3. a) If $Av = \lambda v$ then $\lambda^2 = 6\lambda$, so $\lambda = 0$ or 6.
   Sum evals = trace $A = 12$, so evals must be 6, 6, 0.
   
   b) Basis for nullspace (dim 1); so just find a nullspace vector:
   $$\begin{pmatrix} -2 \\
   1
   \end{pmatrix}.$$ 
   
   c) Column space of a symmetric matrix is orthogonal to its nullspace, so a basis is e.g.
   $$\left\{ \begin{pmatrix} 2 \\
   1 \\
   0
   \end{pmatrix}, \begin{pmatrix} 0 \\
   1 \\
   2
   \end{pmatrix} \right\}.$$ 
   
   c) $M = \frac{1}{6} A$ is symmetric, a projection, singular, Markov and none of the others.
4. a) Elimination:

\[ G = \begin{pmatrix} -1 & 1 & 1 & 1 \\ 0 & -1-i & 0 & 1+i \\ 0 & 0 & 0 & 0 \\ 0 & 1+i & 0 & 1-i \end{pmatrix} \]

2nd row = -4th row, so now clear that \( \text{rank } G = 2 \).

b) Note that \( \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \) is null \( G \), so the constant function

\[ x(t) = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \] works, since \( \frac{d}{dt} x(t) = Gx(t) = 0. \)

5. Already present in pdf.

6. a) \( \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} \) has \( \lambda_1 = \lambda_2 = 0 \).

b) \( \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_{10}^2 = \text{sum of squares of entries of matrix.} \)

All 100 entries square to 1, so sum of squares = 100.
c) The big determinant formula has $5! = 120$ terms. It is not possible for the determinant of a $5 \times 5$ matrix with entries $\pm 1$ to be odd, because then every term in the big formula is $\pm 1$, and so odd. But the sum of an even number of odd numbers is always even.