18.06 FIB EXAM 2 SOLUTIONS

1. a) Subtracting 1806 × 1st row from 2nd row and 2013 × 1st row from 3rd row, matrix becomes

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & -1806 \\ 0 & 1 & 2 \end{pmatrix}$$

Then det = 1806 clearly. (Or any correct method).

b) Subtract 1st row from each other row to get

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & & -1 & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & -1 \end{pmatrix}$$

So $\det A_n = (-1)^{n-1}$. (Or any correct method).

2. a) P is projection onto a 3 dimensional subspace of an $m>3$-dimensional vector space. So $\det(P)=0$.

b) The eigenvalues are 1 (multiplicity 3) and 0 (multiplicity $m-3$).

c) An eigenvector is q_1 with eigenvalue $q_2^T q_1 = q_2 \cdot q_1$.

d) P is symmetric, so M is symmetric.

Left nullspace = right nullspace = column space of P

Column space = row space = orthogonal complemat of column space of P

3. a) Quartic, since the diagonal term in the big formula is $(a_{11}-x)(a_{22}-x)(a_{33}-x)(a_{44}-x)$.

b) Linear, since A_{11} appears at most once in any term of the big formula.

c) Quartic, since

$$\det(xA) = x^4 \det(A).$$

d) Quadratic: we can add the second row to the third row to eliminate some xs. Then each term of the big formula is quadratic in x.

4. a) Let $M = (A\ B)$ be the 3×2 matrix with columns A and B. Since A and B are orthonormal, the projection onto the (A, B)-plane is given by

$$MM^T.$$

Thus the length L is the length of $C - MM^TC$

$$L = \|C - MM^TC\|.$$

b) The volume of a pyramid is

$$\frac{1}{3}(\text{base area})(\text{altitude length})$$

The area of the OAB face is $\frac{1}{2}$, so the volume is

$$V = \frac{1}{6}L = \frac{1}{6}\|C - MM^TC\|.$$