1, so sum of squares = 100.

c) The big determinant formula has $5! = 120$ terms. It is not possible for the determinant of a 5×5 matrix with entries ± 1 to be odd, because then every term in the big formula is ± 1, and so odd. But the sum of an even number of odd numbers is always even.