

Your PRINTED name is: _____

Grading

1

2

3

Please circle your recitation:

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|---|------|-----------------|----------|--------|----------|
| 1 | T 9 | Dan Harris | E17-401G | 3-7775 | dmh |
| 2 | T 10 | Dan Harris | E17-401G | 3-7775 | dmh |
| 3 | T 10 | Tanya Khovanova | E18-420 | 4-1459 | tanya |
| 4 | T 11 | Tanya Khovanova | E18-420 | 4-1459 | tanya |
| 5 | T 12 | Saul Glasman | E18-301H | 3-4091 | sglasman |
| 6 | T 1 | Alex Dubbs | 32-G580 | 3-6770 | dubbs |
| 7 | T 2 | Alex Dubbs | 32-G580 | 3-6770 | dubbs |

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1 (32 pts.) (2 points each)

There are sixteen 2×2 matrices whose entries are either 0 or 1. For each of the sixteen, write down the two singular values. Time saving hint: if you really understand singular values, then there is really no need to compute AA^T or $A^T A$, but it is okay if you must.

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2 (30 pts.) (3 points each: Please circle true or false, and either way, explain briefly.)

a) If A and B are invertible, then so is $(A + B)/2$. True? False? (Explain briefly).

b) If A and B are Markov, then so is $(A + B)/2$. True? False? (Explain briefly).

c) If A and B are positive definite, then so is $(A + B)/2$. True? False? (Explain briefly).

d) If A and B are diagonalizable, then so is $(A + B)/2$. True? False? (Explain briefly).

e) If A and B are rank 1, then so is $(A + B)/2$. True? False? (Explain briefly).

f) If A is symmetric then so is e^A . True? False? (Explain briefly).

g) If A is Markov then so is e^A . True? False? (Explain briefly).

h) If A is symmetric, then e^A is positive definite. True? False? (Explain briefly).

i) If A is singular, then so is e^A . True? False? (Explain briefly).

j) If A is orthogonal, then so is e^A . True? False? (Explain briefly).

3 (38 pts.)

Let $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$.

a) (10 pts.) Find a nonzero solution $y(t)$ in \mathbb{R}^2 to $dy/dt = Ay$ that is independent of t , in other words, $y(t)$ is a constant vector in \mathbb{R}^2 . (Hint: why would a vector in the nullspace of A have this property?)

b) (10 pts.) Show that e^{At} is Markov for every value of $t \geq 0$.

c) (10 pts.) What is the limit of e^{At} as $t \rightarrow \infty$?

d) (8 pts.) What is the steady state vector of the Markov matrix e^A ?