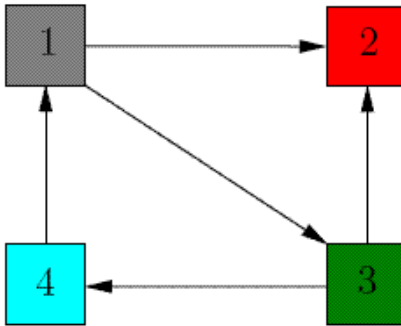


SOLUTIONS, MATLAB Problem, Problem Set 6

18.06 Fall '12

This problem set is due Thursday, October 25, 2012 by 4pm in 2-255. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary('filename')` will start a transcript session, `diary off` will end one, also copy and paste usually work as well.)

10. Q: Use the SVD and compute the hub and authority weights for the following graph:



10. A: First let's construct the adjacency matrix for this graph. An adjacency matrix A has entry $a_{ij} = 1$ if node i links to node j and $a_{ij} = 0$ if not. For this problem, the adjacency matrix is:

$$A = \begin{matrix} & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \\ \mathbf{1} & \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix} \\ \mathbf{2} & \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix} \\ \mathbf{3} & \begin{pmatrix} 0 & 1 & 0 & 1 \end{pmatrix} \\ \mathbf{4} & \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

(I've put node numbers in bold at the border of the matrix. This is to make it easier to read off which nodes are linked. They are just labels, not part of the matrix.)

One can construct this matrix in MATLAB using

```
>> A = [0,1,1,0; 0,0,0,0; 0,1,0,1; 1,0,0,0 ]
```

A =

```
0     1     1     0
0     0     0     0
0     1     0     1
1     0     0     0
```

QUICK RECAP OF HUBS, AUTHORITIES, ETC: The next few paragraphs do a quick synopsis of the theory used in this problem. If you're comfortable with the theory and just want the MATLAB you can skip this part, I won't be offended.

Let \mathbf{x} denote authority weights (i.e. x_n is the authority weight of the n^{th} node), and let \mathbf{y} denote hub weights. The higher the value of x_i ($/y_i$), the better an authority (/hub) node i is.

Given rankings \mathbf{x}^k and \mathbf{y}^k , we can use our adjacency matrix A to update them and get better rankings \mathbf{x}^{k+1} , \mathbf{y}^{k+1} :

$$\begin{cases} \mathbf{x}^{k+1} = A^T \mathbf{y}^k \\ \mathbf{y}^{k+1} = A \mathbf{x}^k \end{cases}$$

The superscripts denote iteration number.

The basic idea is that a node is a good authority if good hubs link to it. A^T encodes whether or not x_i has a link from y_j . The value of y_j tells us how much a link from y_j is worth. For more, see page 369 of your text and in the references cited there.

To get good rankings, we wish find the limiting \mathbf{x} and \mathbf{y} that this iteration converges to.

It helps to consider what happens every two steps. We see our old friends $A^T A$ and AA^T :

$$\begin{cases} \mathbf{x}^{k+2} = A^T A \mathbf{x}^k \\ \mathbf{y}^{k+2} = AA^T \mathbf{y}^k \end{cases}$$

The limiting \mathbf{x} comes from multiplying many times by $A^T A$. We can use eigenvalues for computing these powers. The eigenvector corresponding to the largest eigenvalue of $A^T A$ will eventually dominate. That will be the limiting \mathbf{x} we're looking for.

Likewise, the limiting \mathbf{y} comes from multiplying many times by AA^T . We can find the limiting \mathbf{y} by finding the eigenvector that corresponds to the largest eigenvalue of AA^T .

The SVD factorization gives us these eigenvalues and eigenvectors. So if we can get the SVD, we can get our \mathbf{x} and \mathbf{y} .

END OF RECAP, BACK TO MATLAB:

MATLAB's SVD command is simply `svd()`. It returns, U , S , and V , in that order:

```
>> [U,S,V] = svd(A)
```

$U =$

```
-0.7071      0   -0.7071   -0.0000
 0.0000      0   -0.0000   -1.0000
-0.7071      0    0.7071    0.0000
 0   -1.0000      0          0
```

$S =$

```
1.7321      0      0      0
 0    1.0000      0      0
 0      0    1.0000      0
 0      0      0    0.0000
```

V =

0	-1.0000	0	0
-0.8165	0	0.0000	-0.5774
-0.4082	0	-0.7071	0.5774
-0.4082	0	0.7071	0.5774

The squares of the diagonal terms of \mathbf{S} are the eigenvalues of $A^T A$ and AA^T . The largest is $1.7321^2 = 3.000$, which is the first one listed.

The corresponding eigenvector of $A^T A$ is the first column of \mathbf{V} . The second node is the highest ranked authority; the magnitudes of the weights for nodes 1 through 4 are, respectively 0,0.8165,0.4082,0.4082.

The corresponding eigenvector of AA^T is the first column of \mathbf{U} . The first and third node are tied for highest ranked hub, with weights of magnitude $\frac{\sqrt{2}}{2}$; the weights of other nodes are 0.